UCLA
Department of Electrical Engineering
EEM16 – Fall 2010
Midterm Solutions

1.
a+a'b+a'b'c+a'b'c'd+a'b'c'd'e =

= a+a'b+a'b'c+a'b'c'(d+d'e) = a+a'b+a'b'c+a'b'c'(d+e) = a+a'b+a'b'(c+c'(d+e))
=

= a+a'b+a'b'(c+d+e) = a+a'(b+b'(c+d+e)) = a+a'(b+c+d+e) = a+b+c+d+e

2.
given c=a*b=ab+a'b'. Note c'=(ab+a'b')'=a'b+ab'.

(a) a=b*c ?

b*c = bc+b'c' = b(ab+a'b')+b'(a'b+ab') = ab+~~a'bb'~~+~~a'bb'~~+ab' = a(b+b') = a

So this identity is valid.

(b) a*bc=1 ?

a*bc = abc+a'(bc)' = ab(ab+a'b')+a'(b'+c') = ab(ab+a'b')+a'(b'+a'b+ab') =

= ab(ab+a'b')+a'(b'+a'b+ab') = ab + ~~a'ab'b~~ + a'b' +a'b +~~aa'b'~~ =

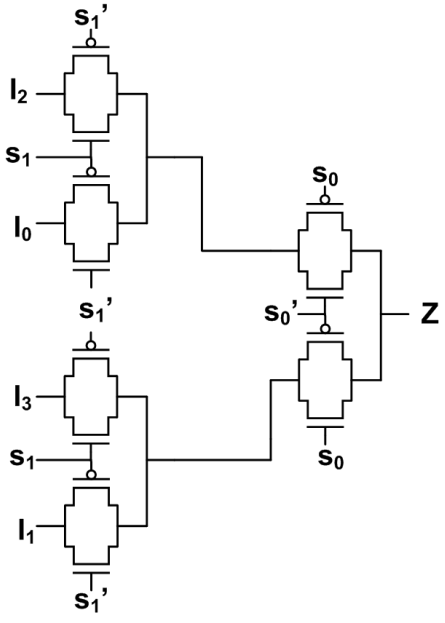
= ab + a'(b+b') = ab+a' = a'+b

So this identity is NOT valid.

3.

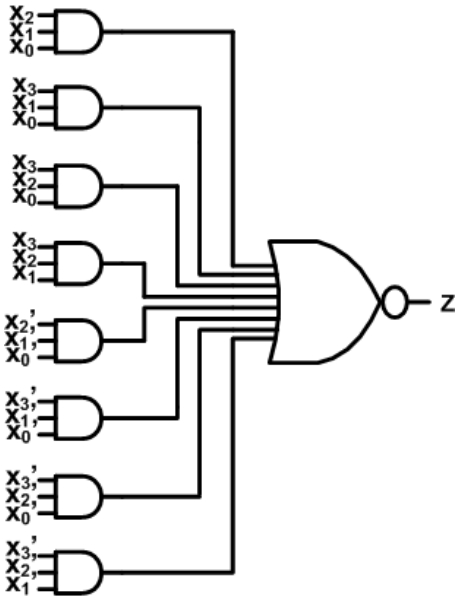
$Z = I_0S_1'S_0' + I_1S_1'S_0 + I_2S_1S_0' + I_3S_1S_0$

$= (I_0S_1' + I_2S_1)S_0' + (I_1S_1' + I_3S_1)S_0$

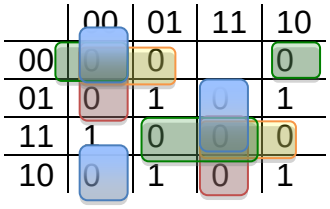


4.
(a)

x_3	x_2	x_1	x_0	z
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0



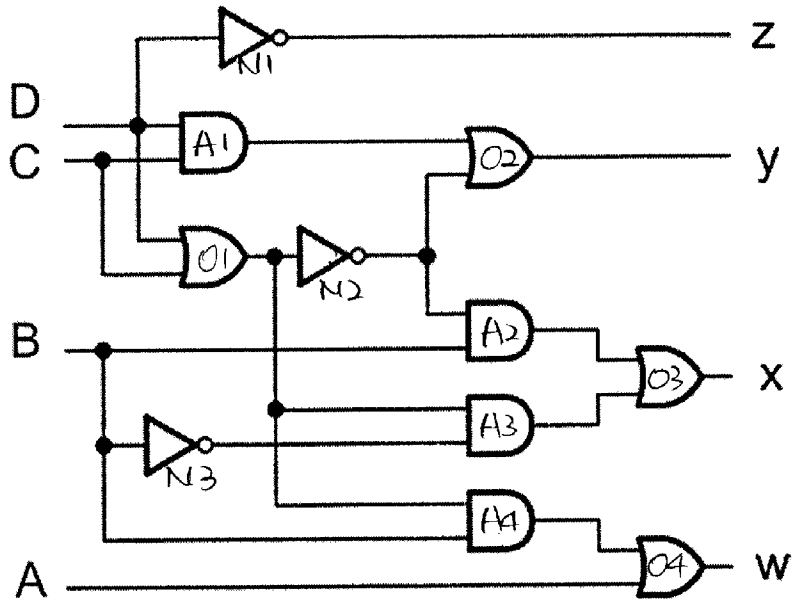
(b) $z=\Sigma m(3,5,6,9,10,12)$
(c) $z=\Pi M(0,1,2,4,7,8,11,13,14,15)$
(d)



$z = (x_2'+x_1'+x_0')(x_3'+x_1'+x_0')(x_3'+x_2'+x_0')(x_3'+x_2'+x_1')$
$(x_2+x_1+x_0)(x_3+x_1+x_0')(x_3+x_2+x_0)(x_3+x_2+x_1)$
$=(((x_2'+x_1'+x_0')(x_3'+x_1'+x_0')(x_3'+x_2'+x_0')(x_3'+x_2'+x_1')$
$(x_2+x_1+x_0)(x_3+x_1+x_0')(x_3+x_2+x_0)(x_3+x_2+x_1))')'$
$= ((x_2'+x_1'+x_0')'+(x_3'+x_1'+x_0')'+(x_3'+x_2'+x_0')'+ (x_3'+x_2'+x_1')'+$
$(x_2+x_1+x_0)'+(x_3+x_1+x_0')'+(x_3+x_2+x_0)'+(x_3+x_2+x_1)')'$
$= (x_2x_1x_0+ x_3x_1x_0+ x_3x_2x_0+ x_3x_2x_1+ x_2'x_1'x_0'+ x_3'x_1'x_0'+ x_3'x_2'x_0'+ x_3'x_2'x_1')'$

…. two-stage AND-NOR (please check the gate network shown above)

5. Analyzing Gate Networks **[20 pts]**



5 a. Find the switching expression of each output.

5 b. Using the given table, find the load factor of each input.

10 c. Determine the critical path and calculate the propagation delay. Assume load of each output is a constant L_0.

Gate type	Fan-In	Propagation delays		Load factor	Size
		t_{pLH} [ns]	t_{pHL} [ns]	[standard loads]	[euiv. gates]
AND	2	$0.15 + 0.037L$	$0.16 + 0.017L$	1.0	2
OR	2	$0.12 + 0.037L$	$0.20 + 0.019L$	1.0	2
NOT	1	$0.02 + 0.038L$	$0.05 + 0.017L$	1.0	1

a. $Z = D'$

$Y = C'D' + CD$

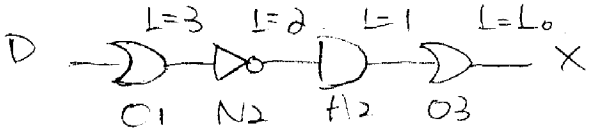
$X = B'C + B'D + BC'D'$

$W = A + BC + BD$

b.

Input	Load Factor
A	1
B	3
C	2
D	3

c. Critical Path



$$t_{pHL} = t_{pHL(O3)} + t_{pHL(A2)} + t_{pHL(N2)} + t_{pLH(O1)}$$

$$= 0.20 + (0.019 \times L_0) + 0.16 + (0.017 \times 1) + 0.03 + (0.017 \times 2) + 0.12 + (0.037 \times 3)$$

$$= 0.692 + 0.019 L_0 \text{ ns}$$

$$t_{pLH} = t_{pLH(O3)} + t_{pLH(A2)} + t_{pLH(N2)} + t_{pHL(O1)}$$

$$= 0.12 + (0.037 \times L_0) + 0.13 + (0.037 \times 1) + 0.02 + (0.038 \times 2) + 0.20 + (0.019 \times 3)$$

$$= 0.66 + 0.037 L_0 \text{ ns}$$

6. For $f(w,x,y,z) = one - set\ (1,5,7,8,9,10,14)$: **[25 pts]**

5 a. Find all prime implicates and prime implicants of f.

5 b. Indicate which of these prime implicates and prime implicants are essential.

5 c. Obtain minimal product of sums and sum of products for f. Are they unique?

5 d. Implement f using 4:1 multiplexers.

5 e. Implement f using PLA.

wx \ yz	00	01	11	10
00	0	1	0	0
01	0	1	1	0
11	0	0	0	1
10	1	1	0	1

wx \ yz	00	01	11	10
00	0	1	0	0
01	0	1	1	0
11	0	0	0	1
10	1	1	0	1

a). Prime Implicants:

$w'yz$, $w'xz$, wxy', wyz', $x'y'z$, $wx'z'$

Prime Implicates:

$w+z$, $w+x+y'$, $x+y+z'$, $x'+y+z$,

$w'+x'+y$, $w'+x'+z'$, $w'+y'+z'$

b). Essential Prime Implicants:

$w'xz$, wyz'

Essential Prime Implicates: $w+z$

c) $f(w,x,y,z) = w'xz + wyz' + wxy' + w'y'z$ (not unique)

$f(w,x,y,z) = (w+z)(w'+x'+y)(w'+x'+z')(x+y+z')$ (not unique)

d. $f(w,x,y,z) = w'xz + wyz' + wx'y' + w'y'z$

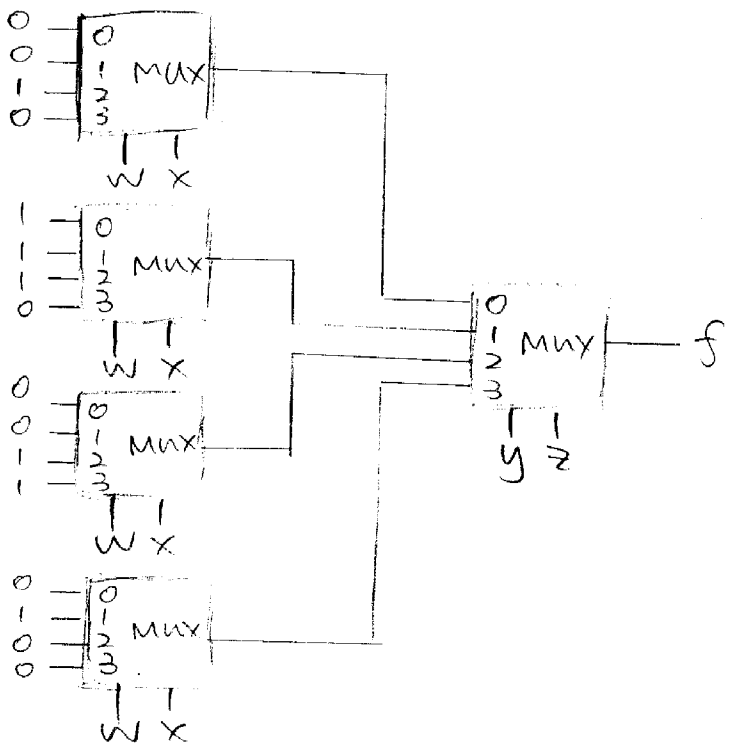
$= (wx')y'z' + (w'x + wx' + w)y'z + (w)yz' + (w'x)y$

$= [(0)w'x' + (0)w'x + (1)wx' + (0)wx]\, y'z'$

$+ [(1)w'x' + (1)w'x + (1)wx' + (0)wx]\, y'z$

$+ [(0)w'x' + (0)w'x + (1)wx' + (1)wx]\, yz'$

$+ [(0)w'x' + (1)w'x + (0)wx' + (0)wx]\, yz$



'0' is ground

'1' is Vdd

e.