SOLUTIONS

UCLA Department of Electrical Engineering

EE2 ~~EE101 – Engineering Electromagnetics~~

Spring 2012

Midterm, May 8 2012, (1:45 minutes)

Name ______________________________ Student number______________________________

This is a closed book exam – you are allowed 1 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	1D crystal	30	
Problem 2	Quantum Mechanics	30	
Problem 3	Carrier Concentrations	40	
Total		100	

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1. 1D crystal (30 points)

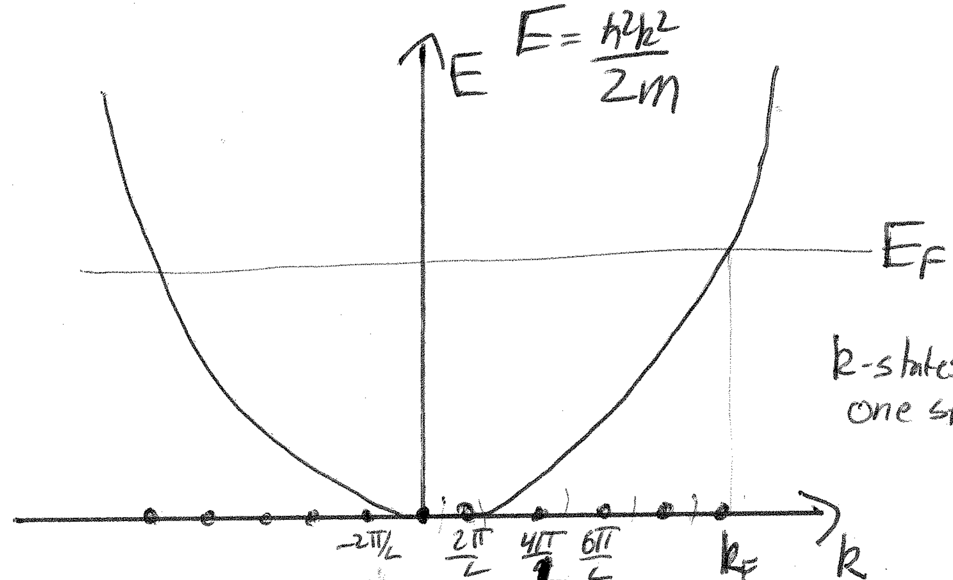
Consider a 1D crystal of total length L, made up of atoms with a lattice constant (spacing) of a.

(a) (15 points) Assume that each atom contributes one valence electron. First, using the free electron model, what is the Fermi level E_F at absolute zero temperature?

In the free electron model, the solutions to the Schrödinger Eq gives: $\psi(x) = \frac{1}{\sqrt{L}} e^{jkx}$

where $k = n \frac{2\pi}{L}$ $n = \ldots -2, -1, 0, 1, 2, \cdots$

$E = \frac{\hbar^2 k^2}{2m}$



k-states spaced one state every $\frac{2\pi}{L}$.

At $T=0$, electrons will fill up the lowest N states in energy where $N = L/a$ is the number of valence electrons.

This will occupy states upto energy E_F and wavenumber k_F. $\frac{\hbar^2 k_F^2}{2m} = E_F$

Each k-state has a spin degeneracy of 2.

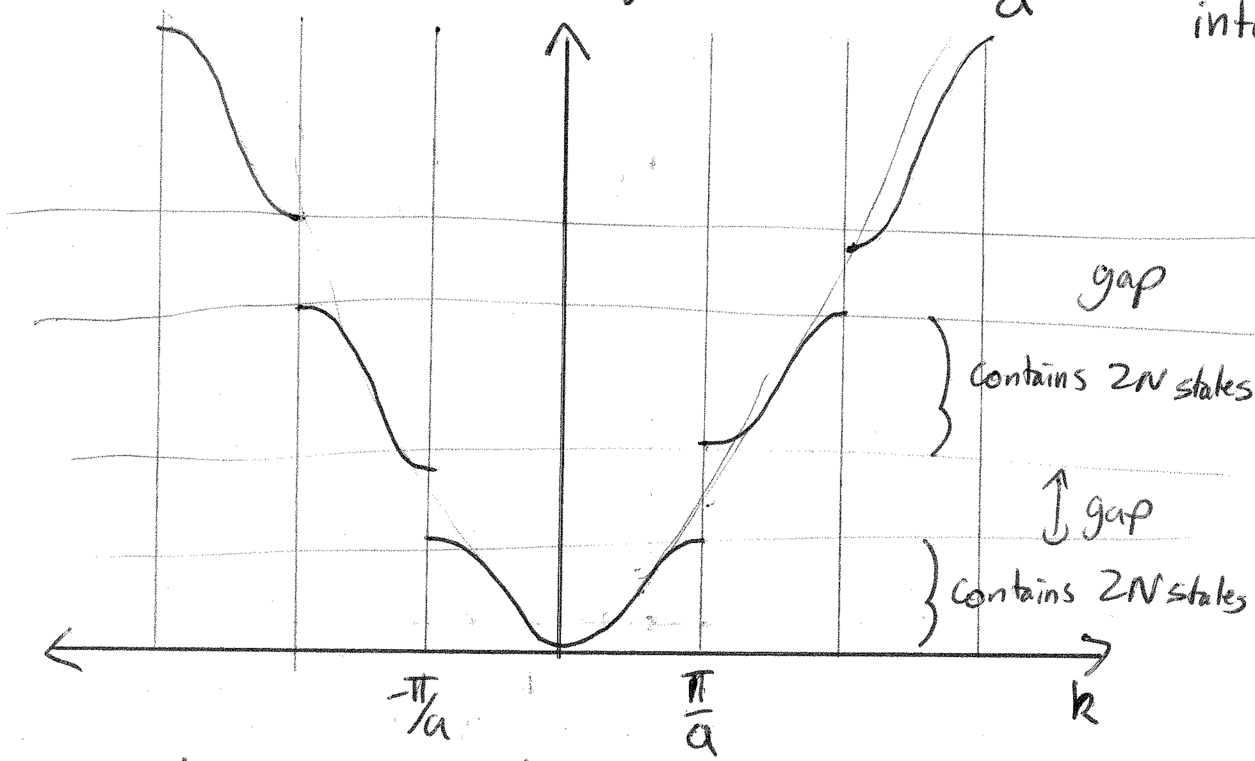
Density of states per unit k is $\underset{\substack{\uparrow \\ \text{spin}}}{\frac{1}{2\pi/L} \times 2} = \frac{2L}{\pi}$

$N = \frac{L}{a} = k_F \frac{2L}{\pi} \Rightarrow k_F = \frac{\pi}{2a}$ (halfway to Brillouin zone edge)

$$\boxed{E_F = \frac{\hbar^2 k_F^2}{2m} = \frac{\hbar^2 \pi^2}{2m\,4a^2} = \frac{\hbar^2 \pi^2}{8ma^2}}$$

(b) (15 points) Explain why a 1D crystal (length L, lattice constant a) made up of a single species of atom can only be a semiconductor if each atom has an even number of valence electrons. While you may cite any necessary equations, you must clearly give an explanation in text.

In a crystal with lattice spacing a, the periodic nature of the lattice potential will open up bandgaps at the Brillioun zone edges at $k = \pm \frac{n\pi}{a}$ where n is an integer.



Each band contains a total of 2N states, where $N = L/a$ is the number of atoms in a crystal. If each atom contains an even number of electrons, at $T=0$ E_F will be in the gap and a band will be completely filled. This is the condition for a semiconductor (or insulator). If each atom has an odd number of electrons E_F will be in the middle of a band, which is the condition for a metal.

2. Quantum Mechanics (30 points)

(a) (15 points) Consider an electron (with effective mass m^*=0.067m_0) in a 1D quantum well of width L=1 nm with infinitely high barriers. If the electron is in state n=2, and makes a transition to state n=1 by emitting a photon, what will the energy (given in units eV) and wavelength (given in units μm) of the photon be?

Infinite well: $E_n = \frac{\hbar^2 \pi^2}{2m^* L^2} n^2 = \frac{(1.06\times10^{-34})^2 \pi^2 \quad n^2}{2\times0.067\times9.1\times10^{-31}\times(10^{-9})^2}$

(in units J)

divide by 1.6×10^{-19} to get units eV.

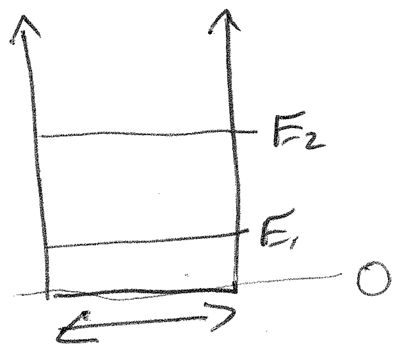
$E_1 = 5.62\ eV$

$E_2 = 22.5\ eV$

$E_2 - E_1 = \boxed{16.9eV = \hbar\omega} = 2.7\times10^{-18}\ J$

$\boxed{\lambda = 0.074\ \mu m}$

(b) (15 points) Explain why an electron in a quantum well cannot have zero kinetic energy ($E_1 \neq 0$). Explain what this has to do with the uncertainty principle (using text, and any relevant equations if needed).



$$E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

Because $E_1 > 0$, the electron has nonzero kinetic energy ($V = 0$ inside well).

An electron with $E_1 = 0$ would have zero kinetic energy. It would also have zero momentum p and zero uncertainty in momentum: $\Delta p = 0$.

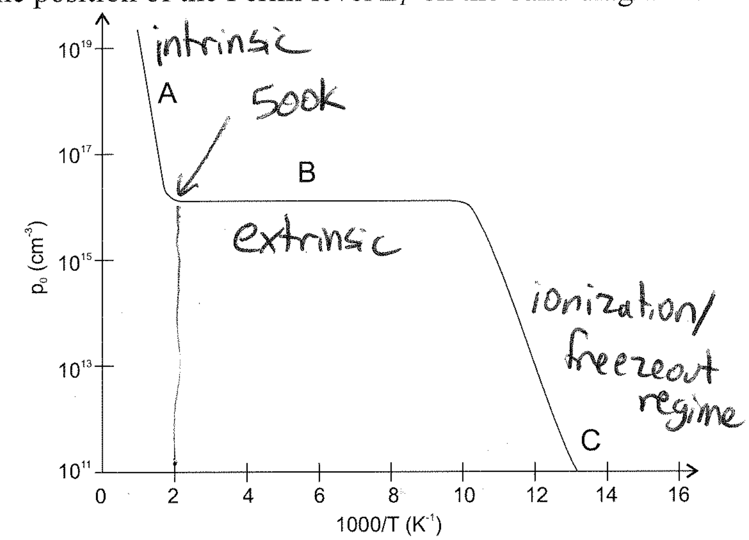
Because we know the electron is localized in the well we know its position uncertainty is finite: $\Delta x \neq \infty$

Therefore $\Delta x \Delta p = 0$ which would violate the Heisenberg uncertainty principle. $E_n = 0$ is a forbidden energy.

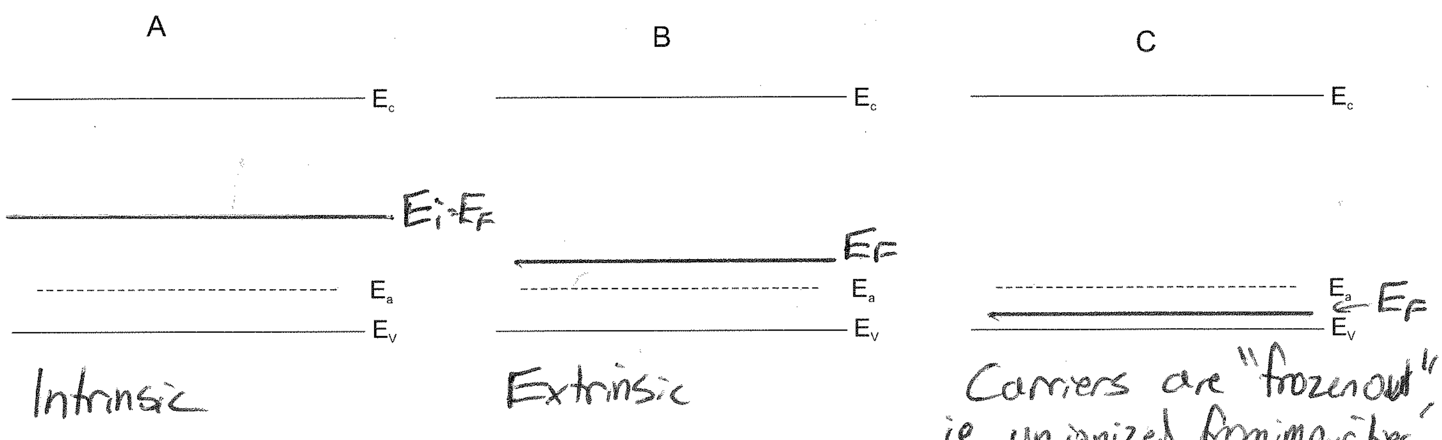
Alternatively, you can argue having $E_n = 0$ would require $n = 0$ and a wave function $\psi(x) = \sqrt{\frac{2}{L}} \sin(0) = 0$. This corresponds to no electron in the well and cannot be a solution.

3. Equilibrium carrier concentrations (40 points)

(a) (10 points) Consider the given plot of the equilibrium hole concentration p_0 versus temperature in Si (plotted logarithmically in terms of inverse temperature 1000/T). For each region of the plot A, B, C, sketch the position of the Fermi level E_F on the band diagram below.



1 of 3 = 4 pts
2 of 3 = 7 pts
3 of 3 = 10 pts



(b) (10 points) Estimate the value of the acceptor doping density N_A.

We can estimate N_A from the hole density in the extrinsic regime where $p_0 \approx N_A$ and the impurities are fully ionized.

$$\boxed{N_A \approx 10^{16}\ cm^{-3}}$$

(c) (10 points) At the temperature T=500 K, what is the energy of the Fermi level E_F (in eV) with respect to the valence band edge?

$$p_0 = N_V e^{-(E_F - E_V)/kT} = 10^{16}\ cm^{-3}$$

$$-kT \ln \frac{p_0}{N_V} = E_F - E_V \quad \leftarrow 4\text{ pts}$$

$kT = 43$ meV at 500 K

$N_V = 1.0 \times 10^{19}\ cm^{-3}$ at $T = 300K$ ← 3 pts

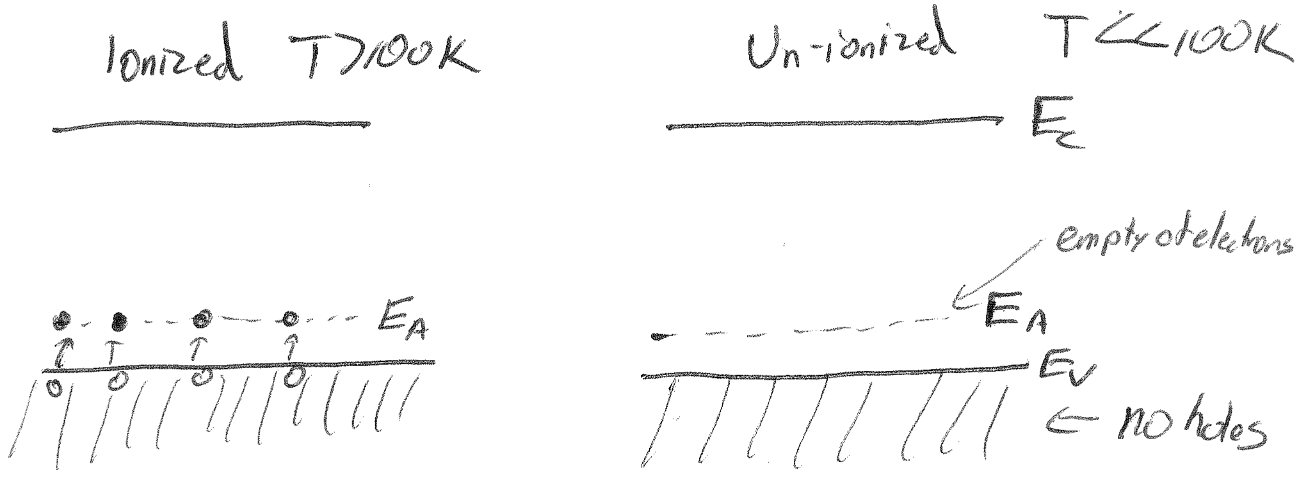
Since $N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$ we must scale by a factor of ... $\left(\frac{500}{300}\right)^{3/2}$ to get N_V at 500K.

$N_V(T=500K) = 2.15 \times 10^{19}\ cm^{-3}$

$$\boxed{E_F - E_V = -(0.043) \times \ln\left(\frac{10^{16}}{2.2 \times 10^{19}}\right) = 0.33\ eV} \quad \leftarrow 3\text{ pts}$$

(d) (10 points) Why does p_0 dramatically decrease in region C?

p_0 decreases as holes are frozen out at low temperatures. In other words electrons fall into the valence band, leaving the impurities (acceptor) un-ionized.



Conductivity of semiconductor: $\sigma = ne\mu_n + pe\mu_p$

Semiconductor electron/hole mobility $\mu_n = \frac{ne\overline{t_n}}{m_n^*}$ $\mu_p = \frac{pe\overline{t_p}}{m_p^*}$

3D free electron Density of States $N(E) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2} E^{1/2}$

Electron Effective mass $m^* = \frac{\hbar^2}{d^2E/dk^2}$

Fermi-Dirac distribution for electrons $f(E) = \frac{1}{e^{(E-E_F)/kT}+1}$

Equilibrium Carrier concentrations $n_0 = \int_{E_C}^{\infty} f(E)N(E)dE$

Equilibrium Carrier concentrations in non-degenerate limit (E_C-$E_F \gg kT$ and E_F-$E_V \gg kT$).

$$n_0 = N_C e^{-(E_C-E_F)/kT} \quad , \; p_0 = N_V e^{-(E_F-E_V)/kT}$$

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \quad , \; N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

$$n_i = \sqrt{n_0 p_0} = \sqrt{N_C N_V}\, e^{-E_g/2kT}$$

Intrinsic Fermi Level $E_i = \frac{kT}{2}\ln\left(\frac{N_V}{N_C}\right) + \frac{E_V+E_C}{2}$

Einstein relation for diffusion coeff: $D = \frac{kT}{e}\mu$ Diffusion length $L = \sqrt{D\tau}$

Debye screening length (for n-type): $L_D = \sqrt{\frac{\epsilon kT}{e^2 n_0}}$

Dielectric relaxation time $\tau_D = \frac{\epsilon}{\sigma} = \frac{\epsilon}{n_0 e\mu_n}$ (n-type) $= \frac{\epsilon}{p_0 e\mu_p}$ (p-type)

Minority carrier rate equations for excess nonequilibrium carrier generation (low-level injection)

$$\frac{d(\delta n)}{dt} = g_{op} - \frac{\delta n}{\tau_n} \text{ (p-type) ,} \qquad \frac{d(\delta p)}{dt} = g_{op} - \frac{\delta p}{\tau_p} \text{ (n-type)}$$

Continuity equations for excess minority carriers (low-level injection, no E-fields, no optical generation)

$$\frac{d(\delta n)}{dt} = D_n\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_n} \text{ (p-type) ,} \qquad \frac{d(\delta p)}{dt} = D_n\frac{d^2(\delta p)}{dx^2} - \frac{\delta p}{\tau_p} \text{ (n-type)}$$

Fundamental constants

Planck's constant:	h=6.63×10^{-34} J s	h=4.14×10^{-15} eV s
	$\hbar$=1.06×10^{-34} J s	$\hbar$=6.58×10^{-16} eV s
Permittivity of free space	ϵ_0=8.85×10^{-12} F/m	ϵ_0=8.85×10^{-14} F/cm
Permeability of free space	μ_0=4π × 10^{-7} Ns^2/C^2	
Conversion from eV to J	1 eV=1.60×10^{-19} J	
Boltzmann's constant	k=1.38×10^{-23} J/K	k=8.62×10^{-5} eV/K
Bare electron mass	m_0=9.11×10^{-31} kg	
Speed of light	c=2.998x10^8 m/s	c=2.998x10^{10} cm/s
Fundamental charge	e = 1.602×10^{-19} C	

1Å=10^{-10} m, 1 nm=10^{-9} m, 1 μm=10^{-6} m.

Material properties

Silicon

All parameters at room temp	Silicon	GaAs
Crystal Structure	Diamond	Zincblende
a	5.43 Å	5.65 Å
Mass density	2.33 g/cm^3	5.31 g/cm^3
ϵ_r	11.8	13.2
E_g	1.11 eV	1.43 eV
μ_n	1350 cm^2/V s	8500 cm^2/V s
μ_p	480 cm^2/V s	400 cm^2/V s
m^*_n	0.26m_0	0.067m_0
m^*_p	0.49m_0	0.5m_0
Effective DOS N_c	2.8×10^{19} cm^{-3}	4.7×10^{17} cm^{-3}
Effective DOS N_v	1.0×10^{19} cm^{-3}	7.0×10^{18} cm^{-3}
n_i	1.5×10^{10} cm^{-3}	2×10^6 cm^{-3}

Useful equations

Electron momentum: $p = mv = \hbar k = h/\lambda$ Planck relation: $E = hf = \hbar\omega$

Time independent Schrodinger's Equation (1D): $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$

Heisenberg uncertainty principle: $(\Delta x)(\Delta p_x) = \frac{\hbar}{2}$

Solution to 1D particle in box (infinite quantum well of width L):

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right) \quad , \ E = \frac{n^2\hbar^2\pi^2}{2mL^2}, \ n = 0,1,2,...$$

Solution to 3D free particle in volume L^3:

$$\psi(x,y,z) = \frac{1}{L^{3/2}}e^{j\mathbf{k}\bullet\mathbf{r}} \ , \ E = \frac{\hbar^2k^2}{2m} \quad k_x = n_x\frac{2\pi}{L}, k_y = n_y\frac{2\pi}{L}, k_z = n_z\frac{2\pi}{L}, \ n_x, n_y, n_z = ... -2,-1,0,1,2,...$$

Ohm's law: $\mathbf{J} = \sigma\boldsymbol{\mathcal{E}}$

Conductivity of free electron gas (i.e. metal): $\sigma = \frac{ne^2\bar{t}}{m}$ $\sigma = \rho^{-1}$

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