

1. Find X, Y such that the following conditions are satisfied:

a) $(2303031022)_4 = X_8$

(5pt)

b) $(236)_7 - (104)_6 = Y_{11}$

(5pt)

$$\begin{aligned} \text{a) LHS} &= \underline{1011} \underline{0011} \underline{0011} \underline{0100} \underline{1010} \\ &= (2631512)_8 \end{aligned}$$

$$\begin{aligned} \text{b) } (236)_7 &= 2 \cdot 7^2 + 3 \cdot 7^1 + 6 \cdot 7^0 \\ &= 98 + 21 + 6 \\ &= \cancel{125} (125)_{10} \end{aligned}$$

$$\begin{aligned} (104)_6 &= 1 \cdot 6^2 + 4 \cdot 6^0 \\ &= 36 + 4 = (40)_{10} \end{aligned}$$

$$(125)_{10} - (40)_{10} = (85)_{10} = (78)_{11}$$

$$\begin{array}{r} \parallel \underline{85} \text{ --- } 8 \\ \parallel \underline{7} \text{ --- } 7 \\ \text{0} \end{array}$$

2. Which of the following functions are equivalent:

(10pt)

$$A = x'y' + x'z'$$

$$B = x'z' + x'y'z$$

$$C = x'y'z' + x'z' + y'z$$

A

x \ yz	00	01	11	10
0	1	1	0	1
1	0	0	0	0

B

x \ yz	00	01	11	10
0	1	1	0	1
1	0	0	0	0

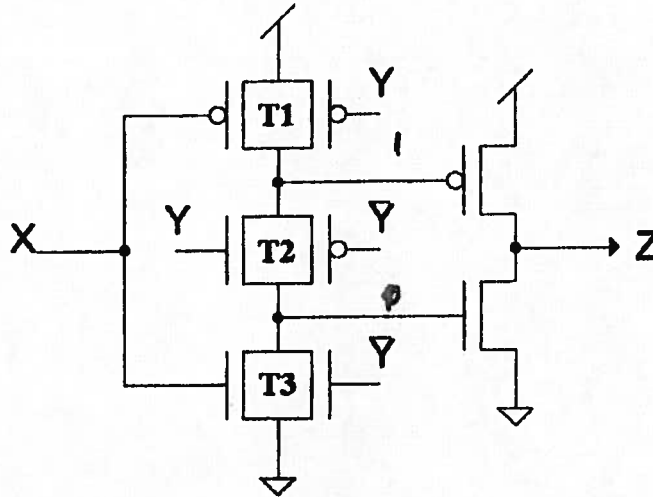
C

x \ yz	00	01	11	10
0	1	1	0	1
1	0	1	0	0

$$\Rightarrow A=B.$$

3. For the two-input gate given by the following

(10pt)



complete the table (for $T_i = \text{on/off}$) provided below:

(Note that only T_2 is a transmission gate.)

X	Y	T_1	T_2	T_3	Z
0	0	ON	OFF	ON	0
0	1	ON	ON	OFF	0
1	0	ON	OFF	ON	0
1	1	OFF	ON	ON	1

2.5 each

4. Use a * gate that implements the following logic:

(10pt)

X	Y	X * Y
0	0	1
0	1	1
1	0	0
1	1	1

to implement the gate network of the function:

$$f(x, y, z) = (((x + y)' + z)' + y + z)' + x'$$

Hint: Simplify first and then draw the gate network.

$$x * y = (xy) = x' + y = E(x, y).$$

$$f = \left((x'y + z)' + y + z \right) + x'$$

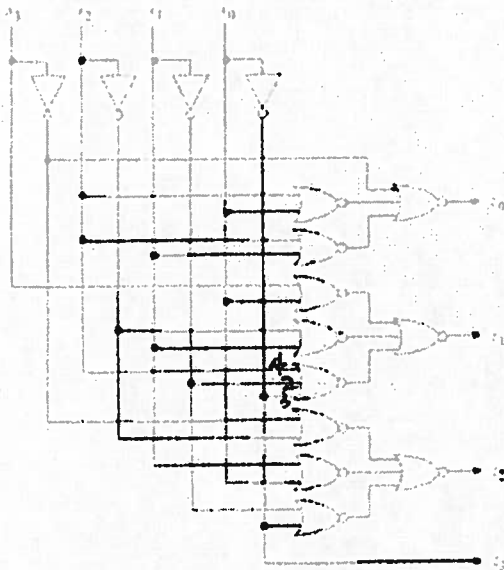
$$= \left((x'y)' \cdot z' + y + z \right) + x'$$

$$= (x + y + z)' + x'$$

$$= x'y'z' + x' = x'$$

$$f = E(x, 0).$$

5. Analyzing gate networks



Gate type	Fan-in	Propagation delays		Load factor [standard loads]	Size [equiv. gates]
		t_{pLH} [ns]	t_{pHL} [ns]		
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$	1.0	2
AND	3	$0.20 + 0.038L$	$0.18 + 0.018L$	1.0	2
AND	4	$0.28 + 0.039L$	$0.21 + 0.019L$	1.0	3
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$	1.0	2
OR	3	$0.12 + 0.038L$	$0.34 + 0.022L$	1.0	2
OR	4	$0.13 + 0.038L$	$0.45 + 0.025L$	1.0	3
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0	1
NAND	2	$0.05 + 0.038L$	$0.08 + 0.027L$	1.0	1
NAND	3	$0.07 + 0.038L$	$0.09 + 0.030L$	1.0	2
NAND	4	$0.10 + 0.037L$	$0.12 + 0.051L$	1.0	2
NAND	5	$0.21 + 0.038L$	$0.34 + 0.019L$	1.0	1
NAND	6	$0.24 + 0.037L$	$0.36 + 0.019L$	1.0	5
NAND	8	$0.24 + 0.038L$	$0.42 + 0.019L$	1.0	6
NOR	2	$0.06 + 0.075L$	$0.07 + 0.016L$	1.0	1
NOR	3	$0.16 + 0.111L$	$0.08 + 0.017L$	1.0	2
NOR	4	$0.23 + 0.149L$	$0.08 + 0.017L$	1.0	1
NOR	5	$0.18 + 0.038L$	$0.23 + 0.018L$	1.0	1
NOR	6	$0.16 + 0.037L$	$0.24 + 0.018L$	1.0	5
NOR	8	$0.54 + 0.038L$	$0.23 + 0.018L$	1.0	6
NOR	2*	$0.30 + 0.036L$	$0.30 + 0.021L$	1.1	3
		$0.16 + 0.036L$	$0.15 + 0.020L$	2.0	

Using the table above, find the:

- Load factor of each primary input of the gate network (2pt)
- Network size in equivalent gates (Hint: See last column) (3pt)
- Show the critical path of the gate network and calculate the corresponding delays t_{pLH} and t_{pHL} . Assume the load on each primary output of the gate network is $L = 6$. (5pt)

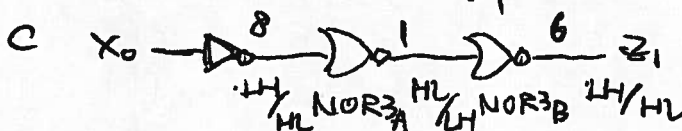
a. X_3 2; X_2 4; X_1 4; X_0 4

b. NOT $\times 4$ $1 \times 4 = 4$

NOR2 $\times 7$ $1 \times 7 = 7$

NOR3 $\times 4$ $2 \times 4 = 8$

$$\begin{array}{r} + \\ \hline 19 \end{array}$$



$$t_{pLH}(X_0, Z_1) = t_{pLH}(\text{NOR3B}) + t_{pHL}(\text{NOR3A}) + t_{pLH}(\text{NOT})$$

$$= 0.16 + 0.111 \cdot 6 + 0.08 + 0.017 \cdot 1 + 0.02 + 0.028 \times 8$$

$$= 1.247$$

$$= 1.25 \text{ ns}$$

$$= 1.25 \text{ ns}$$

$$t_{PHL}(X_0, Z_1) = t_{PHL}(NOR3B) + t_{PHL}(NOR3A) + t_{PHL}(NOT)$$

$$= 0.08 + 0.017 \cdot 6 + 0.16 + 0.111 \cdot 1 + \\ 0.05 + 0.017 \cdot 8$$

$$= 0.639 \text{ ns}$$

6. For $f(w, x, y, z) = \text{one-set}(0, 1, 2, 3, 5, 7, 8, 10, 11, 15)$

- Find all the prime implicants.
- Indicate which of these prime implicants are essential.
- Obtain a minimal sum of products for f . Is it unique?

(3pt)

(3pt)

(4pt)

wx \ yz	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	1	0
10	1	0	1	0

$y x'$
 $x' y$

PI

a. $wx', yz, wz, xz', \cancel{wxz}, x'y$

b. EPI. yz, wz, xz'

c. $f = yz + wz + xz'$

unique.

a) $x_3' x_2', x_1 x_0, x_3' x_0, x_2' x_0', x_2' x_1$

b) $x_1 x_0, x_3' x_0, x_2' x_0'$

