Problem 1 (12 pts): Number conversion

(a) (6 pts) $X = 85$ in decimal, convert it to Radix-2, Radix-8, Radix-16.
(b) (3 pts) $X = -47$ in decimal, convert it into 2's complement with 8-bit length.
(c) (3 pts) $X = 100110$ is the 2's complement representation of a number, convert this number to a decimal signed number (radix-10).

Answer:

(a) $(85)_{10} = (1010101)_2 = (125)_8 = (55)_{16}$

(b) To convert -47 into 2's complement.

first, $+47$ in 2's complement $= (00101111)_{2's}$

Second, -47 shall be $(11010001)_{2's}$

(c) X must be a negative number.

Its magnitude shall be $(011010)_{2's} = 26$

$\therefore (100110)_{2's} = -26$ in decimal.

Problem 2 (16 pts) Boolean Algebra:

(a) (8 pts) An operation $*$ is defined for binay variables a and b as follows:

$$a * b = ab + a'b'$$

Let $c = a * b$. Determine which of the following identities are valid:

a1. $a = b * c$

a2. $a * bc = 1$

(b) (8 pts) Reduce the following Boolean expression to a minimum number of literals:

$$xyz + x'y + xyz'$$

Answer:

(a) $c = a * b = ab + \bar{a}\bar{b}$

a1. $b * c = bc + \bar{b}\bar{c} = b(ab + \bar{a}\bar{b}) + \bar{b}\,\overline{ab + \bar{a}\bar{b}}$

$= abb + \bar{a}\bar{b}b + \bar{b}(\overline{ab} \cdot \overline{\bar{a}\bar{b}})$

$= ab + \bar{b}(\bar{a} + \bar{b})(a + b)$

$= ab + \bar{b}(a\bar{a} + \bar{a}b + a\bar{b} + b\bar{b})$

$= ab + a\bar{b}\bar{b}$

$= a(b + \bar{b})$

$= a$ Proves a1 is valid

a2. $a * bc = abc + \bar{a}\,\overline{bc} = abc + \bar{a}(\bar{b} + \bar{c})$

$= ab(ab + \bar{a}\bar{b}) + \bar{a}\bar{b} + \bar{a}\,\overline{ab + \bar{a}\bar{b}}$

$= ab + \bar{a}\bar{b} + \bar{a}(\overline{ab} \cdot \overline{\bar{a}\bar{b}})$

$= ab + \bar{a}\bar{b} + \bar{a}(\bar{a} + \bar{b})(a + b)$

$= ab + \bar{a}\bar{b} + \bar{a}(a\bar{a} + \bar{a}b + a\bar{b} + b\bar{b})$

$= ab + \bar{a}\bar{b} + \bar{a}b$

$= ab + \bar{a}$

$= \bar{a} + b$ When $a = 1$, $b = 0$, $a * bc = 0$. Invalid

(b) $xyz + \bar{x}y + xy\bar{z}$

$= y(\bar{x} + xz) + xy\bar{z}$

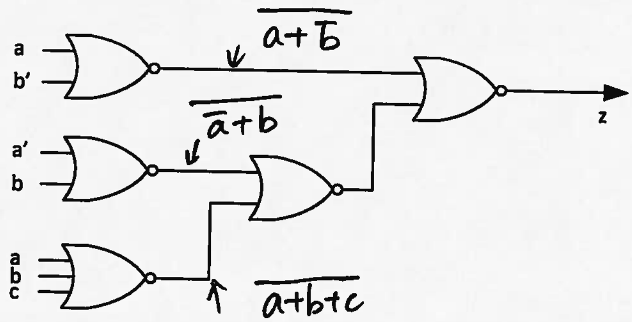
$= y(\bar{x} + z) + xy\bar{z}$

$= y(\bar{x} + z + x\bar{z})$

$= y(\bar{x} + z + x) = y$

Problem 3 (10 pts):

Obtain a minimal two-level NAND-NAND network for the function implemented by the network shown below.



Answer:

$$Z = \overline{\overline{a+\bar b} + \overline{\overline{\bar a+b} + \overline{a+b+c}}}$$

$$= (a+\bar b)(\overline{\bar a+b} + \overline{a+b+c})$$

$$= (a+\bar b)(a\bar b + \bar a\bar b\bar c)$$

$$= a\bar b + a\bar a\bar b\bar c + a\bar b + \bar a\bar b\bar b\bar c$$

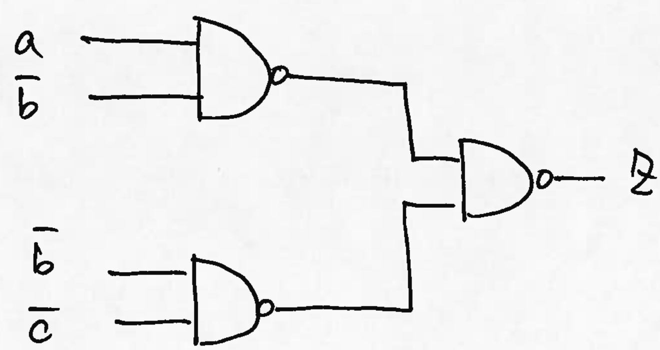
$$= a\bar b + \bar a\bar b\bar c$$

$$= (a+\bar a\bar c)\bar b$$

$$= (a+\bar c)\bar b$$

$$= a\bar b + \bar b\bar c$$

$$= \overline{\overline{a\bar b} \cdot \overline{\bar b\bar c}}$$



Problem 4 (18 pts):

Design an efficient multiple-of-3 circuit that takes in a decimal digit 0-9 represented as a 4-bit binary number a[3:0], i.e. a circuit whose output is logic 1 when the input a[3:0] represents number 3, 6, or 9.

(a) (3 pts) Write a truth table for the function.
(b) (3 pts) Draw a Karnaugh map of the function.
(c) (3 pts) Identify the prime implicants of the function.
(d) (3 pts) Identify which of the prime implicants (if any) are essential.
(e) (3 pts) Find a switching expression of the function with minimal gate inputs and minimal number of gates.
(f) (3 pts) Draw a logic gate diagram of the function **using NAND gates only**.

Answer:

(a) Write your truth table in the table below:

a[3]	a[2]	a[1]	a[0]	output
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

X is don't-care-bit that can either be 1 or 0

(b) Draw a Karnaugh map of the function.

Fill the following Karnaugh map template:

a_3a_2 \ a_1a_0	00	01	11	10
00	0	0	1	0
01	0	0	0	1
11	X	X	X	X
10	0	1	X	X

a[1] (columns 11, 10); a[2] (rows 01, 11); a[3] (rows 11, 10); a[0] (columns 01, 11)

$a_3a_2 \backslash a_1a_0$	00	01	11	10
00	0	0	1	0
01	0	0	0	1
11	X	X	X	X
10	0	1	X	X

(c) Identify the prime implicants of the function.
List the prime implicants:

$a_3 a_0, \; a_2 a_1 \overline{a_0}, \; \overline{a_2} a_1 a_0$

(d) Identify which of the prime implicatns (if any) are essential.
List the essential prime implicants:

$a_3 a_0, \; a_2 a_1 \overline{a_0}, \; \overline{a_2} a_1 a_0$ are all essential.

(e) Find a switching expression of the function with minimal gate inputs and minimal number of gates.
Show the work on the Karnaugh map on the previous page and then list the implicants you would use and the corresponding Boolean logic equation here.

$F = (a_3 + a_1)(a_2 + a_0)(\overline{a_2} + \overline{a_0})$

$a_3a_2 \backslash a_1a_0$	00	01	11	10
00	0	0	1	0
01	0	0	0	1
11	X	X	X	X
10	0	1	X	X

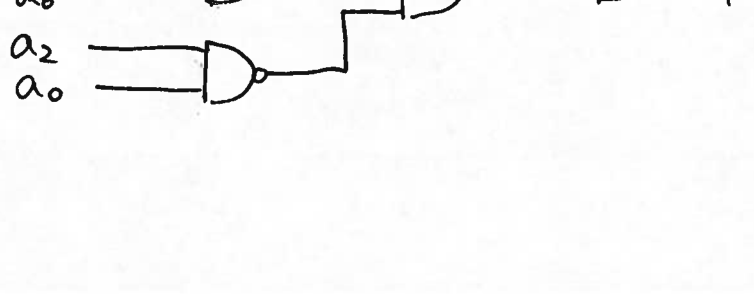
Need 3 OR GATES, 1 AND GATE
It has 6 gate inputs

(f) Draw a logic gate diagram of the function (remember, only NAND gates)

$F = (a_3 + a_1)(a_2 + a_0)(\overline{a_2} + \overline{a_0})$

$= \overline{\overline{a_3} \cdot \overline{a_1}} \cdot \overline{\overline{a_2} \cdot \overline{a_0}} \cdot \overline{a_2 \cdot a_0}$

$= \overline{\overline{\overline{\overline{a_3} \cdot \overline{a_1}} \cdot \overline{\overline{a_2} \cdot \overline{a_0}} \cdot \overline{a_2 \cdot a_0}}}$



Problem 5 (12 pts):
Among the following three gates, which ones are universal and prove them. (you may use constants 0 and 1 as inputs wherever necessary)

X	Y	X*Y	XΔY	XΦY
0	0	0	1	0
0	1	0	1	0
1	0	1	0	1
1	1	0	1	1

Answer:

$X * Y = X\overline{Y}$ NOT GATE $\overline{X} = 1 * X = 1 \cdot \overline{X} = \overline{X}$

AND GATE $X \cdot Y = X * \overline{Y} = X * (1 * Y) = X \cdot \overline{\overline{Y}} = X \cdot Y$

$X \Delta Y = \overline{X} + Y$ NOT GATE $\overline{X} = X \Delta 0 = \overline{X} + 0 = \overline{X}$

AND GATE $XY = \overline{\overline{XY}} = \overline{\overline{X} + \overline{Y}} = (\overline{X} + \overline{Y}) \Delta 0$

$= (X \Delta \overline{Y}) \Delta 0$

$= (X \Delta (Y \Delta 0)) \Delta 0$

$X \phi Y = X$ Complement of NOT GATE

In conclusion, $X * Y$ and $X \Delta Y$ is universal operation

$X \phi Y$ is not.

Problem 6 (12 pts):
Implement the following Boolean expression with MUX 2:1 gates:

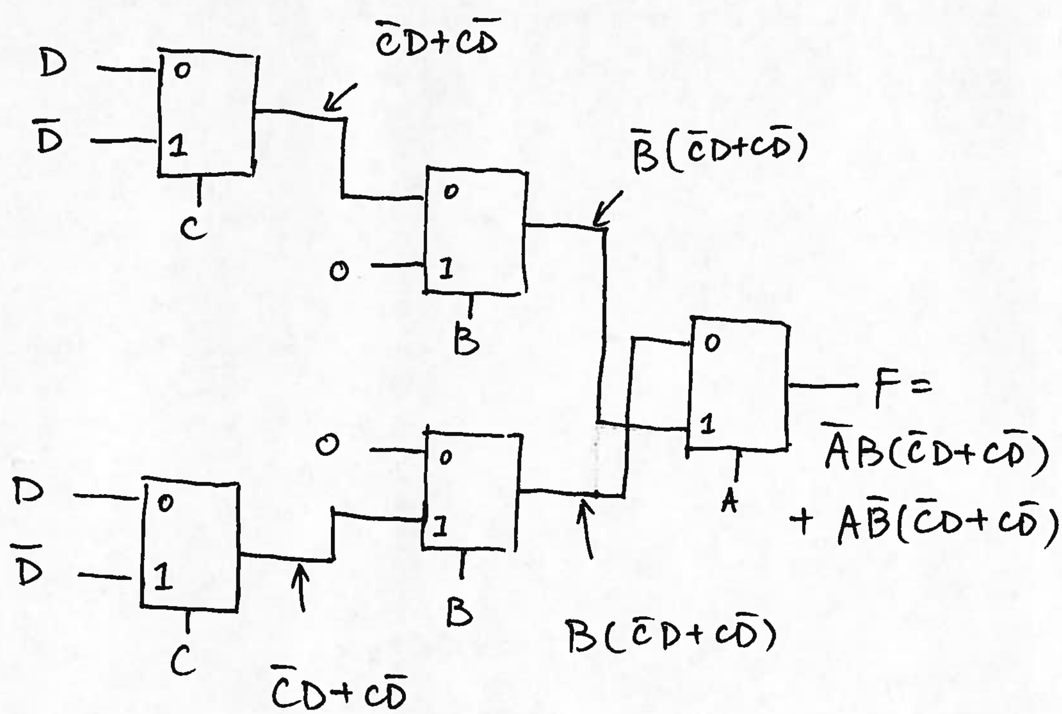
$$F = AB'CD' + A'BCD' + AB'C'D + A'BC'D$$

Answer:

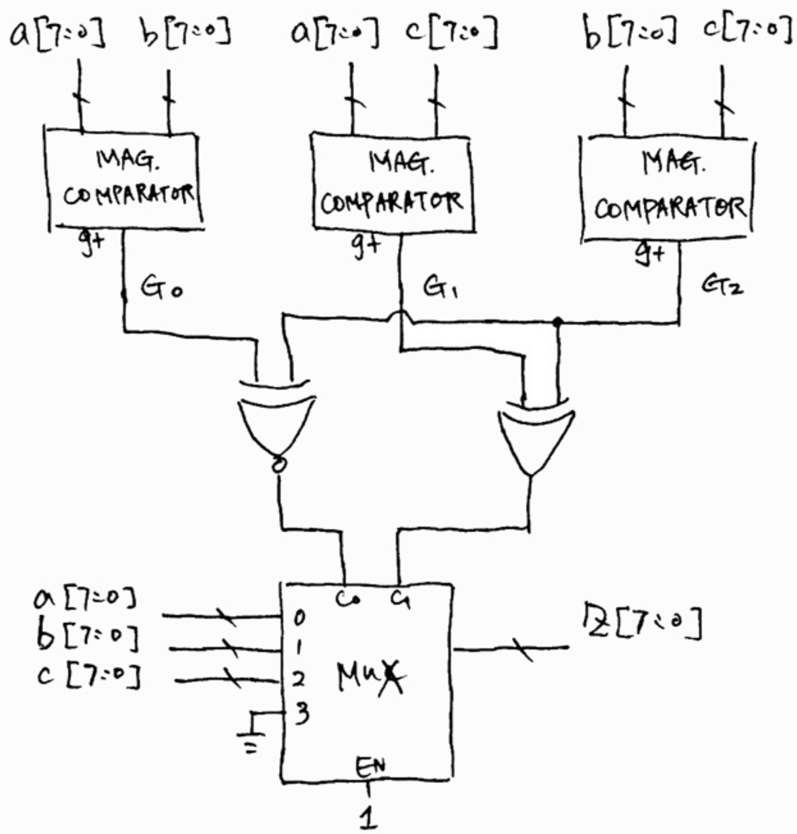
$$F = A\bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + \bar{A}B\bar{C}D$$

$$= A(\bar{B}C\bar{D} + \bar{B}\bar{C}D) + \bar{A}(BC\bar{D} + B\bar{C}D)$$

$$= A(B\cdot 0 + \bar{B}(C\bar{D} + \bar{C}D)) + \bar{A}(B(C\bar{D} + \bar{C}D) + \bar{B}\cdot 0)$$



Problem 7 (20 pts):
Design a combinational circuit with three 8-bit inputs a[7:0], b[7:0], and c[7:0] representing binary numbers that outputs the median value of the three input number (i.e. the middle value if the input numbers a, b, and c are ordered by their value). You may use any of the primitive gates (NOT, AND OR, NAND, NOR, XOR, XNOR) or any of the building blocks discussed in the lecture (decoder, multiplexer, encoder, arbiter, priority encoder, comparator, adder).



Magnitude Comparator will take in 2 8-bit inputs & output true if a>b.

Define 00 – a
01 – b
10 – c

$G_0G_1G_2$	Explanation	C_1C_0
0 0 0	a≤b, a≤c, b≤c	01
0 0 1	a≤b, a≤c, b>c	10
0 1 0	a≤b, a>c, b≤c	XX
0 1 1	a≤b, a>c, b>c	00
1 0 0	a>b, a≤c, b≤c	00
1 0 1	a>b, a≤c, b>c	XX
1 1 0	a>b, a>c, b≤c	10
1 1 1	a>b, a>c, b>c	01

$C_1 = G_1 \oplus G_2$

$C_0 = \overline{G_0 \oplus G_2}$