EE2: Physics for Electrical Engineers

Practice Midterm Spring 2016

Actual midterm: April 27th 2016, 2 pm to 3:50 pm, 1102 Perloff Hall

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Closed book, but with 1-sheet (2-sides of 8.5" × 11" paper) of notes.
Please use calculator.

Question 1. (20 points) Neamen 2.25

2.25 An elecron is bound in a one-dimesional infinite potential well with a width of 75 Å. Determine the electron energy levels (in eV) for $n = 1, 2, 3$.

$$E = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{(1.054\times10^{-34})^2\pi^2 n^2}{2\times(9.11\times10^{-31})\times(75\times10^{-10})^2}$$

$E = 1.070 \text{ x } 10^{-21}\, n^2 (J)$

or

$E = 6.69 \text{ x } 10^{-3}\, n^2 (eV)$

Then

$E_1 = 6.69 \text{ x } 10^{-3} (eV)$
$E_2 = 2.67 \text{ x } 10^{-2} (eV)$
$E_3 = 6.02 \text{ x } 10^{-2} (eV)$

Question 2. (20 points) Neamen 3.14

3.14 Two possible valence bands are shown in the E versus k diagram given in Figure P3.14. State which band will result in the heavier hole effective mass; state why.

The effective mass for a hole is given by

$$m_p^* = \left(\frac{1}{\hbar^2}\cdot\left|\frac{d^2E}{dk^2}\right|\right)^{-1}$$

We have

$$\left|\frac{d^2E}{dk^2}\right|(curve\ A) > \left|\frac{d^2E}{dk^2}\right|(curve\ B)$$

So

$$m_p^*(curve\ A) < m_p^*(curve\ B)$$

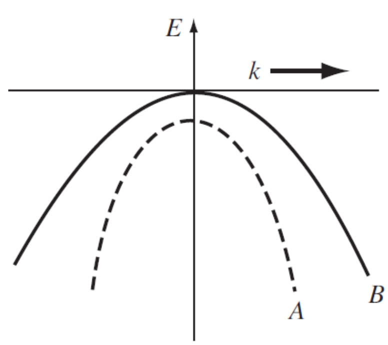

Figure P3.14 | Valence bands for Problem 3.14.

Question 3. (20 points) Neamen 3.45

3.45 Assume that the Fermi energy level is exactly in the center of the bandgap energy of a semiconductor at $T = 300$ K. (*a*) Calculate the probability that an energy state in the bottom of the conduction band is occupied by an electron for Si, Ge, and GaAs. (*b*) Calculate the probability that an energy state in the top of the valence band is empty for Si, Ge, and GaAs.

(a) When $E_F = E_{midgap}$

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} = \frac{1}{1+\exp\left(\frac{E_g}{2kT}\right)}$$

Si: $E_g = 1.12\ eV$,

$$f(E) = \frac{1}{1+\exp\left[\frac{1.12}{2(0.0259)}\right]}$$

or

$$\underline{f(E) = 4.07x10^{-10}}$$

Ge: $E_g = 0.66\ eV$,

$$f(E) = \frac{1}{1+\exp\left[\frac{0.66}{2(0.0259)}\right]}$$

or

$$\underline{f(E) = 2.93x10^{-6}}$$

GaAs: $E_g = 1.42\ eV$,

$$f(E) = \frac{1}{1+\exp\left[\frac{1.42}{2(0.0259)}\right]}$$

or

$$\underline{f(E) = 1.24x10^{-12}}$$

(b) We now prove that the probability of a state at $E_2 = E_F - \Delta E$ being empty equals to the probability of a state at $E_1 = E_F + \Delta E$ being occupied:

We have

$$f_1(E_1) = \frac{1}{1+\exp\left(\frac{E_1 - E_F}{kT}\right)} = \frac{1}{1+\exp\left(\frac{\Delta E}{kT}\right)}$$

and

$$1 - f_2(E_2) = 1 - \frac{1}{1+\exp\left(\frac{E_2 - E_F}{kT}\right)}$$

$$= 1 - \frac{1}{1+\exp\left(\frac{-\Delta E}{kT}\right)} = \frac{\exp\left(\frac{-\Delta E}{kT}\right)}{1+\exp\left(\frac{-\Delta E}{kT}\right)}$$

or

$$1 - f_2(E_2) = \frac{1}{1+\exp\left(\frac{+\Delta E}{kT}\right)}$$

Hence the result for part (b) is the same with that in part (a).

Question 4. (20 points) Neamen 4.19

4.19 The electron concentration in silicon at $T = 300$ K is $n_0 = 2 \times 10^5$ cm^{-3}. (*a*) Determine the position of the Fermi level with respect to the valence band energy level. (*b*) Determine p_0. (*c*) Is this n- or p-type material?

$$n_i = \sqrt{N_c N_v \exp\left(-\frac{E_g}{kT}\right)} = \sqrt{\left(2.8\times10^{19}\right)\left(1.04\times10^{19}\right)\exp\left(-\frac{1.12}{0.0259}\right)} = 6.95\times10^{9}\ \text{cm}^{-3}$$

$$p_0 = \frac{n_i^2}{n_0} = 2.415\times10^{14}\ \text{cm}^{-3}$$

(p_0 can be slightly different if you use different value of $n_i = 0.7 \sim 1.5\times10^{10}$)

Since $p_0 > n_0$, it is *p*-type material.

Instead of holes, we use electron density to find a more accurate value of Fermi level because $E_C - E_F \gg kT$.

$$n_0 = N_c \exp\left[\frac{E_F - E_c}{kT}\right]$$

$$N_c = 2.8\times10^{19}$$

$$E_F - E_c = kT\ln\left(\frac{n_0}{N_c}\right) = 0.0259\ln\left(\frac{2\times10^5}{2.8\times10^{19}}\right) = -0.845\text{ eV}$$

$$E_F - E_v = (E_F - E_c) + (E_c - E_v) = (E_F - E_c) - E_g = 0.2764\text{ eV}$$

Question 5. (20 points) Neamen 4.50

4.50 A silicon device is doped with donor impurity atoms at a concentration of 10^{15} cm^{-3}. For the device to operate properly, the intrinsic carriers must contribute no more than 5 percent to the total electron concentration. (*a*) What is the maximum temperature that the device may operate? (*b*) What is the change in $E_c - E_F$ from the $T = 300$ K value to the maximum temperature value determined in part (*a*). (*c*) Is the Fermi level closer or further from the intrinsic value at the higher temperature?

(a)

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \qquad (4.60)$$

and $n_i = 0.05 n_o$

yielding $n_0 = 1.0025\times10^{15}$ cm^{-3}
so $n_i = 5.0125\times10^{13}$ cm^{-3}

so from

$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right] \qquad (4.23)$$

$$(5.0125\times10^{13})^2 = (2.8\times10^{19})\times(1.04\times10^{19})\times(\frac{T}{300})^3\times\exp[\frac{-1.12}{0.0259\times(T/300)}]$$

So $T \approx 482\ K$

(b)

When T=300K

$$n_i = \sqrt{N_c N_v \exp\left[-\frac{(E_c - E_v)}{kT}\right]} = 6.95\times10^9\ \text{cm}^{-3}$$

$$n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} = 1.0000\times10^{15}\ \text{cm}^{-3}$$ (almost the same as n_i at 482K)

from

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right) \qquad (4.63)$$

The change of Fermi level position is

$$E_{F,T=482K} - E_{F,T=300K}$$

$$= 0.0259 \times \frac{482}{300} \times \ln\left[\frac{2.8\times10^{19}\left(\frac{482}{300}\right)^{3/2}}{1.0025\times10^{15}}\right] - 0.0259 \times \ln\left[\frac{2.8\times10^{19}}{1.0000\times10^{15}}\right]$$

$$= 0.4556 - 0.2652 = 0.1904 \text{ eV}$$

(c) Take n-type semiconductor as an example, as the temperature increases, n_i increases, that is, the majority carrier electron concentration in the conduction band increases; as the probability of occupancy of allowed energy states by electrons and holes are equal, the majority carrier hole concentration in the valance band also increases. As a result, the crystal behaves more like an intrinsic semiconductor (losing its extrinsic characteristics), meaning that E_F moves closer to the intrinsic Fermi level. The same for p-type semiconductor.
The variation of Fermi level with temperature is shown in Fig. 4.19.

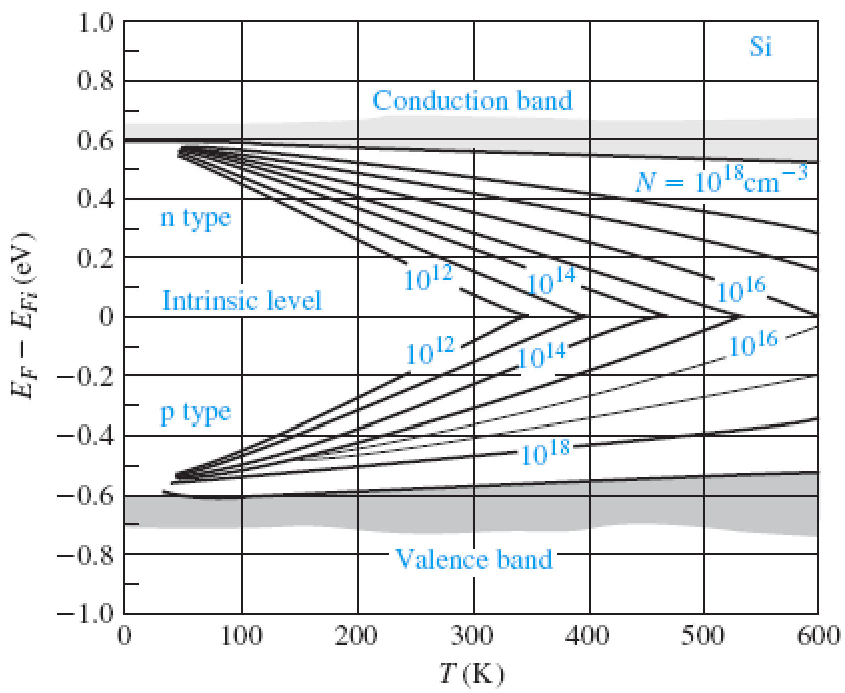

Figure 4.19 | Position of Fermi level as a function of temperature for various doping concentrations.
(From Sze [14].)