UCLA Department of Electrical Engineering
EE2H – Physics for Electrical Engineers
Winter 2018
Midterm, February 7 2018, (1:45 minutes)

Name

Student number

This is a closed book [illegible] 1 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at least you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Carrier concentrations	30	30
Problem 2	Quantum Mechanics	30	30
Problem 3	Bands and Fermi levels	40	~~28~~ 35
Total		100	~~88~~ 95

RW

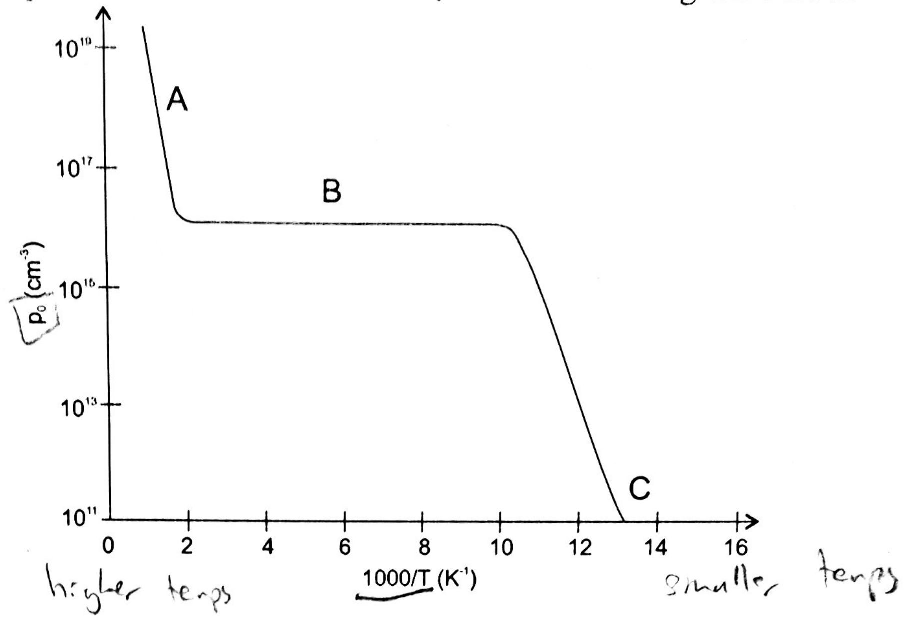
EE2H – Physics for Electrical Engineers

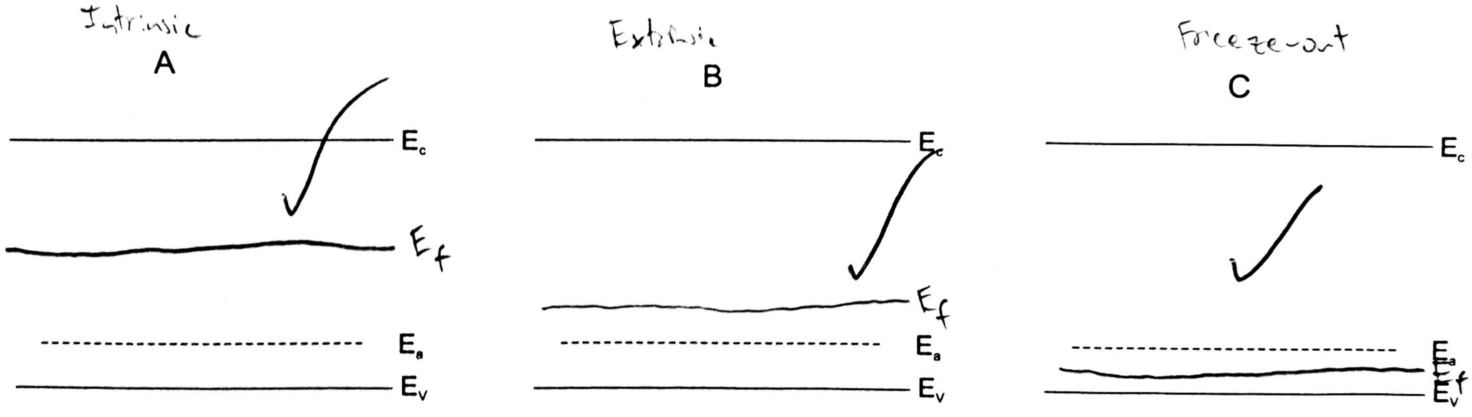
Midterm

1. Carrier concentrations (30 points)

30/30

(a) (15 points) Consider the given plot of the equilibrium hole concentration p_0 versus temperature in Si (plotted logarithmically in terms of inverse temperature 1000/T). For each region of the plot A, B, C, sketch the position of the Fermi level E_F on the band diagram below.





15/15

(b) (7 points) Estimate the value of the acceptor doping density N_A.

In region B it is in the extrinsic region. As T isn't large enough for a lot electrons to jump out of the valence band into the conduction band, almost all of the holes are due to electrons jumping from E_V to E_A. thus N_A is about p_0 in the extrinsic region B.

Therefore $\boxed{N_A \simeq 10^{16}\ cm^{-3}}$ ✓

(c) (8 points) Why does p_0 dramatically decrease in region C?

Region C is the freeze-out zone. ✓ At these small temperatures the electrons in the valence band have so little thermal energy they can't even make the jump to the E_A zone to create a hole in the valence band. Because of this the effect of the p-type doping is drastically reduced, leading to such a low p_0.

2. Quantum Mechanics (30 points)

10

(a) (10 points) The Pauli exclusion principle states that for particles such as electrons, only one electron may occupy each quantum state. Explain how this rule is relevant to understanding why a semiconductor exists (i.e. how a semiconductor is different than a metal or insulator).

A semiconductor a) has its E_F in the bandgap between E_v + E_c (unlike a metal) and b) has a bandgap that is small enough to reasonably allow an electron to jump the gap (unlike an insulator). By adjusting E_F one can make a semiconductor conduct electricity or act like an insulator. The Pauli Exclusion principle (PEP) [illegible] [illegible] the PEP allows us to generally know which bands are full. Without PEP the electrons could share states, making it impossible to effectively determine the amount of energy needed for an electron to jump across the bandgap. Without this we wouldn't be able to explain why or how semiconductors exist. It also helps explain that full bands can't conduct as there is no "room" for electrons to be "pushed" by an electric field.

(b) (10 points) Under what condition might an electron wavepacket might have momentum and velocity of opposite sign?

As $p = mv$, if an electron wavepacket's momentum + velocity have opposite sign, that means that the mass of the electron wavepacket must be negative.

As $m^* = \frac{\hbar^2}{d^2E/dk^2}$, that means that $\frac{d^2E}{dk^2}$ must be negative at k_0 (the average k of the wavepacket.

10

(c) (10 points) Consider the 3D Hydrogen atom, with state energies $E_n = -13.6/n^2$ eV, and compare it to a free electron in a 1D quantum well of width L with infinitely high barriers. What should the width L be so that $E_2 - E_1$ is equal for both the Hydrogen atom and quantum well? Give a number.

3D Hydrogen Atom: $E_2 - E_1 = \frac{-13.6}{2^2} - \frac{-13.6}{1^2} = 10.2$ eV

1D Quantum Well: $E_2 - E_1 = \frac{2^2 \hbar^2 \pi^2}{2mL^2} - \frac{1^2 \hbar^2 \pi^2}{2mL^2} = \frac{3\hbar^2 \pi^2}{2mL^2}$

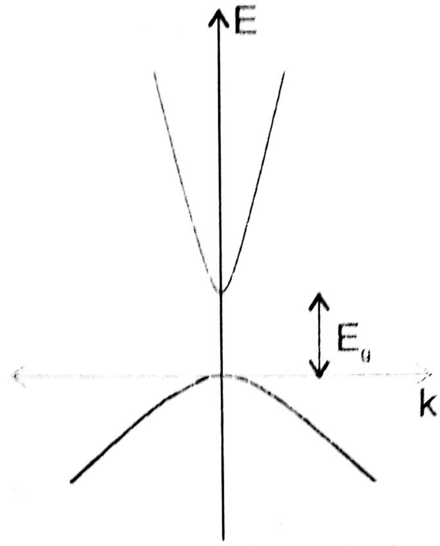
$\frac{3\hbar^2 \pi^2}{2mL^2} = 10.2 \text{ eV} = 1.632 \times 10^{-18}$ J

$L = 3.34 \times 10^{-10}$ m

$\boxed{L = 0.334 \text{ nm}}$

10

3. Bands and Fermi levels (40 points)



Consider the semiconductor represented with the conduction and valence bands shown above.

(a) (10 points) Does this material likely have larger electron or hole mobility? Why?

$\left|\frac{d^2E}{dk^2}\right|$ is greater for the electrons. As $m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$, electrons tend to have a smaller effective mass. As $\mu = \frac{e\bar{t}}{m^*}$ the electrons likely have a larger mobility than the holes.

10/10

(b) (10 points) Consider that this semiconductor is extrinsic, doped with donors such that $N_D=10^{15}$ cm^{-3}. If the temperature increases from 200K to 300K, will E_F increase, decrease, or stay the same? Why? You may use any combination of text, equations, and sketches to argue your answer.

As $f(E) = \frac{1}{e^{(E-E_F)/kT}+1}$, as T increases, the head and tail of the f(E) will "smear" out as shown in Figure A. As $n \approx N_D$ for extrinsic semiconductor & $n = \int_{E_c}^{\infty} f(E)N(E)dE$, in order for n to remain constant while the f(E) "smears" out (with a higher probability of more electrons in conduction band and less in valence band), the f(E) must shift left (i.e. E_f must decrease), as shown in Figure B.

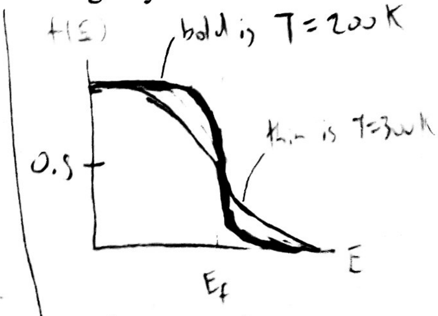


Figure A

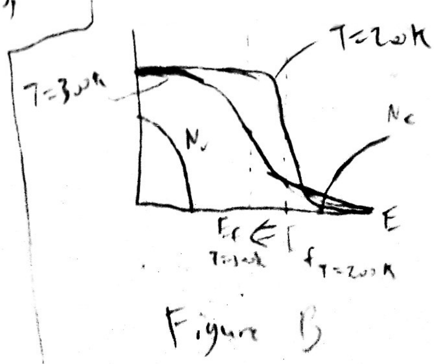


Figure B

~~3/10~~

10/10

(c) (10 points) Now consider that this semiconductor is intrinsic, and in equilibrium at T=300 K. If the temperature increases to 400 K, will the intrinsic Fermi level energy increase, decrease, or stay the same? Why? You may use any combination of text, equations, and sketches to argue your answer.

Intrinsic Fermi Level is $E_i = \frac{kT}{2} \ln\left(\frac{N_v}{N_c}\right) + \frac{E_v + E_c}{2}$. As E_v + E_c are independent of T, only first term matters. Looking at the N_v and N_c, N_c is directly proportional to m_n^* and N_v is directly proportional to m_p^*. As proved in part ⓐ, the electrons have a smaller effective mass than the holes (i.e, $m_n^* < m_p^*$). Thus $N_v > N_c$. Therefore $\ln\left(\frac{N_v}{N_c}\right)$ is a positive number, so when T increases, the first term increases, meaning that E_i increases.

10/10

(d) (10 points) Consider that the Fermi level E_F of this material is fixed at 0.1 eV above the valence band edge, i.e. E_F-E_V=0.1 eV. If the temperature increases from 200 K to 300 K, what is the ratio of the hole concentrations $p(300K)/p(200K)$? Give a number.

$E_F - E_v = 0.1$ eV
$= 1.6 \times 10^{-20}$ J

$$p = N_v e^{-(E_F - E_v)/kT}$$

$$\frac{p(300)}{p(200)} = \frac{N_v e^{-1.6\times10^{-20}/300k}}{N_v e^{-1.6\times10^{-20}/200k}} = \frac{e^{\frac{-1.6\times10^{-20}}{300k}}}{e^{-\frac{1.6\times10^{-20}}{200k}}} = \boxed{6.9}$$

5/10