EE 3 W'13 Final Exam Name ____________

UID ____________

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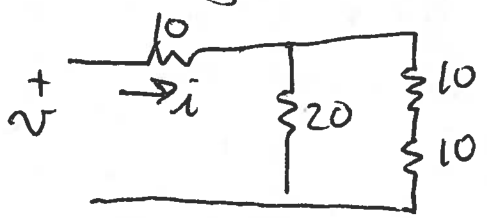
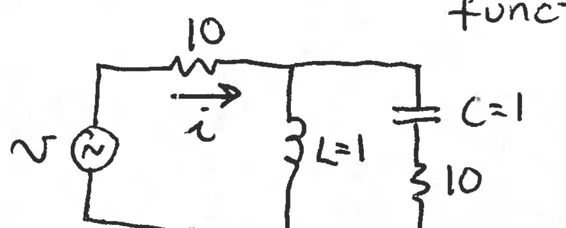
Instructions: Attempt all three questions. You have two hours. Calculators may be used. This is a closed book exam. Two sheets of formulae are attached.

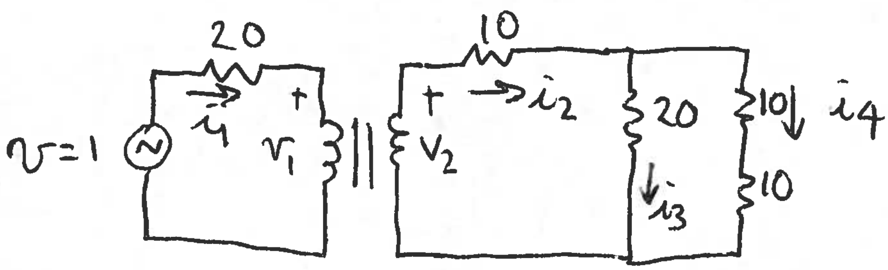
1. a) Determine the equivalent resistance to the following circuit:



b) Determine i for the following circuit as a function of ω and v.

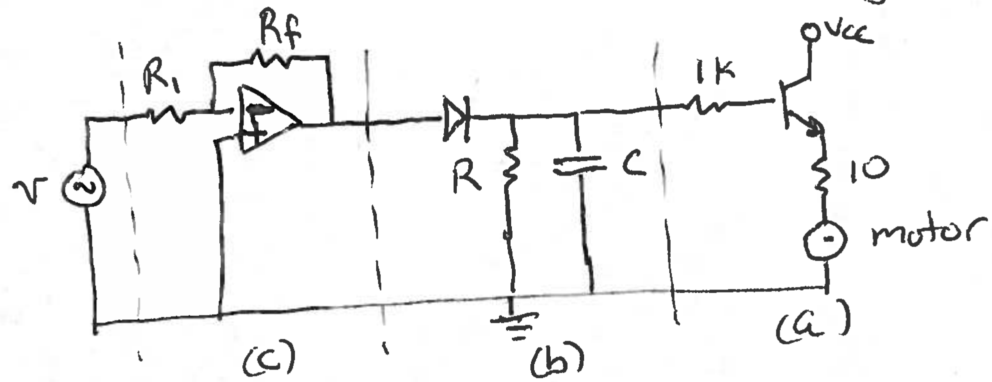


c) Determine voltages and currents in the following circuit, where for the ideal transformer $N_1 = \frac{N_2}{2}$



d) Describe a practical application of transformers in the electrical grid.

2. A small sinusoidal signal is received (e.g. sound or radio). Its presence is used to control whether a motor turns on. The following circuit is used.



a) Motor drive circuit. What are the purposes of the resistors in the path to the base and between the transistor and the motor?
Under what conditions does current flow through the motor?

b) Rectifier circuit. Suppose the frequency of the sinusoid is $f_c = 1\,kHz$. For it to drive the motor, why must it be converted to DC?
If $R = 1\,k\Omega$, suggest an appropriate value for C so that the current flowing into the base has only a small ripple.

c) What factors cause attenuation of communication signals (consider for example your experiments with sound).
Suppose we need an amplification of a factor of 10 in voltage to enable driving the motor. What are suitable values for R_1 and R_f?

d) Describe two applications of transistors with broad societal impact.

3. In a communications system, received signal power is attenuated as the square of distance in free space. That is, transmission loss $L = \left(\frac{d}{d_o}\right)^2$ where $L = 1$ when $d = d_o$.

a) Suppose $d = 100 d_o$. What power gain G is needed in an amplifier to restore the signal power to the same level as at d_o?

b) We have available the following amplifiers:
(i) $F = 8, G = 10$; (ii) $F = 10, G = 100$
If we can use at most 3 amplifier stages to achieve the gain computed in part (a), what sequence of amplifiers will produce the lowest noise figure?

c) Radio and TV signals were historically relayed across the country using analog repeaters. Suppose each link has the transmission loss of part (a) and the amplifier of part (b). After how many links is the signal to noise ratio degraded by a factor of 10 compared to the first link?

d) What advantages do digital repeaters have over analog systems? Why were analog systems used up until the 1980's?

Transmission Lines

Heaviside conditions for distortionless transmission: $\frac{R}{G} = \frac{j\omega L}{j\omega C}$

Power losses in resistive transmission line:

$P_{in} = v_{in} i$;

$P_{out} = (v_{in} - v_{loss})i = (v_{in} - iR)i = v_{in}i - i^2R$

$= P_{in} - P_{loss}$

Ideal Transformers

$v_1 i_1 = v_2 i_2$ (Power conservation)

$\frac{v_1}{v_2} = \frac{i_2}{i_1} = \frac{N_1}{N_2}$

Motors

$ei = T\omega_m$(electrical power=mechanical power); $\omega_m = \frac{v}{r}$ = angular velocity

$T = Fr = ZBlir$

Linear Approximation

Taylor: $f(x^* + h) = f(x^*) + \frac{df(x^*)}{dx^*}h + \frac{1}{2}\frac{d^2 f(\xi)}{d^2 x^*}h^2$; Thus $f(x^* + h) \approx f(x^*) + \frac{df(x^*)}{dx^*}h$

Feedback principle: if error is small, even though the overall system is nonlinear, a linear feedback controller can be successful.

Diodes

Ideal: when voltage drop across diode in direction of arrow is positive, has zero resistance; when voltage is negative, has infinite resistance

Practical: when voltage exceeds turn-on voltage, resistance is very low; when voltage is negative, until some breakdown level negligible current can flow

Transistors

Logic Usage: when V_{BE} is high for an npn transistor or low for pnp transistor, current flows through CE path with minimal resistance; if V_{BE} is low for an npn transistor or high for a pnp transistor, the CE path has near infinite resistance. In either case, minimal current flows into the base.

Small signal approximation: when V_{BE} is just above the turn-on voltage, the transistor can act as an amplifier

Noise Figure

$F = \frac{SNR_{in}}{SNR_{out}}$; G = Gain = amplification of power

$F = F_1 + \frac{F_2 - 1}{G_1}$; $G = G_1 G_2$

For a line with power attenuation (loss) *L*, *F=L*, *G=1/L*

Digital Repeater

For an error probability of *p* on one link, with *N* repeaters, error probability=*Np*

Amplitude Modulation

$s(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t$

Radio Propagation

Signal power is proportional to d^a; $2<a<4$ are typical.

Equations

Circuit Elements and Solution Methods

$v = iR$

$i = C\frac{dv}{dt}; v = \frac{1}{C}\int_{-\infty}^{t} idt$

$v = L\frac{di}{dt}; i = \frac{1}{L}\int_{-\infty}^{t} vdt$

$Z=v/i$; For steady state sinusoidal response use $Z_R = R$; $Z_L = j\omega L$; $Z_C = 1/j\omega C$ where $\omega = 2\pi f$

If $v = iZ = i(Z_r + jZ_i)$ then $|i| = \frac{|v|}{\sqrt{Z_r^2 + Z_i^2}}; \theta = -\tan^{-1}\frac{Z_i}{Z_r}$

KVL: The voltage drops around a loop sum to zero

KCL: The currents leaving a junction sum to zero

Mesh Currents: apply KVL with independent currents in each loop

Node Voltages: apply KCL at each junction

Series Resistors: $R_{eq} = R_1 + R_2 + ... + R_n$; Series Inductors: $L_{eq} = L_1 + L_2 + ... + L_n$

Voltage Divider: $v_2 = \frac{R_2}{R_1 + R_2}v$

Parallel Capacitors: $C_{eq} = C_1 + C_2 + ... + C_n$

Parallel Resistors: $R_{eq} = \frac{R_1R_2}{R_1 + R_2}; i_2 = \frac{R_1}{R_1 + R_2}i; i_1 = \frac{R_2}{R_1 + R_2}i$

Ideal Op Amp: infinite input resistance, zero output resistance, infinite gain

Non-inverting mode: $v_0 \approx \left(1 + \frac{R_f}{R_1}\right)v_i$; Inverting mode: $v_0 \approx -\frac{R_f}{R_1}v_i$

Adder: $v_0 \approx -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2\right)$; Integrator: $v_0 = \frac{1}{RC}\int_{-\infty}^{t} v_i dt$

Time-Frequency Relations

Complex Exponentials: $e^{jx} = \cos x + j\sin x$; $\cos x = \frac{e^{jx} + e^{-jx}}{2}; \sin x = \frac{e^{jx} - e^{-jx}}{2j}$

Interpretation of Frequency Domain: signals can be written as combinations of sinusoids (either complex exponentials, or combinations of cosines and sines) of different frequencies. The Fourier transform values denote the relative content at the different frequencies. The inverse transform is used to go from the frequency to time domain.

Fourier Transform: $X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$; Inverse transform: $x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df$

Convolution in time=multiplication in frequency domain: $\int_{-\infty}^{\infty} x_1(t)x_2(t-\tau)d\tau \leftrightarrow X_1(f)X_2(f)$

Filters: used to wipe out particular ranges of frequencies (e.g., low pass attenuates high frequencies); easier to work in frequency domain due to above relation.

Frequency translation by f_c: $e^{j2\pi f_c t}x(t) \leftrightarrow X(f - f_c)$