[CS M51A Fall 14] Solutions for Midterm exam

Date: 11/04/14

Problem 1 (10 points)

X = (x, y, z) is a 3-digit weighted mixed-radix number system: x is a radix-16 digit, y a is radix-3 digit, and z is a radix-12 digit.

1. **(5 points)** Convert X=(6, 1, 11) to a decimal number.

   Solution $X = 6 \times 3 \times 12 + 1 \times 12 + 11 = 216 + 12 + 11 = 239$

2. **(5 points)** What is the largest number of X in decimal?

   Solution $LargestX = (15, 2, 11) = 15 \times 3 \times 12 + 2 \times 12 + 11 = 575$

Problem 2 (10 points)

Simplify the following boolean expression by using postulates of Boolean Algebra.

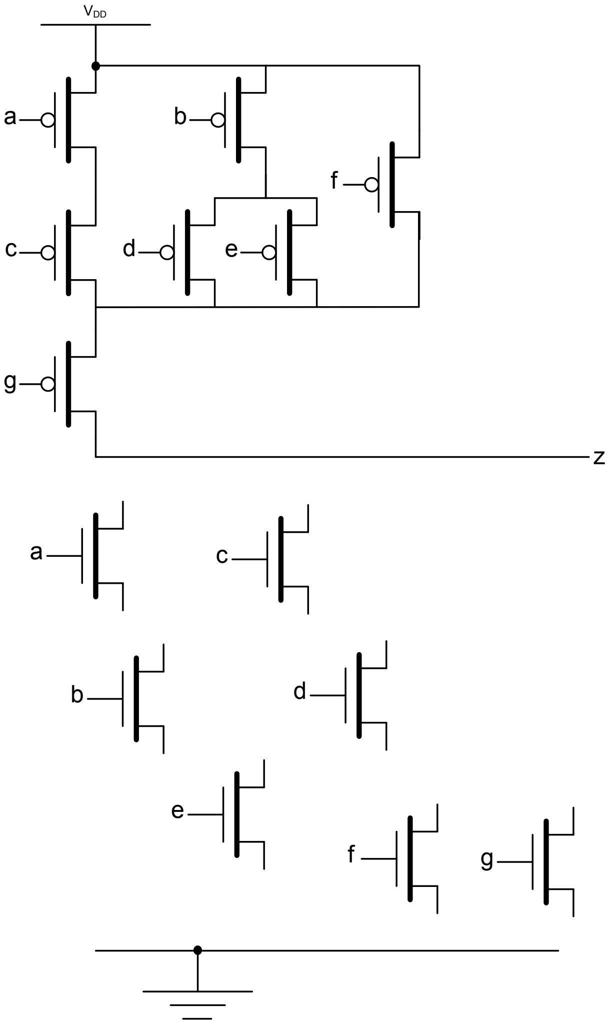
$$(a'b' + c)(a + b)(b' + a'c')'$$

Solution

$$\begin{aligned}
& (a'b' + c)(a + b)(b' + a'c')' \\
= \;& (a'ab' + ac + a'b'b + bc)(b' + a'c')' \\
= \;& (ac + bc)(b' + a'c')' \\
= \;& (a + b)c \cdot b(a + c) \\
= \;& (a + b)(a + c)bc \\
= \;& (a + ab + ac + bc)bc \\
= \;& (a(1 + b + c) + bc)bc \\
= \;& (a + bc)bc \\
= \;& abc + bc \\
= \;& bc(a + 1) \\
= \;& bc
\end{aligned}$$

Problem 3 (20 points)

We are given the following partial CMOS network.



1. **(10 points)** Write the expression for the pull-up network. From this, derive the expression for the pull-down network using switching algebra.

   Solution From the given network we can directly write for the pull-up network

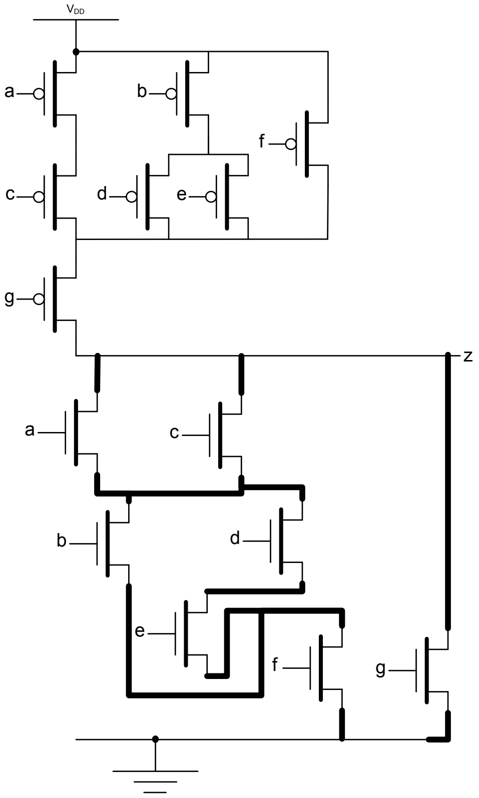
$$z = (a'c' + b'(d' + e') + f')g'$$

   Therefore, the expression for the pull-down network is

$$\begin{aligned} z' &= \left[(a'c' + b'(d' + e') + f')g'\right]' \\ &= (a'c' + b'(d' + e') + f')' + g \\ &= (a'c')'(b'(d' + e'))'f + g \\ &= (a + c)(b + (d' + e')')f + g \\ &= (a + c)(b + de)f + g \end{aligned}$$

2. **(10 points)** Connect the NMOS transistors to complete the pull-down network so that it corresponds to the expression obtained from part 1 and drives the output z to a valid output - i. e. either V_{DD} or ground - for any combination of inputs. Please only add additional wiring.

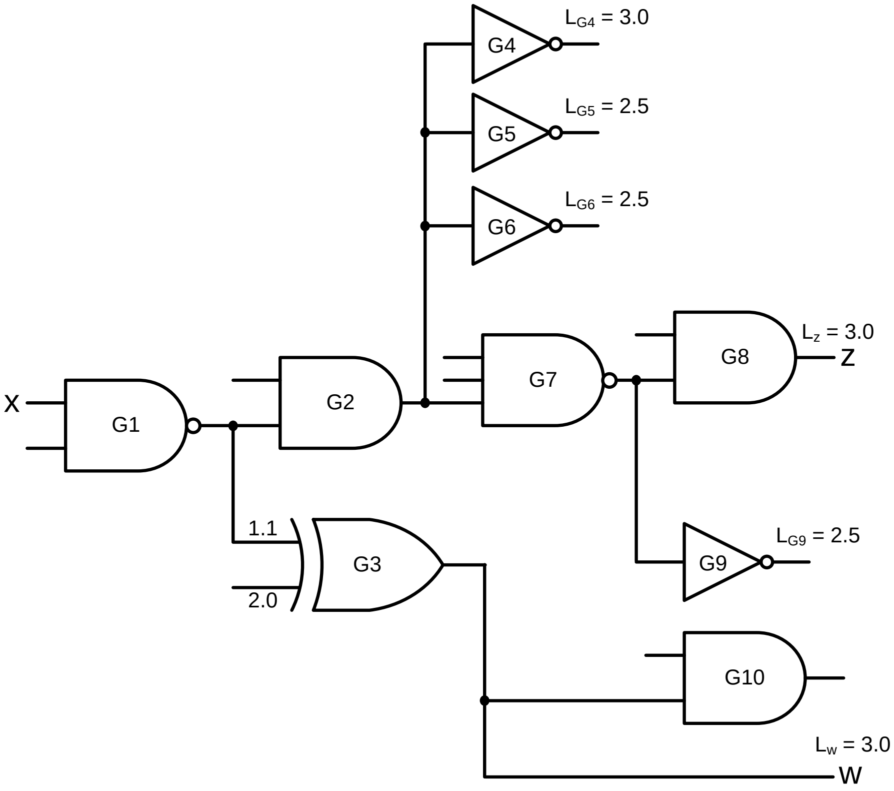
   Solution Using the expression for z', we can connect the circuit as shown. The additional lines are marked in bold.



Problem 4 (20 points)

We would like to determine the propagation delay of the gate network shown here. The output is z with a input x. The necessary gate characteristics are given in the table below.

Gate	Fan-	Propagation Delays (ns)		Load Factor
Type	in	t_{pLH}	t_{pHL}	I
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$	1.0
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0
NAND	3	$0.07 + 0.038L$	$0.09 + 0.039L$	1.0
XOR	2	$0.30 + 0.036L$	$0.30 + 0.021L$	1.1
		$0.16 + 0.036L$	$0.15 + 0.020L$	2.0



L_{G4} = 3.0
G4
L_{G5} = 2.5
G5
L_{G6} = 2.5
G6
L_z = 3.0
G8
Z
G7
X
G1
G2
1.1
G3
2.0
L_{G9} = 2.5
G9
G10
L_w = 3.0
W

1. **(10 points)** (a) Determine the output load of gate G3.

   Solution
   The output load of gate G3 is 4, since this output is connectd to G10 (load 1), and W (load 3).

   b) If the fanout factor of gate G3 is 8, then how many additional gate inputs with load factor of 2 can be connected to the output of gate G3?

   Solution
   The output load of gate G3 is 4. The remaining fanout of gate G3 is 4 loads. Thus, 2 gate inputs with load factor 2 can be connected.

2. **(10 points)** Find the worst case value of $t_{pLH}(x \to z)$. Fill in the blanks below with the appropriate values.

Solution

Gate type and fan-in	G1: NAND2 → G2: AND2 → G7: NAND3 → G8: AND2
LH / HL	G1: HL → G2: HL → G7: LH → G8: LH
Output load L	G1: 2.1 → G2: 4.0 → G7: 2.0 → G8: 3.0

For the propagational delay values:

$$\begin{array}{ll} \text{G1:} & 0.08 + 0.027L = 0.08 + 0.027 \cdot 2.1 = 0.1367 \\ \text{G2:} & 0.16 + 0.017L = 0.16 + 0.017 \cdot 4.0 = 0.228 \\ \text{G7:} & 0.07 + 0.038L = 0.07 + 0.038 \cdot 2.0 = 0.146 \\ \text{G8:} & 0.15 + 0.037L = 0.15 + 0.037 \cdot 3.0 = 0.261 \end{array}$$

$t_{pLH}(x \to z) = 0.1367 + 0.228 + 0.146 + 0.261 = 0.7417$ (ns)

Problem 5 (10 points)

Decide whether the function E defined below is a universal function. If Yes, show your proof. You can use constant 0 or 1 as your input.

(a) **(5 points)**

x	y	$E(x,y)$
0	0	1
0	1	1
1	0	0
1	1	1

Solution Yes, E is a universal function.

$E(x,y) = x' + y$

NOT gate: $E(x,0) = x'$

OR gate: $E(E(x,0),y) = E(x',y) = x + y$

(b) **(5 points)**

x	y	z	E(x,y,z)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Solution Yes, E is a universal function.

$E(x,y,z) = x'yz + xy'z' + xyz' + xyz = zy + z'x$

NOT gate: $E(1,0,z) = z'$

AND gate: $E(0,y,z) = yz$

Problem 6 (30 points)

F is a function that accepts inputs $x \in \{0, 1, 2\}$, $y \in \{0, 1, 2, 3\}$, and outputs $z = x^2 + y$. Suppose you use binary code to encode x, y, and z. x is encoded as x_1x_0, y is encoded as y_1y_0, z is encoded as $z_2z_1z_0$.

(a) **(6 points)** Fill in the following truth table.

x_1	x_0	y_1	y_0	z_2	z_1	z_0
0	0	0	0			
0	0	0	1			
0	0	1	0			
0	0	1	1			
0	1	0	0			
0	1	0	1			
0	1	1	0			
0	1	1	1			
1	0	0	0			
1	0	0	1			
1	0	1	0			
1	0	1	1			
1	1	0	0			
1	1	0	1			
1	1	1	0			
1	1	1	1			

(b) **(6 points)**Based on the truth table in (a), draw the K-map for z_2, z_1, and z_0.

(c) **(6 points)**Use the K-maps in (b) to find all the prime implicants for z_2, z_1, and z_0 respectively.

(d) **(6 points)**Use the K-maps in (b) to find all the essential prime implicants for z_2, z_1, and z_0 respectively.

(e) **(6 points)**Implement z_2, z_1, and z_0 using minimal NAND-NAND networks. Note that each output has a separate gate network. You can directly use x_1, x_0, y_1, y_0 as inputs.

Solution

(a).

x_1	x_0	y_1	y_0	z_2	z_1	z_0
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	1	0
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	1
0	1	1	1	1	0	0
1	0	0	0	1	0	0
1	0	0	1	1	0	1
1	0	1	0	1	1	0
1	0	1	1	1	1	1
1	1	0	0	–	–	–
1	1	0	1	–	–	–
1	1	1	0	–	–	–
1	1	1	1	–	–	–

(b)

z_0

y_0

0	1	1	0
1	0	0	1
–	–	–	–
0	1	1	0

x_0

x_1

y_1

z_1

y_0

0	0	1	1
0	1	0	1
–	–	–	–
0	0	1	1

x_0

x_1

y_1

z_2

y_0

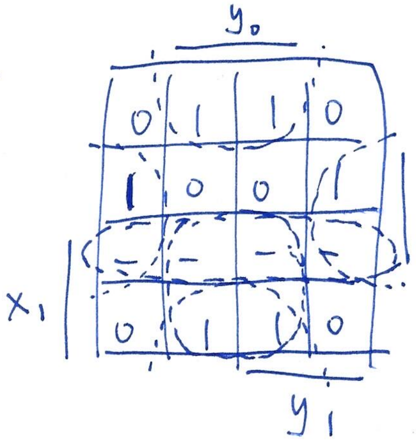
0	0	0	0
0	0	1	0
–	–	–	–
1	1	1	1

x_0

x_1

y_1

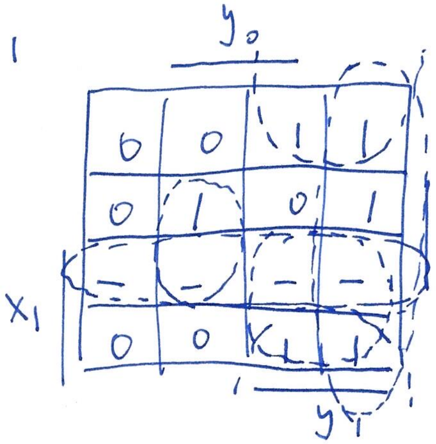
(c). z_0.



0	1	1	0
1	0	0	1
–	–	–	–
0	1	1	0

$y_0 x_0'$
$y_0' x_0$
$x_1 y_0$
$x_1 x_0$

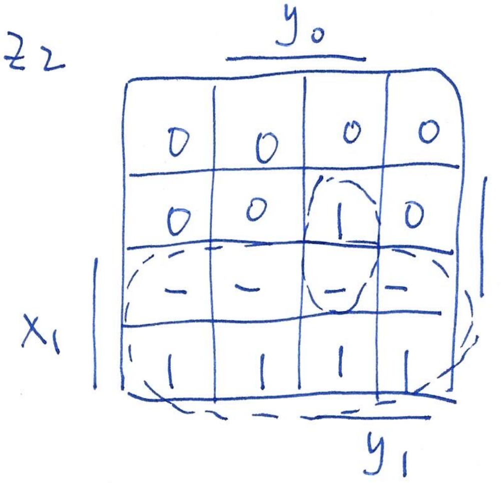
z_1



0	0	1	1
0	1	0	1
–	–	–	–
0	0	1	1

$y_0 x_0 y_1'$
$x_1 x_0$
$x_1 y_1$
$x_0' y_1$
$y_0' y_1$

z_2



0	0	0	0
0	0	1	0
–	–	–	–
1	1	1	1

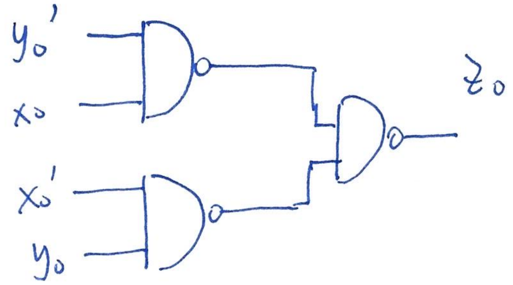
x_1
~~$x_1' x_0$~~
$x_0 y_0 y_1$

(d). z_0: $y_0' x_0$, $x_0' y_0$

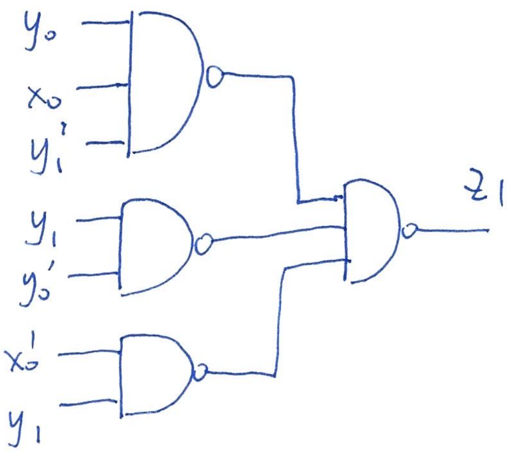
z_1: $y_0 x_0 y_1'$, $y_1 y_0'$, $x_0' y_1$

z_2: x_1, $x_0 y_0 y_1$

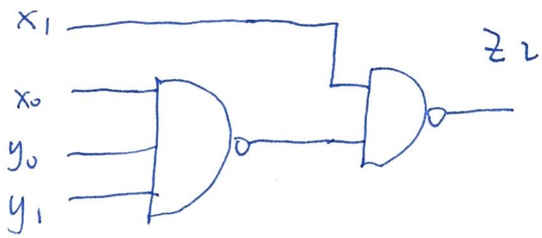
(e). z_0



z_1



z_2


x_1
x_0
y_0
y_1
z_2