EE2: Physics for Electrical Engineers

Midterm Spring 2016

April 27th 2016, 2 to 4 pm, 1102 Perloff Hall

Instructors: Prof. Chee Wei Wong, Jinghui Yang and Yi-Ping Lai
Closed book, but with 1-sheet (2-sides of 8.5” × 11” paper) of notes.
Please use calculator.

Question 1. (35 points) Chapter 1 and 2: The Crystal Structure of Solids & Introduction to Quantum Mechanics

1.A. (10 points). What are the fourteen Bravias lattice of crystal solids found in Nature?

1.B. (10 points). The de Broglie wavelength of an electron is 85 Å. Determine the electron energy (in eV), momentum and velocity.

1.C. (15 points). Write down the one-dimensional non-relativistic form of the time-dependent Schrödinger equation. For the one-electron atom wavefunction (hydrogen atom), draw the radial probability density function $\Psi_{nlm} \cdot \Psi^{*}_{nlm}$ for the principal quantum number n =1 and n = 2 states.

Question 2. (30 points) Chapter 3: Introduction to the Quantum Theory of Solids

2.A. (15 points). Extending from the discretized energy levels in hydrogen, we taught the Kronig-Penny model. Describe briefly the key points in the Kronig-Penny model in the formation of the allowed and forbidden bands. Include notes on the Bloch function and the k-space description.

2.B. (15 points). With the formation of the conduction-valence bands, density of states, and the Fermi-Dirac distribution function, determine the probability that an energy level is occupied by an electron if the state is $5kT$ above the Fermi level.

Question 3. (35 points) Chapter 4: The Semiconductor in Equilibrium

3.A. (15 points). For a particular semiconductor, E_g = 1.50 eV, m_p^*= 10 m_n^*, T = 300K and n_i = 10^5 cm^{-3}. Determine the position of the intrinsic Fermi level E_{fi} with respect to the center of the band gap.

3.B. (20 points). Impurity atoms are added so that the Fermi energy level is 0.45 eV below the center of the band gap. (i) Are acceptors or donor atoms added? (ii) What is the concentration of impurity atoms added?

Helpful constants:
Boltzmann’s constant $k = 1.38 \times 10^{-23}$ J/K
Planck’s constant $h = 6.625 \times 10^{-34}$ J-s
Electronic charge $e = 1.60 \times 10^{-19}$ C

[redacted]

EE 2 Midterm.

$45 + 24 + 29 = 98$

1. A.

abc denotate length

α, β, γ is angle.

(Simple cubic,
Body centered
face centered.

+9

with all the same length, same angle (90°) ③

with one side length is different. $a=b\neq c$. ② (no face centere)

with two side length is different. ④

$a\neq b\neq c$

$a=b\neq c$
$\alpha=\beta=90°$
$\gamma=120°$ ① simple ~~cubic~~. primitive

$a\neq b\neq c$
$\alpha=\gamma=90°$
$\beta=120°$ ② no Face-centered

which two?

$a\neq b\neq c$
$\alpha\neq\beta\neq\gamma\neq 90°$ ① simple cubic

$a=b=c$
$\alpha=\beta=\gamma\neq 90°$ ① simple cubic.

1.B $\lambda = 85\,\text{Å} = 85\times10^{-10}\,m$

$$\lambda = \frac{h}{p}$$

$$\Rightarrow p = \frac{h}{\lambda} = \frac{6.625\times10^{-34}\,J\cdot s}{85\times10^{-10}\,m} = \boxed{7.79\times10}$$

$$E = \frac{1}{2}mv^2 = \frac{1}{2m}m^2v^2 = \frac{p^2}{2m}$$

$$= \frac{(7.79\times10^{-26}\frac{kg\,m}{s})^2}{2\times9.11\times10^{-31}\,kg} = \boxed{3.334\times10^{-21}\,J}$$

$$= \boxed{2.08\times10^{-2}\,eV}$$

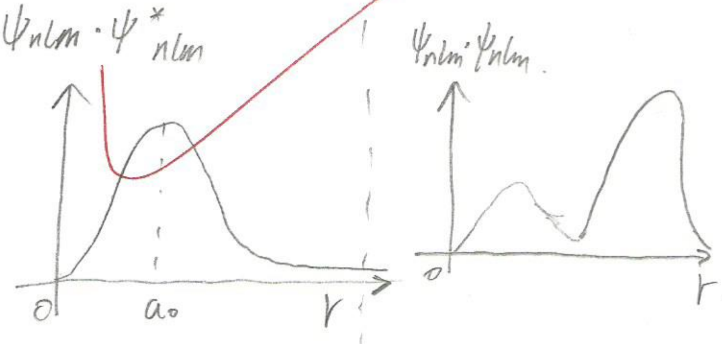
$$V = \frac{p}{m} = \frac{7.79\times10^{-26}\,\frac{kg\cdot m}{s}}{9.11\times10^{-31}\,kg} = \boxed{8.56\times10^{4}\,\frac{m}{s}}$$

Electron energy is $2.08\times10^{-2}\,eV$
momentum is $7.79\times10^{-26}\,\frac{kg\cdot m}{s}$
velocity is $8.56\times10^{4}\,\frac{m}{s}$

1C. +15

$$\boxed{-\frac{\hbar}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = j\hbar\frac{\partial\psi(x,t)}{\partial t}}$$

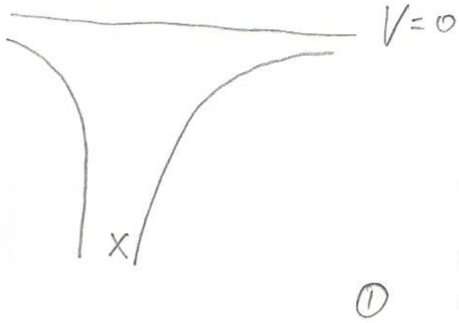
for $n=1$. for $n=2$.



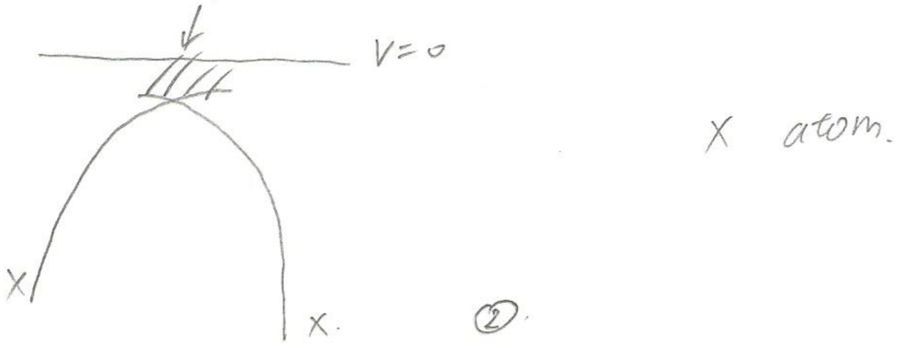
Bohr radius.

1

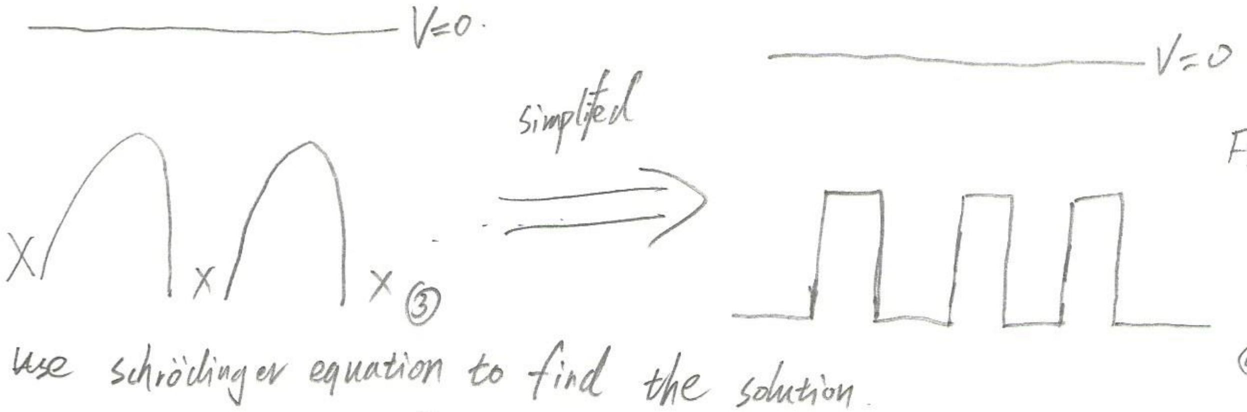
For single atom, | For two atom.



forbidden because the exclusive principle.



For lattice, we can simplfied to



Finite period potential well.

Use schrödinger equation to find the solution.

$\therefore \psi(x) = U(x) e^{jkx}$

periodic function.

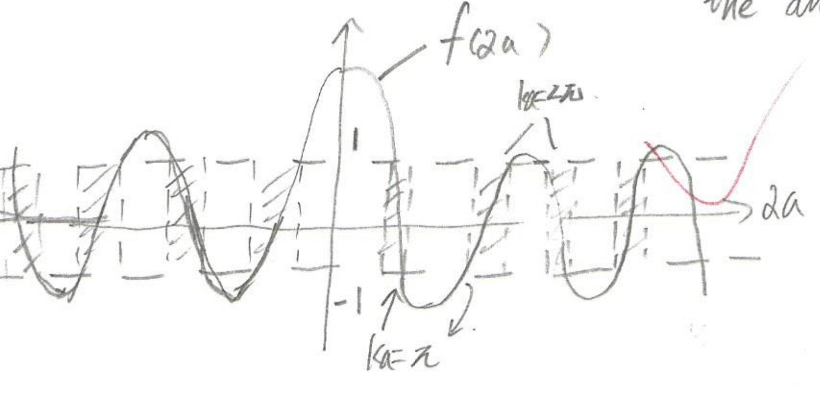
To solve the schrödinger equation with boundary condition: $U(x)$ and $\frac{dU(x)}{dx}$ continuous, finite, single valued.

And the we try to find the energy ~~of the electron~~

We leads to the Bloch function:

$$P' \frac{\sin 2a}{2a} + \cos 2a = \cos ka \Rightarrow \text{to solve this, we need graphing methods}$$

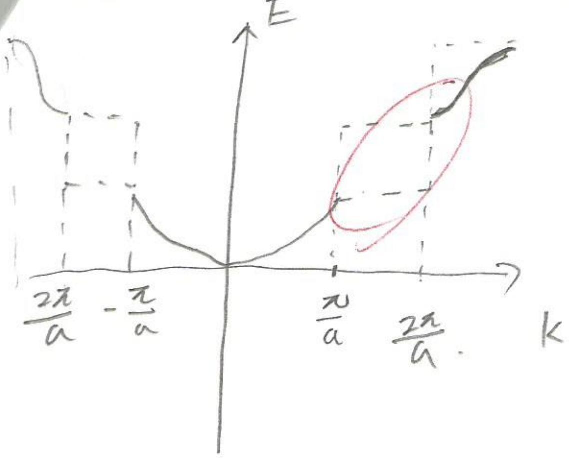
let $f(2a)$ and draw it; But $\cos ka$ only between $[-1, 1]$, which leads to the allowed band.



The solution can only inside the shaded area. (allow bands).

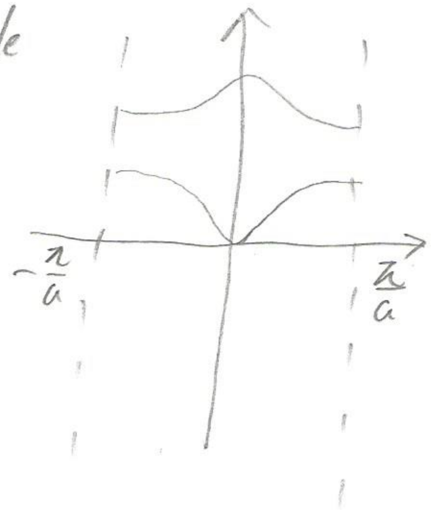
Forbidden bands is the place where solution doesn't exist (Forbidden bands

We can draw the graph in different way



can shift them inside

$\Longrightarrow \frac{\pi}{a}$



2.B. $f_F(E) = \dfrac{1}{1+\exp(\frac{E-E_F}{kT})} = \dfrac{1}{1+e^5} = \boxed{0.00669285}$

$E - E_F = 5kT$

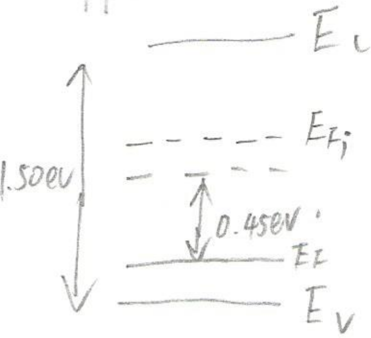
3A. $E_g = 1.50\ eV$, $m_p^* = 10 m_n^*$ $T = 300k$. $n_i = 10^5\ cm^{-3}$

$E_{Fi} = E_{midagp} + \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$.

$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln(10) = \boxed{0.04468\ eV}$

3.B i) $\boxed{\text{Acceptors}}$ are added so that the Fermi energy level is below the center of the band gap.

ii)



It's a p-type doping.

$$N_a \approx p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$$

$$= 10^5 \exp\left[\frac{0.45+0.04468}{0.02579}\right)$$

$$= \boxed{2.14 \times 10^{13} \ \text{cm}^{-3}}$$