SOLUTIONS

UCLA Department of Electrical Engineering
ECE2H – Physics for Electrical Engineers
Winter 2020
Midterm, February 12 2020, (1:45 minutes)

Name ______________________ Student number______________________

This is a closed book exam – you are allowed 1 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at least you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Carrier concentrations	35	
Problem 2	Quantum Mechanics	20	
Problem 3	Bands 1	15	
Problem 4	Bands 2	30	
Total		100	

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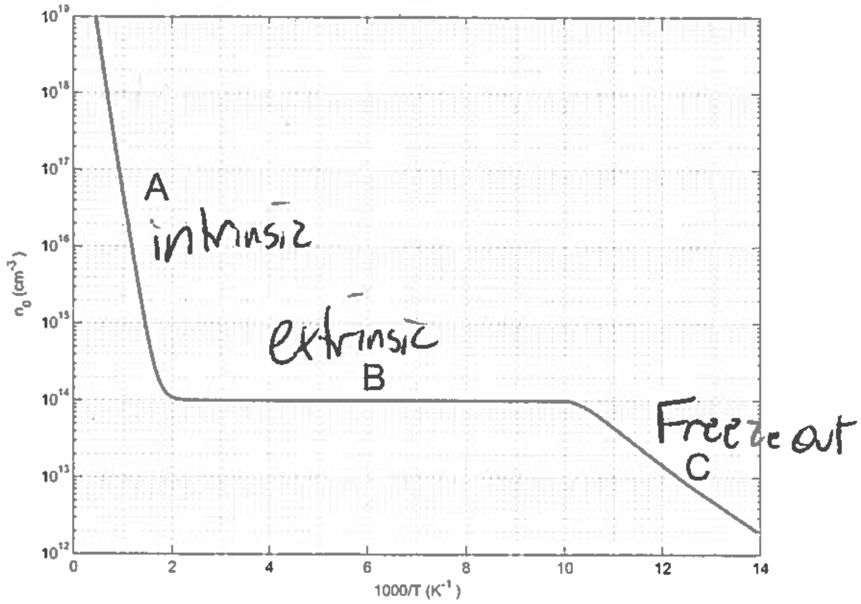
1. **Carrier concentrations** **(35 points)**

For this problem, consider the given plot of the equilibrium electron concentration n_0 versus temperature in a uniformly doped piece of silicon (plotted logarithmically in terms of inverse temperature 1000/T).



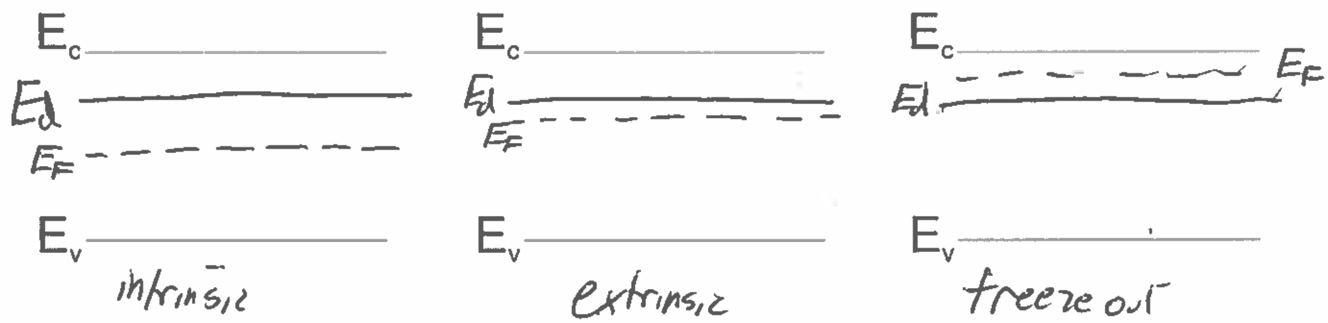
(a) (10 points) Label each region A, B, C as either intrinsic, extrinsic, or freeze-out.

(b) (5 points) Is this an n–type or p-type piece of semiconductor? What is the donor or acceptor density? (give a number)

n-type.

$N_D = 10^{14}$ cm^{-3}

(c) (10 points) On the band diagrams below, sketch the Fermi-level E_F for each case. Please also add a line which indicates the energy level of either the donors (E_d) or acceptor states (E_a) (depending on which type of semiconductor it is).



(a) (b) (c)

(d) (5 point) At T=300 K, what is the equilibrium hole concentration p_0? (give a number)

$n_i^2 = 2.3\times10^{20}$ cm^{-3} at 300K

$$p_0 = \frac{n_i^2}{n_0} = \frac{2.3\times10^{20}}{10^{14}} = 2.3\times10^{6}\ \text{cm}^{-3}$$

(e) (5 point) At T=300 K, what is the Fermi energy E_F? Express this in units of eV, relative to its distance from either one of the band edges E_c or E_v. (give a number)

Extrinsic. $n_0 = 10^{14} = N_c e^{-(E_c-E_F)/kT}$

$$-kT \ln\left(\frac{n_0}{N_c}\right) = E_c - E_F$$

$$-kT \ln\left(\frac{10^{14}}{2.8\times10^{19}}\right) = E_c - E_F = 0.326\ \text{eV}$$

2. Quantum Mechanics (20 points)

(a) (5 points) Consider a 1D metal of length L with free-electrons ($V(x)$=0) and periodic boundary conditions. What is the expectation value for momentum $<p>$ for a free electron with wavefunction $\psi_1(x) = \sqrt{\frac{1}{L}} e^{jk_0 x}$?

$\langle p \rangle = \hbar k_0$

(b) (5 points) Now consider an electron in a state with wavefunction $\psi(x) = \sqrt{\frac{1}{L}}\left(e^{jk_0 x} + e^{-jk_0 x}\right)$.

What is the expectation value for momentum $<p>$?

$\langle p \rangle = 0$

(c) (5 points) With an electron in the same state with wavefunction $\psi_2(x) = \sqrt{\frac{1}{L}}\left(e^{jk_0x} + e^{-jk_0x}\right)$, if we perform a measurement of momentum, list the possible momentum values that might be measured along with their probability that they are measured?

2 possible values: $+\hbar k_0$
$-\hbar k_0$

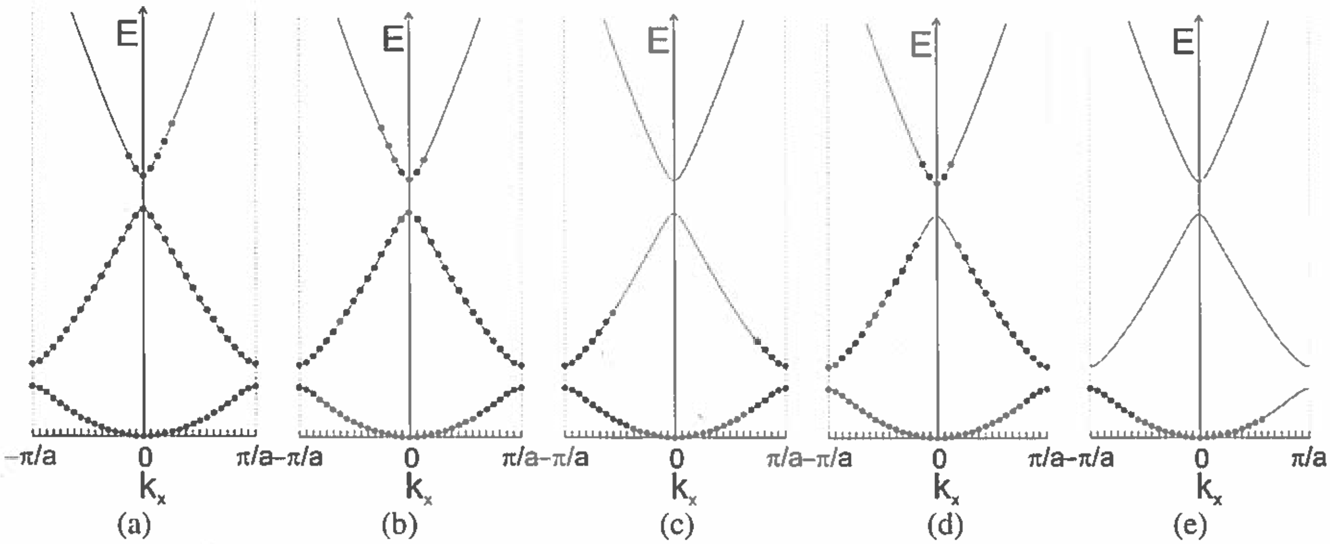
50% probability of each,

(d) (5 points) Which state, ψ_1 or ψ_2, has a larger uncertainty in momentum Δp ? Why?

State $\psi_2(x)$ has larger uncertainty, since it can have 2 possible measured values.

State $\psi_1(x)$ has $\Delta p = 0$.

3. Bands 1 (15 points)



Consider the following 1D electronic band structures with filled electronic states as indicated by the closed circles. For each case, write whether the electrical current density in the x-direction J_x is zero, positive, or negative.

(a) Neg

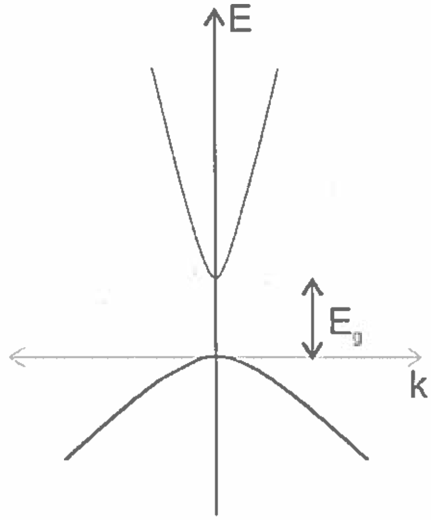
(b) Pos

(c) Neg

(d) Zero

(e) ~~An~~ Pos

4. (30 points) Bands 2



Consider the semiconductor represented with the conduction and valence bands shown above.

(a) (5 points) Does this material likely have larger electron or hole mobility? Why?

Larger electron mobility $\mu = \frac{e\bar{\tau}}{m^*}$ due to its smaller eff mass. $m^* = \hbar^2 / \frac{d^2E}{dk^2}$

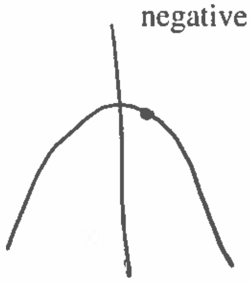
(b) (5 points) Does this material likely have a larger electron or hole effective density of states? Why?

Larger hole density of states due to its larger eff mass.

(c) (10 points) Now consider that this semiconductor is intrinsic, and in equilibrium at T=300 K. If the temperature increases to 400 K, will the intrinsic Fermi level energy increase, decrease, or stay the same? Why? You may use any combination of text, equations, and sketches to argue your answer.

The effective DOS is larger for holes. To maintain the condition that $n_0 = p_0$ for intrinsic semiconductors, the Fermilevel must increase as temp increases.

(d) (10 points) Explain physically and qualitatively. Under what conditions can an electron have a negative effective mass? What is the physical meaning of a negative effective mass?



Since $m^* = \hbar^2 / \frac{d^2E}{dk^2}$ an electron has a negative eff mass when it occupies a set of k-states with negative E curvature vs. k. This is because ~~a~~ while a positive force will increase the k-number, it will move it into a state with negative group velocity. This occurs fundamentally because the electron is a wave packet, which reflects from the distributed lattice potential.

5. Fundamental constants

Planck's constant:	h=6.63×10^{-34} J s	h=4.14×10^{-15} eV s
	$\hbar$=h/2π =1.06×10^{-34} J s	$\hbar$= h/2π =6.58×10^{-16} eV s
Permittivity of free space	ϵ_0=8.85×10^{-12} F/m	ϵ_0=8.85×10^{-14} F/cm
Permeability of free space	μ_0=4π × 10^{-7} Ns^2/C^2	
Conversion from eV to J	1 eV=1.60×10^{-19} J	
Boltzmann's constant	k=1.38×10^{-23} J/K	k=8.62×10^{-5} eV/K
Bare electron mass	m_0=9.11×10^{-31} kg	
Speed of light	c=2.998x10^8 m/s	c=2.998x10^{10} cm/s
Fundamental charge	e = 1.602×10^{-19} C	

1 Å=10^{-10} m, 1 nm=10^{-9} m, 1 μm=10^{-6} m.

Material properties

Silicon

All parameters at room temp	**Silicon**	**GaAs**
Crystal Structure	Diamond	Zincblende
a	5.43 Å	5.65 Å
Mass density	2.33 g/cm^3	5.31 g/cm^3
ϵ_r	11.8	13.2
E_g	1.11 eV	1.43 eV
μ_n	1350 cm^2/V s	8500 cm^2/V s
μ_p	480 cm^2/V s	400 cm^2/V s
m^*_n	0.26m_0	0.067m_0
m^*_p	0.49m_0	0.5m_0
Effective DOS N_c	2.8×10^{19} cm^{-3}	4.7×10^{17} cm^{-3}
Effective DOS N_v	1.0×10^{19} cm^{-3}	7.0×10^{18} cm^{-3}
n_i	1.5×10^{10} cm^{-3}	2x10^6 cm^{-3}

Useful equations

Electron momentum: $p = mv = \hbar k = h/\lambda$ Planck relation: $E = hf = \hbar\omega$

Time independent Schrödinger's Equation (1D): $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$

Expectation value for position $\langle x\rangle = \int_{-\infty}^{\infty} xP(x)dx = \int_{-\infty}^{\infty} x|\psi(x)|^2\,dx$, $\langle x^2\rangle = \int_{-\infty}^{\infty} x^2|\psi(x)|^2\,dx$.

Expectation value for momentum $\langle p\rangle = -j\hbar\int_{-\infty}^{\infty}\psi^*(x)\dfrac{\partial}{\partial x}\psi(x)dx = \hbar\int_{-\infty}^{\infty} k|\psi(k)|^2\,dk$,

Uncertainty $\Delta x = \sqrt{\left\langle (x-\langle x\rangle)^2\right\rangle} = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$ $\Delta p_x = \sqrt{\left\langle (p_x-\langle p_x\rangle)^2\right\rangle} = \sqrt{\langle p_x^2\rangle - \langle p_x\rangle^2}$

Heisenberg uncertainty principle: $(\Delta x)(\Delta p_x) \geq \dfrac{\hbar}{2}$

Solution to 1D particle in box (infinite quantum well of width L with boundary condition $\psi(0)=\psi(L)=0$)

$$\psi(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right) \text{ for } 0<x<L,\ \ \psi(x)=0 \text{ otherwise} \quad , E_n=\frac{n^2\hbar^2\pi^2}{2mL^2},\ \ n=1,2,...$$

Solution to 3D free particle in volume L^3: (with periodic boundary conditions)

$$\psi(x,y,z)=\frac{1}{L^{3/2}}e^{j\mathbf{k}\bullet\mathbf{r}}\ ,\ \ E=\frac{\hbar^2k^2}{2m}\quad k_x=n_x\frac{2\pi}{L},k_y=n_y\frac{2\pi}{L},k_z=n_z\frac{2\pi}{L},\ \ n_x,n_y,n_z=...-2,-1,0,1,2,...$$

Ohm's law: $\mathbf{J}=\sigma\boldsymbol{\mathcal{E}}$

Conductivity of free electron gas with one carrier type (i.e. metal): $\sigma=\dfrac{ne^2\overline{\tau}}{m}$ $\qquad \sigma=\rho^{-1}$

Conductivity of semiconductor (with two carrier types): $\sigma=ne\mu_n+pe\mu_p$

Semiconductor electron/hole mobility $\qquad \mu_n=\dfrac{e\overline{t}_n}{m_n^*} \qquad \mu_p=\dfrac{e\overline{t}_p}{m_p^*}$

3D free electron Density of States $\qquad N(E)=\dfrac{1}{2\pi^2}\left(\dfrac{2m^*}{\hbar^2}\right)^{3/2}E^{1/2}$

Electron Effective mass $\qquad m^*=\dfrac{\hbar^2}{d^2E/dk^2}$

Fermi-Dirac distribution for electrons: $f(E)=\dfrac{1}{e^{(E-E_F)/kT}+1}$ and for holes: $f_h(E)=1-f(E)$

Equilibrium Carrier concentrations $\qquad n_0=\displaystyle\int_{E_C}^{\infty}f(E)N(E)dE$

Equilibrium Carrier concentrations in non-degenerate limit (E_C-$E_F \gg kT$ and E_F-$E_V \gg kT$).

$$n_0=N_Ce^{-(E_C-E_F)/kT}\quad ,\ p_0=N_Ve^{-(E_F-E_V)/kT}$$

Effective Density of States $\qquad N_C=2\left(\dfrac{2\pi m_n^*kT}{h^2}\right)^{3/2}\quad ,\ N_V=2\left(\dfrac{2\pi m_p^*kT}{h^2}\right)^{3/2}$

Intrinsic carrier density $\qquad n_i=\sqrt{n_0p_0}=\sqrt{N_CN_V}e^{-E_g/2kT}$

Intrinsic Fermi Level $\qquad E_{Fi}=\dfrac{kT}{2}\ln\left(\dfrac{N_V}{N_C}\right)+\dfrac{E_V+E_C}{2}$

Einstein relation for diffusion coeff: $D_n=\dfrac{kT}{e}\mu_n,\quad D_p=\dfrac{kT}{e}\mu_p$ $\quad$ Diffusion length $\qquad L=\sqrt{D\tau}$

Debye screening length (for n-type): $\qquad L_D=\sqrt{\dfrac{\epsilon kT}{e^2n_0}}$

Dielectric relaxation time $\qquad \tau_D=\dfrac{\epsilon}{\sigma}=\dfrac{\epsilon}{n_0e\mu_n}$ (n-type) $=\dfrac{\epsilon}{p_0e\mu_p}$ (p-type)

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