UCLA Department of Electrical Engineering
ECE2H – Physics for Electrical Engineers
Winter 2020
Midterm, February 12 2020, (1:45 minutes)

Nam[redacted] Student number[redacted]

This is a closed book exam – you are allowed 1 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

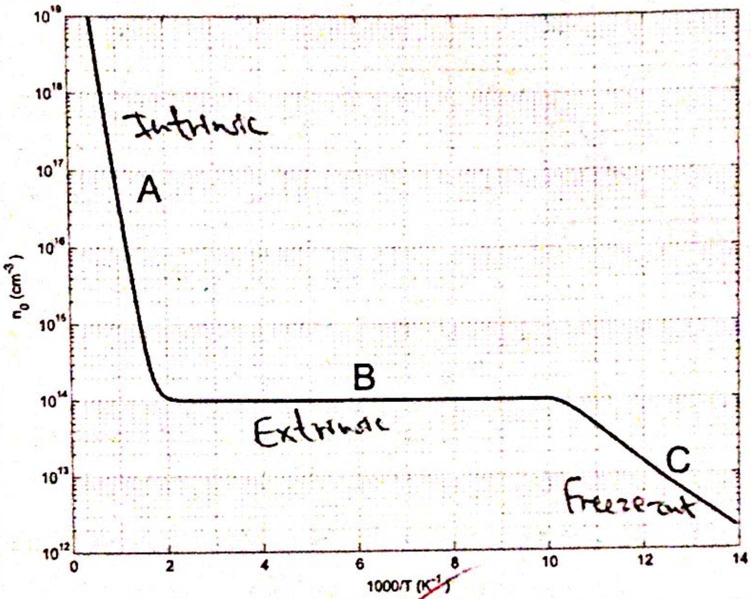
If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at least you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Carrier concentrations	35	35
Problem 2	Quantum Mechanics	20	10
Problem 3	Bands 1	15	15
Problem 4	Bands 2	30	26
Total		100	86

1. Carrier concentrations (35 points)

For this problem, consider the given plot of the equilibrium electron concentration n_0 versus temperature in a uniformly doped piece of silicon (plotted logarithmically in terms of inverse temperature 1000/T).



(a) (10 points) Label each region A, B, C as either intrinsic, extrinsic, or freeze-out.

A – Intrinsic

B – Extrinsic

C – Freeze-out

(b) (5 points) Is this an n–type or p-type piece of semiconductor? What is the donor or acceptor density? (give a number)

This is an n-type semiconductor.

$N_D = 10^{14}\ cm^{-3}$

Justification: In extrinsic state at room temperature

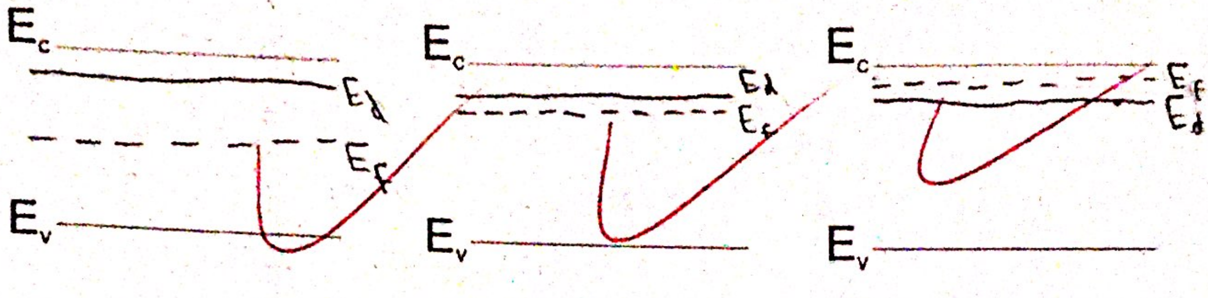
$n_0 = 10^{14}\ cm^{-3}$

$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5\times10^{10}\ cm^{-3})^2}{10^{14}\ cm^{-3}}$

$p_0 = 2.25\times10^{6}$

$p_0 \ll n_0$

(c) (10 points) On the band diagrams below, sketch the Fermi-level E_F for each case. Please also add a line which indicates the energy level of either the donors (E_d) or acceptor states (E_a) (depending on which type of semiconductor it is).



(a) Intrinsic

(b) Extrinsic

(c) Freeze-out

(d) (5 point) At T=300 K, what is the equilibrium hole concentration p_0? (give a number)

$\frac{1000}{300K} = 3.33\ K^{-1}$

$p_0 n_0 = n_i^2$

$p_0 = \frac{n_i^2}{n_0}$

$p_0 = \frac{(1.5\times10^{10}\ cm^{-3})^2}{10^{14}\ cm^{-3}}$

$\boxed{p_0 = 2.25\times10^{6}\ cm^{-3}}$

(e) (5 point) At T=300 K, what is the Fermi energy E_F? Express this in units of eV, relative to its distance from either one of the band edges E_c or E_v. (give a number)

$\boxed{E_c - E_F = 0.324\ eV}$

$n_0 = N_c e^{-(E_c-E_F)/kT}$

$\frac{n_0}{N_c} = e^{-(E_c-E_F)/kT}$

$\ln\left(\frac{n_0}{N_c}\right) = -(E_c - E_F)/kT$

$kT\ln\left(\frac{N_c}{n_0}\right) = E_c - E_F$

$k = 8.62\times10^{-5}\ eV/K$

$T = 300K$

$N_c = 2.8\times10^{19}\ cm^{-3}$

$n_0 = 10^{14}\ cm^{-3}$

2. Quantum Mechanics (20 points)

(a) (5 points) Consider a 1D metal of length L with free-electrons ($V(x)=0$) and periodic boundary conditions. What is the expectation value for momentum $<p>$ for a free electron with wavefunction $\psi_1(x)=\sqrt{\frac{1}{L}}e^{jk_0x}$?

$\langle p\rangle = \hbar \int_{-\infty}^{\infty} k|\Psi_1(k)|^2 dk$

Using periodic boundary conditions:

$\langle p\rangle = \hbar \int_0^L k|\Psi_1(k)|^2 dk$

$\langle p\rangle = \hbar \int_0^L k(\Psi_1(k))(\Psi_1^*(k))\,dk$

$\langle p\rangle = \hbar \int_0^L k\left(\frac{1}{L}\right) dk$

$\langle p\rangle = \frac{\hbar}{L}\left[\frac{k^2}{2}\right]_0^L$

-3

$\langle p\rangle = \frac{\hbar L^2}{2L}$

$\boxed{\langle p\rangle = \frac{\hbar L}{2}}$

(b) (5 points) Now consider an electron in a state with wavefunction $\psi(x)=\sqrt{\frac{1}{L}}(e^{jk_0x}+e^{-jk_0x})$.

What is the expectation value for momentum $<p>$?

$\langle p\rangle = \hbar \int_{-\infty}^{\infty} k|\Psi(k)|^2 dk$

-3

$|\Psi(k)|^2 = \Psi(k)\Psi^*(k)$

$= \sqrt{\frac{1}{L}}(e^{jk_0k}+e^{-jk_0k})\sqrt{\frac{1}{L}}(e^{-jk_0k}+e^{jk_0k})$

$= \frac{1}{L}(2+e^{-2jk_0k}+e^{2jk_0k})$

$\langle p\rangle = \hbar \int_0^L k(2+e^{-2jk_0k}+e^{2jk_0k})\,dk$

Periodic terms integrate to 0:

$\langle p\rangle = \frac{2\hbar}{L}\int_0^L k\,dk$

$\langle p\rangle = \frac{2\hbar}{L}\left(\frac{k^2}{2}\right)\Big|_0^L = \hbar L$

$\boxed{\langle p\rangle = \hbar L}$

(c) (5 points) With an electron in the same state with wavefunction $\psi_2(x) = \sqrt{\frac{1}{L}}\left(e^{jk_0x} + e^{-jk_0x}\right)$, if we perform a measurement of momentum, list the possible momentum values that might be measured along with their probability that they are measured?

3

We know that $p = \hbar k$.

Procedure: find all possible k-states using $k = n\frac{2\pi}{L}$ (integer n), then calculate all possible momentum states and their probabilities.

To calculate momentum probability: use $|\psi_2(x)|^2 dx$ to calculate the probability an electron is in a position then calculate the k-state (then momentum) corresponding to that position.

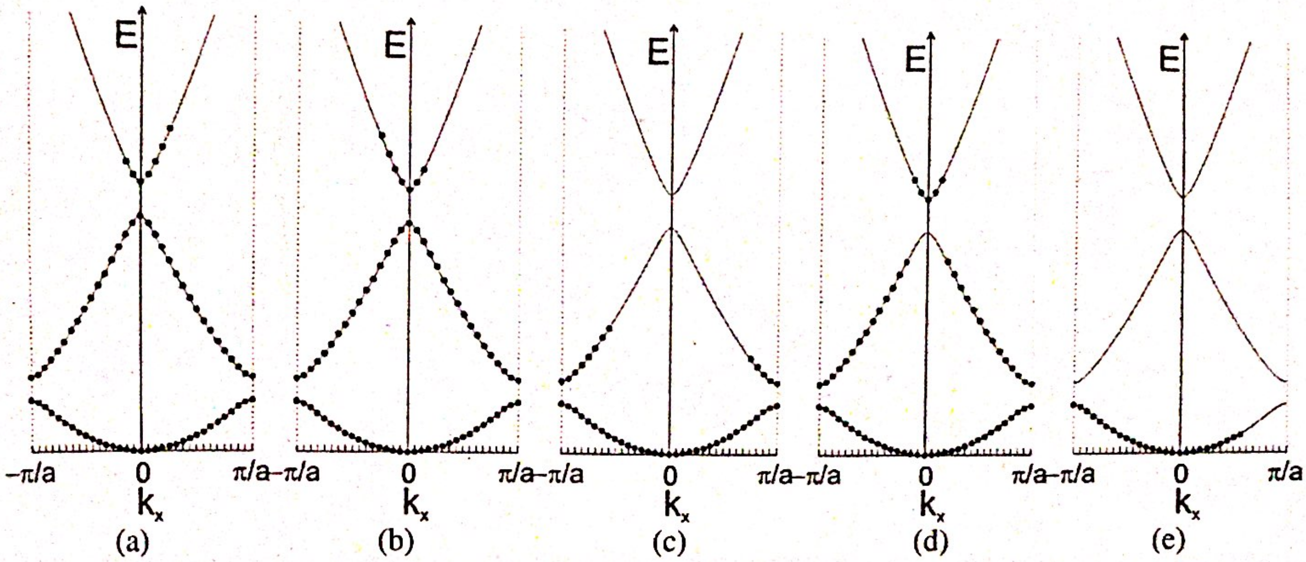
(d) (5 points) Which state, ψ_1 or ψ_2, has a larger uncertainty in momentum Δp ? Why?

~~ψ_1 has a larger uncertainty in momentum. Since ψ_2 has higher expectation of momentum than ψ_1, ψ_2 is more likely to take on~~

$\boxed{\psi_2(x) \text{ has greater uncertainty in momentum.}}$ The probability of finding an electron with wavefunction $\psi_2(x)$ between $x=0$ to $x=\frac{L}{2}$ is given by $\int_0^{L/2} |\psi_2(x)|^2 dx = 1$. The probability of finding an electron with wavefunction $\psi_1(x)$ between $x=0$ and $x=L$ is given by $\int_0^L |\psi_1(x)|^2 = 1$. 1

Thus, for an electron with wavefunction $\psi_2(x)$, we have a higher certainty in position than an electron with wavefunction $\psi_1(x)$. Thus the uncertainty in momentum for an electron with wavefunction $\psi_2(x)$ must be greater than the uncertainty in momentum for an electron with wavefunction $\psi_1(x)$.

3. Bands 1 (15 points)



Consider the following 1D electronic band structures with filled electronic states as indicated by the closed circles. For each case, write whether the electrical current density in the x-direction J_x is zero, positive, or negative.

(a) negative ✓

(b) positive ✓

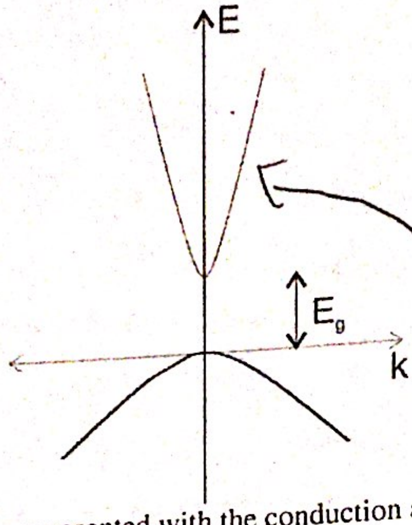
(c) negative ✓

(d) 0 ✓

(e) positive ✓

4. (30 points) Bands 2

26



Higher $\left|\frac{d^2E}{dk^2}\right| \Rightarrow$ Lower m^* $\Rightarrow$ Higher mobility

Consider the semiconductor represented with the conduction and valence bands shown above.

(a) (5 points) Does this material likely have larger electron or hole mobility? Why?

This material likely has a larger electron mobility. $\left|\frac{\partial^2 E}{\partial k^2}\right|$ is larger for the conduction band than for the conduction band. $m^* = \frac{\hbar^2}{\partial^2 E/\partial k^2}$, so the electrons in the conduction band have a smaller effective mass than the holes in the valence band. Since semiconductor mobility is inversely proportional to effective mass the mobility of electrons in the conduction band is likely larger than the mobility of holes in the valence band.

(b) (5 points) Does this material likely have a larger electron or hole effective density of states? Why?

This material likely has a larger hole effective density of states. As explained in part a, electrons in the conduction band have a smaller effective mass than the holes in the valence band. $N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$, and $N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$. Differences between N_c and N_v must arise from differences in effective mass; since the effective mass of holes in the valence band is greater than the effective mass of electrons in the conduction band, and since $N_v \propto (m_p^*)^{3/2}$ and $N_c \propto (m_n^*)^{3/2}$, the hole effective density of states is likely larger than the electron effective density of states.

(c) (10 points) Now consider that this semiconductor is intrinsic, and in equilibrium at T=300 K. If the temperature increases to 400 K, will the intrinsic Fermi level energy increase, decrease, or stay the same? Why? You may use any combination of text, equations, and sketches to argue your answer.

The intrinsic Fermi level energy will increase. Consider the equation $E_{Fi} = \frac{kT}{2}\ln\left(\frac{N_v}{N_c}\right) + \frac{E_v+E_c}{2}$. The term $\frac{E_v+E_c}{2}$ will be the same after the increase in temperature as it was before the increase in temperature. From part b, we know that $N_v > N_c$, so $\ln\left(\frac{N_v}{N_c}\right) > 0$. Thus an increase in temperature will cause the intrinsic Fermi level energy to increase, so the intrinsic Fermi level energy will increase if the temperature increases to 400 K.

(d) (10 points) Explain physically and qualitatively. Under what conditions can an electron have a negative effective mass? What is the physical meaning of a negative effective mass?

An electron can have negative effective mass if $\frac{d^2E}{dk^2} < 0$; in other words, if the energy of the band decreases as the magnitude of the wave number increases, the electron has negative effective mass. Physically, this means that if we apply an electric field, the electron will move in the direction of the electric field in k-space. (It should be noted that an electron with a positive effective mass will move opposite the direction of the electric field in k-space.)

Why?
Group vel?
Wave Packet?
Lattice?

6/10