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Problem#1

For a 2D semiconductor system, the E-k relationship near the conduction band edge is parabolic, namely $E(k) = E_c + \frac{\hbar^2 k^2}{2m_e}$, where E_C is the conduction band minimum, and m_e is the electron effective mass. However, for 2D system, the k is also 2D (i.e. k_x and k_y) with spacing of $\frac{2\pi}{L_{x,y}}$. That is the "unit area" of a k is $\frac{(2\pi)^2}{A}$, as opposed to $\frac{(2\pi)^3}{V}$ in the 3D semiconductor.

(a) What is the total number of states for energy $E \leq E_0$?
(b) Derive the density of states at energy level E_0, note the number of states between E_0 to $E_0 + \Delta E$ is $\approx D(E_0)\Delta E$
(c) Derive the equation for electron concentration n for a fermi-energy E_{f0}?

(Note that the number of states and electron concentration have the units of cm^{-2})

a) $E(k) = E_c + \frac{\hbar^2 k^2}{2m_e}$; $k = \frac{2\pi}{L_{x,y}} \approx k = \frac{(2\pi)^2}{A}$ where $E = E(k) - E_c$

$N(k) = 2 \cdot \frac{1}{A} \cdot \frac{\pi k^2}{\frac{(2\pi)^2}{A}} = \frac{2\pi k^2}{(2\pi)^2} = \frac{k^2}{2\pi} \rightarrow k = \sqrt{\frac{2mE}{\hbar^2}}$

$N(E) = \frac{2mE}{2\pi\hbar^2} = \frac{mE}{\pi\hbar^2} \rightarrow N(E) = \frac{m_e(E - E_c)}{\pi\hbar^2}$

b) $g(E)$ = density of states $\Rightarrow \frac{dN(E)}{dE} = \frac{m_e}{\pi\hbar^2} \rightarrow g(E)\Delta E \cancel{\rightarrow} \frac{m_e}{\pi\hbar^2}\Delta E$

c) $n = \int_{E_c}^{\infty} g(E)\,F(E)\,dE \rightarrow n = N_c \int \frac{E - E_c}{1 + \exp\left(\frac{E - E_f}{kT}\right)} dE$

using boltzman's approximation

$n = N_c \exp\left(\frac{E_{f0} - E_c}{kT}\right)$

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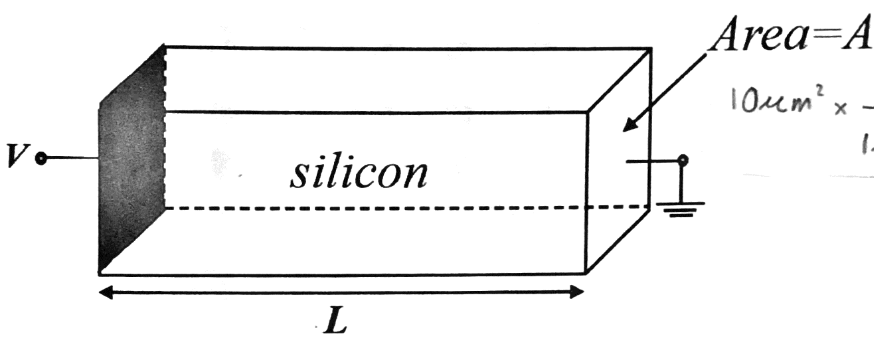
$1 \mu m = 10^{-4} cm$ *check* calculator acting up

$\cancel{1 \mu m^2} = 10^{-9} cm^2$

Problem#2

A block of Si has a length of $L = 100\mu m$ ~~and~~ an area $A = 10\mu m^2$. $100 \mu m \times \frac{10^{-4} cm}{1 \mu m} = 10^{-3} cm$



$10 \mu m^2 \times \frac{10^{-9} cm^2}{1 \mu m^2} = 1 \times 10^{-8}$

(a) If the silicon is doped with arsenic (n-type) at a concentration $N_{D1} = 5 \times 10^{16} cm^{-3}$, what are the electron and hole density in the silicon block. In addition, what is the resistance between two faces of the block, as shown in the figure.

(b) Additional phosphorus has been introduced in to the silicon in part (a) with uniform doping concentration of $N_{D2} = 5 \times 10^{16} cm^{-3}$ by solid state diffusion. Calculate the new electron and hole density in the silicon block as well as the resistance between two faces of the block.

(c) Additional boron has been introduced to the silicon in part (b) with uniform doping concentration of $N_A = 1.5 \times 10^{17} cm^{-3}$ by solid state diffusion. Calculate the new electron and hole density in the silicon block as well as the resistance between two faces of the block.

a) $n \approx N_D = 5 \times 10^{16} cm^{-3} \rightarrow np = (n_i)^2 \rightarrow p = \frac{(n_i)^2}{n} = \frac{(10^{10})^2}{5 \times 10^{16}} = 2.0 \times 10^{3} cm^{-3}$

$J = \frac{I}{A}$; $\mathcal{E} = \frac{V}{L}$ $R = \frac{V}{I} \rightarrow R = \frac{\mathcal{E} \cdot L}{J \cdot A} \rightarrow \frac{\mathcal{E} \cdot L}{\sigma \mathcal{E} \cdot A} \rightarrow R = \rho \frac{L}{A}$

$\rho = \frac{1}{q(\mu_n n)} = \frac{1}{q(5 \times 10^{16} \cdot 980)} = 0.1276 \ \Omega \cdot cm$ $R = \frac{(0.1276)(10^{-3})}{(1 \times 10^{-8})} = 12760 \ \Omega \Rightarrow 12.8 k\Omega$

b) New $N_{D_n} = N_{D1} + N_{D2} \rightarrow 5 \times 10^{16} + 5 \times 10^{16} \rightarrow 1 \times 10^{17} cm^{-3}$ since is not compensated $n \approx N_{D_n} = 1 \times 10^{17} cm^{-3}$

$p = \frac{(n_i)^2}{n} = \frac{(1 \times 10^{10})^2}{1 \times 10^{17}} = 1 \times 10^{3} cm^{-3}$

$\rho = \frac{1}{q(1 \times 10^{17} \cdot 800)} = 0.078 \ \Omega cm$ $R = \frac{(0.078)(10^{-3})}{(1 \times 10^{-8})} = 7800 \rightarrow 7.8 k\Omega$

continued in back

$$\frac{dJ_p}{dx} = -q D_p \frac{d^2(\Delta p)}{dx^2}$$

Problem#3

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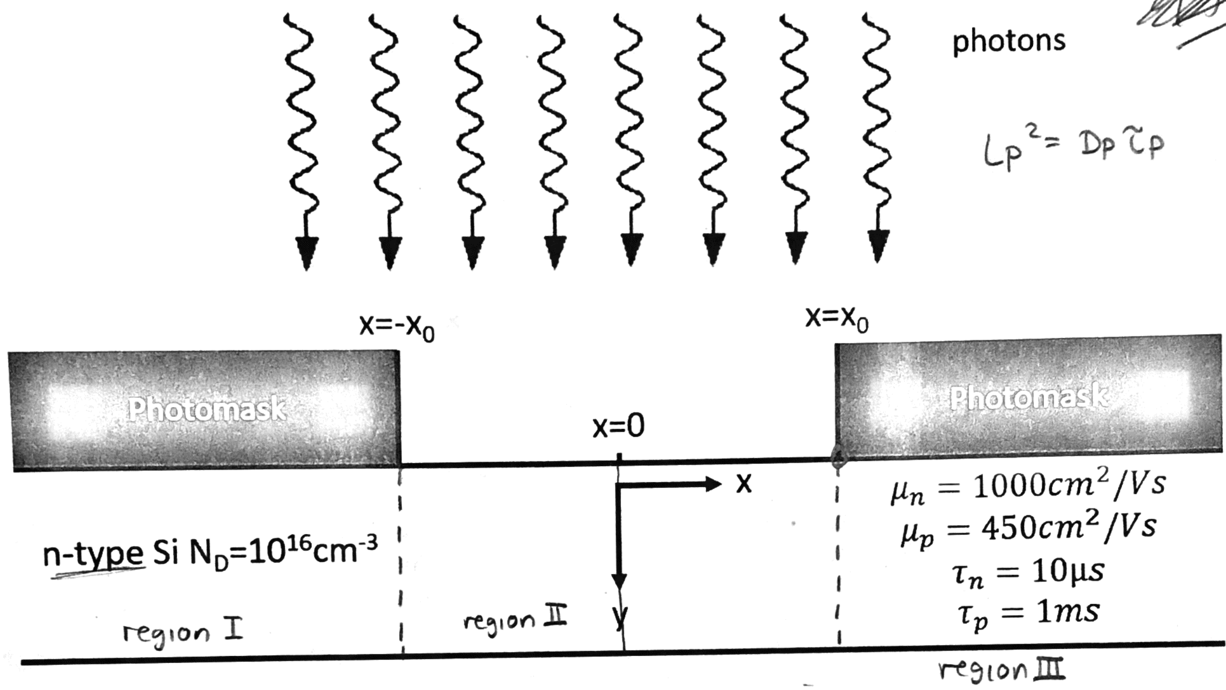

$L_p^2 = D_p \tau_p$

Photons are incident onto silicon wafer. In the shaded area (i.e. not obstructed by the photomask), there is uniform electron/hole generation with generation rate $G_L\ (cm^{-3}s^{-1})$. The lateral dimensions of the wafer can be treated as it extends to infinity.

(a) Derive Δn as a function of the lateral position (i.e. x direction), note that $\Delta n \approx \Delta p$ under steady state as per discussed in the lecture.

(b) If $x_0 = 50\mu m$, what is value of $\Delta n \approx \Delta p$ at x_0.

$\Delta n = \Delta p$ under steady state $\quad \frac{\partial(\Delta P)}{\partial t} = \frac{-1}{q}\frac{\partial J_p}{\partial x} - \frac{\Delta P}{\tau_p} + G_L \rightarrow 0 = \frac{-1}{q}\left(-q D_p \frac{d^2(\Delta P)}{dx}\right) - \frac{\Delta P}{\tau_p} + G_L$

Region I: $\Delta P_1 = D_p \tau_p \frac{d^2(\Delta P)}{dx} \rightarrow \Delta p_1(x) = A_1 \exp(x/L_p) + B_2 \exp\left(\frac{-x}{L_p}\right)$

$0 = D_p \cdot \tau_p \frac{d^2(\Delta P)}{dx} - \Delta P + G_L \cdot \tau_p$

at $x = -\infty$ $B_2 = 0$, so $\Delta P_1(x) = A_1 \exp(x/L_p)$

$\Delta P = D_p \cdot \tau_p \frac{d^2(\Delta P)}{dx} + G_L \tau_p$

Region III: $\Delta P_3 = D_p \tau_p \frac{d^2(\Delta P)}{dx} \rightarrow \Delta P_3(x) = C_1 \exp(x/L_p) + C_2 \exp(-x/L_p)$ at $x = \infty$ $C_1 = 0$ $\Delta P_3 = C_2 \exp\left(\frac{-x}{L_p}\right)$

Region II: $\Delta P_2 = G_L \cdot \tau_p$

Problem 2:

$N_d = 1\times10^{17}\ cm^{-3}$

$N_A = 1.5\times10^{17} cm^{-3}$

now we have more acceptors so we are a p-type

compensated p-type

$p = N_A - N_D = 1.5\times10^{17} - 1\times10^{17} = 5\times10^{16}\ cm^{-3}$

$$n = \frac{(n_i)^2}{p} = \frac{(1\times10^{10})^2}{5\times10^{16}} = 2\times10^{3}\ cm^{-3}$$

μ now depends on both of them so $N_i = N_A + N_D$

$$\rho = \frac{1}{q(\mu_p p)} = \frac{1}{q(240)(5\times10^{16})} = 0.5208\,\Omega cm$$

$$R = \frac{(0.5208)(10^{-3})}{(1\times10^{-8})} = 52080 \Rightarrow 52.1k\Omega$$

Problem 3:

boundary conditions

$\Delta P_1(-x_0) = \Delta P_2(-x_0) \rightarrow A_1 \exp\left(\frac{-x_0}{L_P}\right) = G_L\tau_p \rightarrow A_1 = \frac{G_L\tau_p}{\exp\left(\frac{x_0}{L_P}\right)}$

$\Delta P_2(x_0) = \Delta P_3(x_0) \rightarrow G_L\tau_p = C_2 \exp\left(\frac{-x_0}{L_P}\right) \rightarrow C_2 = \frac{G_L\tau_p}{\exp\left(\frac{-x_0}{L_P}\right)}$

$$\Delta P_1 = \frac{G_L\tau_p}{\exp\left(\frac{-x_0}{L_P}\right)} \exp(x/L_P)$$

$$\Delta P_2 = G_L\tau_p$$

$$\Delta P_3 = \frac{G_L\tau_p}{\exp\left(\frac{-x_0}{L_P}\right)} \exp\left(\frac{-x}{L_P}\right)$$

$\rightarrow$

$\Delta n_1 = \Delta P_1 = G_L\tau_p \exp\left(\frac{x+x_0}{L_P}\right)$ $\quad x < -x_0$ $\quad$ — ω

$\Delta n_2 = \Delta P_2 = G_L\tau_p$ $\quad -x_0 \le x \le x_0$

$\Delta n_3 = \Delta P_3 = \tau_p G_L \exp\left(\frac{x_0 - x}{L_P}\right)$ $\quad x > x_0$

$$\Delta n_1 = G_L \tau_p \exp\left(\frac{x + x_0}{L_p}\right)$$

$$L_p = \sqrt{D_p \tau_p}$$

$$\Delta n_2 = G_L \tau_p$$

$$\Delta n_3 = G_L \tau_p \exp\left(\frac{x_0 - x}{L_p}\right)$$

Einstein's Relation

$$\frac{D_p}{\mu_p} = \frac{kT}{q}$$

$$D_p = \frac{(kT)\mu_p}{q}$$

$$D_p = \mu_p \frac{(8.617\times10^{-5})(300)}{1} \frac{eV\,K}{eV\,K}$$

$$D_p = \left(450 \frac{cm^2}{V\cdot s}\right)\left(\frac{300k}{1.6\times10^{-19}}\right)$$

$$D_p = 11.65 \frac{cm^2}{s}$$

if $x_0 = 50\mu m$

$$\begin{cases} \Delta n_1 = G_L \tau_p \exp\left(\frac{x + 50\mu m}{0.108}\right) & x \leq -50\mu m \\ \Delta n_2 = G_L \tau_p & -50\mu m < x < 50\mu m \\ \Delta n_3 = G_L \tau_p \exp\left(\frac{50\mu m - x}{0.108}\right) & x \geq 50\mu m \end{cases}$$

$$L_p = \sqrt{(1\times10^{-3}s)\left(11.6 \frac{cm^2}{s}\right)}$$

$$L_p = 0.108\ cm$$

@ $x_0 = 50\mu m$

$$\Delta n_3(50\mu) = G_L \tau_p \exp\left(\frac{50\mu - 50\mu}{0.108\times10^{-2}}\right)$$

$$= (1\times10^{-3}) G_L \exp(0)$$

$$\Delta n_3(50\mu m) = (1\times10^{-3}) G_L$$

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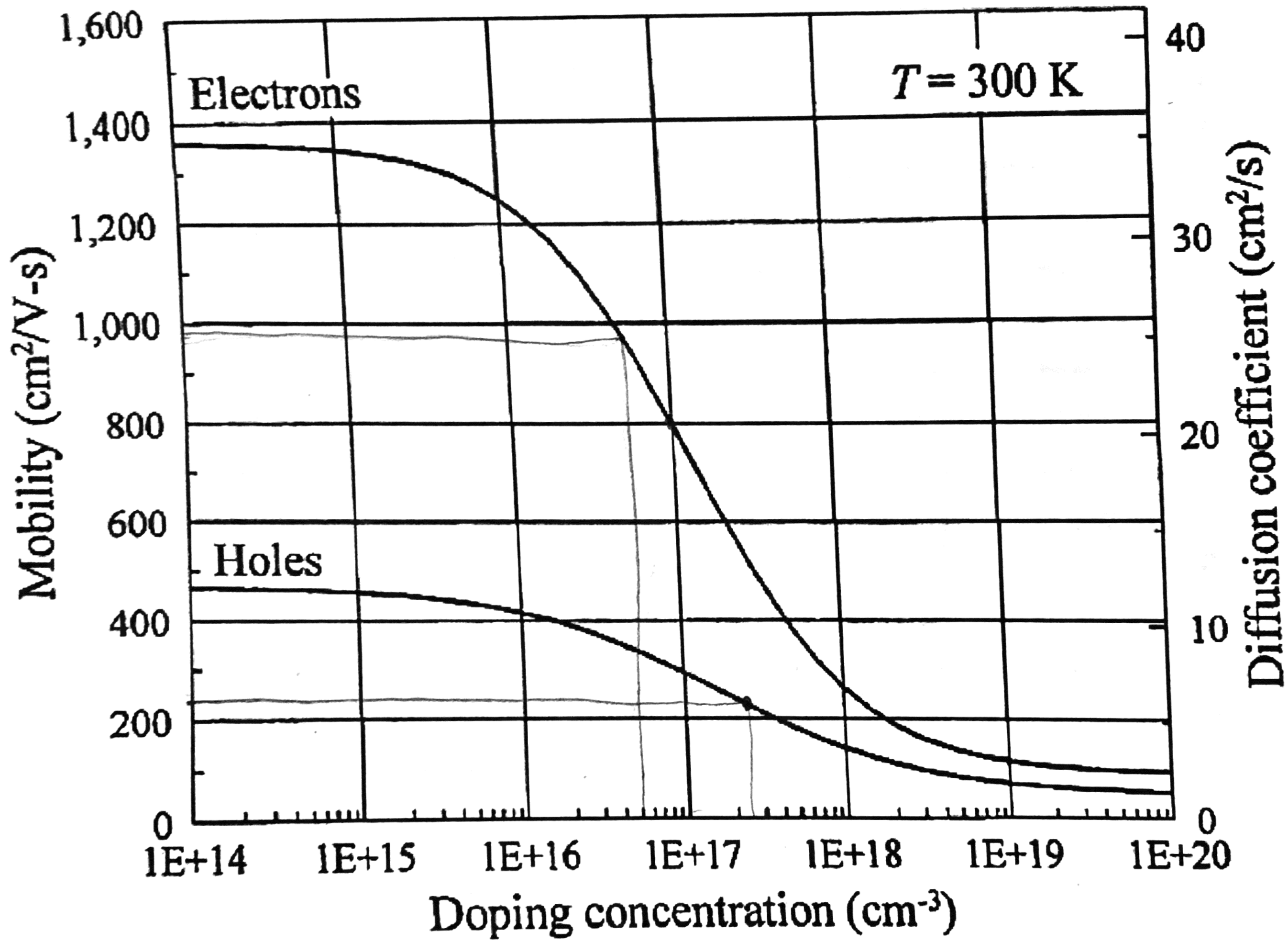

Electron and hole mobilities in bulk silicon at 300 K