SOLUTIONS

UCLA Department of Electrical Engineering
EE2 – Physics for Electrical Engineers
Final Exam 2013
June 12 2013, (3 hours)

Name ________________________ Student number ________________________

This is a closed book exam – you are allowed 2 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	p-n diode basics	25	
Problem 2	Conduction	20	
Problem 3	Equilibrium Carrier Concentrations	25	
Problem 4	Switching Dynamics	30	
Problem 5			
Total		100	

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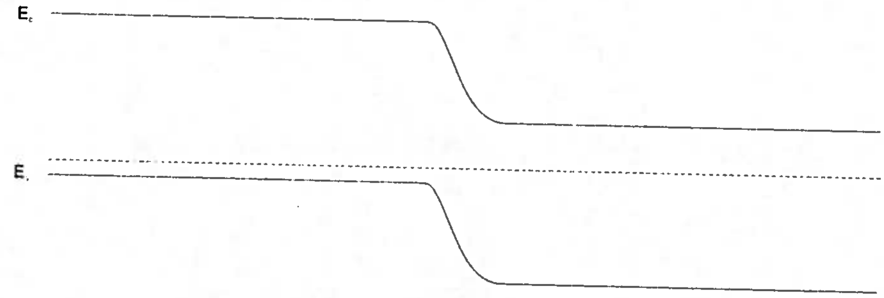
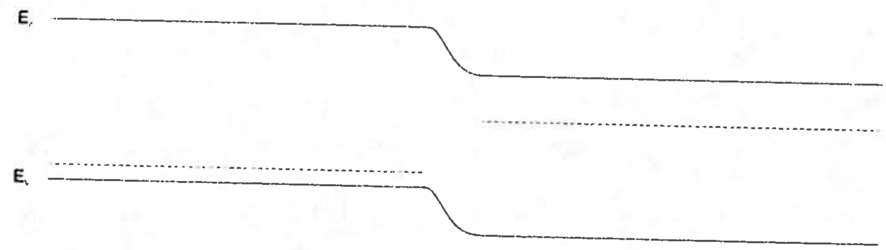
1. *p-n* diode basics (~~20~~ 25 points)

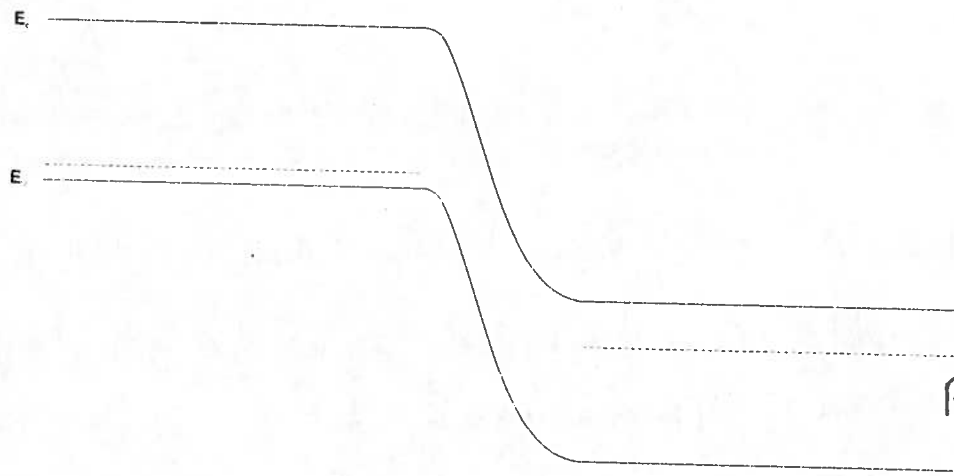
(a) (6 points) Label next to each of the 3 diagrams, whether this shows a *p-n* diode in forward, reverse, or zero bias.



Zero



Forward



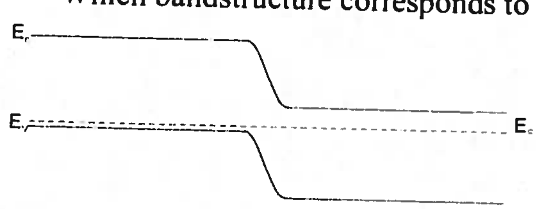
Reverse

(b) (4 points) Consider two possible p-n diodes: p+/n and p/n+, which have either doping:

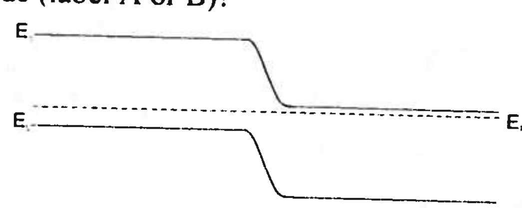
Device A: p+/n: $N_A=10^{17}$ cm^{-3}, $N_D=10^{15}$ cm^{-3}.

Device B: p/n+: $N_A=10^{15}$ cm^{-3}, $N_D=10^{17}$ cm^{-3}.

Which bandstructure corresponds to which diode (label A or B)?

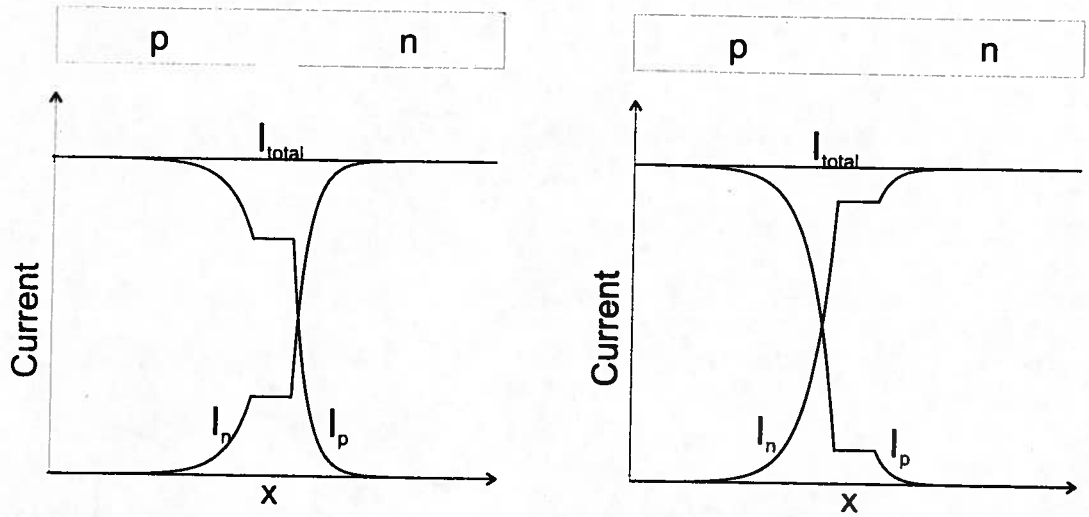


A: p+/n



B: p/n+

(c) (~~10~~ 7 points) Below is a figure which plots the current carried by electrons and holes separately as a function of position for two silicon p-n diodes with different doping levels. Indicate, which diode has a larger doping ratio N_D/N_A? (where N_D is the donor doping on the n-side, and N_A is acceptor doping on p-side).



This structure has a larger ratio of $I_n(-x_{po})/I_p(x_{no})$, which means $\frac{N_D}{N_A}$ is larger.

(d) (~~10~~ 8 points) Explain qualitatively: If you wish to minimize the depletion region generation current in reverse bias, is it better to choose low doping or high doping for your p-n diode? Why?

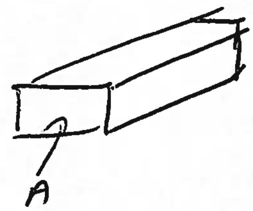
It is better to choose high doping, since this will mimize the depletion region thickness. Since the generation rate per volume is approximately independent, a thinner region gives less volume for generation of electron hole pairs, and less generation current.

2. Conduction in semiconductor (20 points)
   (a) (10 points) At room temperature, a silicon bar that is 1 cm long, with a width of 10 μm and a height of 1 μm is doped uniformly with donors such that N_D=10^{14} cm^{-3}. A voltage of 10 V is applied across the length of the bar. Calculate the current I that flows.

$\sigma = ne\mu_n$ $\mu_n = 1350\ cm^2/Vs$

$n = N_D$

$R = \frac{L}{A\sigma}$ $A = 10^{-7}\ cm^2$



$I = \frac{V}{R} = \frac{VAne\mu}{L} = 2.16\times10^{-8} A = 21.6 nA$

   (b) (10 points) Now imagine that this bar is uniformly illuminated with light causing a generation rate g_{op}=10^{15} cm^{-3}. Will the current I change appreciably (say by more than 1%)?

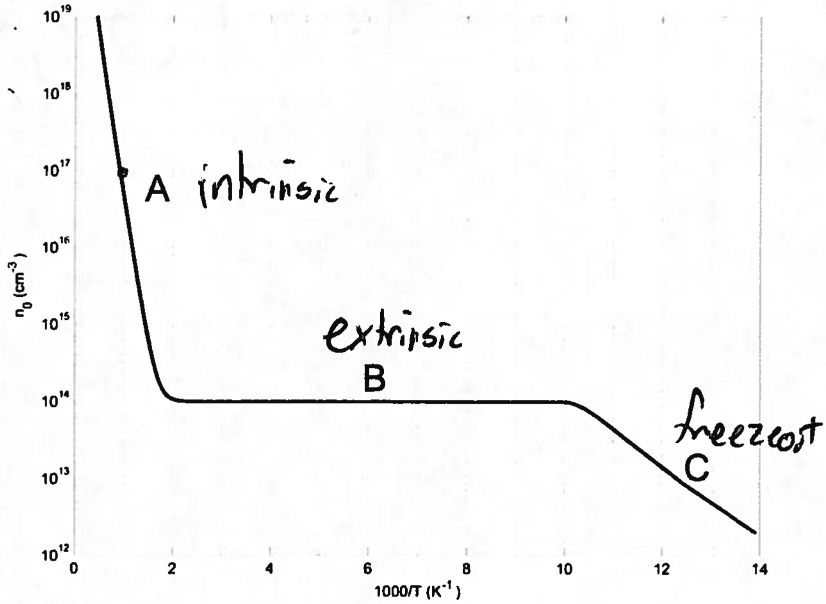
Assume low level injection: Chart shows $\tau_n \approx 3\times10^{-4} s$

$g_{op} = \frac{\delta n}{\tau_n}$ $\delta n = 10^{15} \times 3\times10^{-4} \approx 3\times10^{11} cm^{-3}$

The carrier concentration changes by less than 1%, so I will not change by more than 1%.

3. Equilibrium carrier concentrations (25 points)

For this problem, consider the given plot of the equilibrium electron concentration n_0 versus temperature in a uniformly doped piece of silicon (plotted logarithmically in terms of inverse temperature 1000/T).



(a) (7 points) Label each region A, B, C as either intrinsic, extrinsic, or freeze-out.

(b) (5 points) Estimate the equilibrium hole density p_0 at 1000/T=3.33 .

$T \approx 300K$ extrinsic regime

$p_0 n_0 = n_i^2$

$p_0 = \frac{n_i^2}{10^{14}\,cm^{-3}} = 2.3\times10^6\,cm^{-3}$

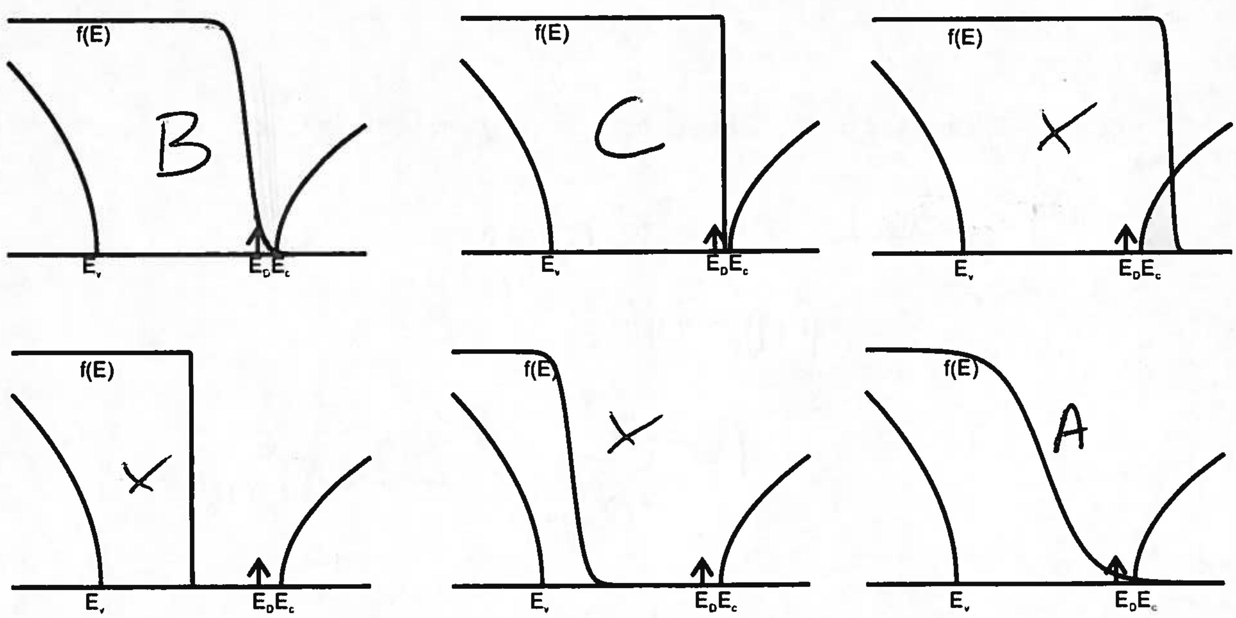
(c) (5 points) Estimate the equilibrium hole density p_0 at $1000/T=1$.

At T=1000K, we are in intrinsic regime

$p_0 = n_0 = n_i \approx 10^{17} cm^{-3}$

from plot

(d) (8 points) Below are plotted 6 Fermi-Dirac distribution functions, plotted on top of the density of states $N(E)$ for the conduction and valence band states (x-axis is energy). The donor energy state is indicated with E_D. For each plot below, label which correspond to regions A, B, and C in the plot above, and which don't correspond at all?



4. Switching dynamics (30 points)

Consider a silicon $p+/n$ diode in forward bias, where the current $i(t)$ is switched from I_1 to I_2 at time t=0, where both I_1 and I_2 are forward currents. You may use the dynamic charge equation:

$$i(t) = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p(t)}{dt}.$$

(a) (6 points) What is Q_p(t) at t=0, and at t=∞?

Initial condition at $t=0$. Steady state $\frac{d}{dt}=0$ $\quad I_1 = \frac{Q_p}{\tau_p}$ $\quad Q_p(t=0) = I_1 \tau_p$

At $t=\infty$ steady state

$I_2 = \frac{Q_p}{\tau_p}$ $\quad Q_p(t=\infty) = I_2 \tau_p$

(b) (10 points) Derive an expression for Q_p(t), in terms of τ_p, I_1, and I_2.

$\frac{dQ_p}{dt} + \frac{Q_p}{\tau_p} = I_2$ with above initial & final conditions

$Q_p(t)_{homo} = a e^{-t/\tau_p}$ $\quad Q_p(t)_{part} = b I_2$

$Q_p(t) = a e^{-t/\tau_p} + b I_2$

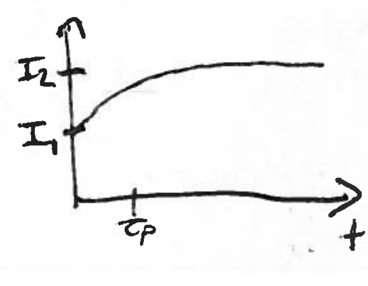
$b = \tau_2$ to satisfy $Q_p(t=\infty) = I_2 \tau_p$

$Q_p(t) = I_2 \tau_p + a e^{-t/\tau_p}$ at $t=0$

$I_1 \tau_p = I_2 \tau_p + a$

$a = \tau_p (I_1 - I_2)$

$$\boxed{Q_p(t) = I_2 \tau_p + \tau_p (I_1 - I_2) e^{-t/\tau_p}}$$



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(c) (7 points) Assume that the p-side acceptor density is $N_A=10^{18}\text{cm}^{-3}$. If we have the choice of the making the donor density $N_D=10^{15}$ cm^{-3} or $N_D=10^{17}$ cm^{-3}, which should we choose achieve the fastest switching speed, and why?

The switching speed is limited by τ_p. We should choose the higher doping $N_D = 10^{17}\,\text{cm}^{-3}$, since τ_p decreases with increasing doping.

(d) (7 points) What effect will choosing $N_D=10^{15}$ cm^{-3} versus $N_D=10^{17}$ cm^{-3} have on the reverse breakdown voltage magnitude V_{br}?

Let's write the expression for the maximum electric field within a p-n junction.

$$\mathcal{E}_{max} = \sqrt{\frac{2e(V_0 - V)}{\epsilon}\left(\frac{N_A N_D}{N_A + N_D}\right)}$$

Since $N_A \gg N_D$ for both cases we approximate:

$$\mathcal{E}_{max} \approx \sqrt{\frac{2e(V_0 - V)}{\epsilon} N_D}$$

As N_D is increased, $\mathcal{E}_{max}$ at a given bias V increases, Therefore $|V_{br}|$ will decrease as N_D increases.

Solution to 3D free particle in volume L^3:

$$\psi(x,y,z)=\frac{1}{L^{3/2}}e^{j\mathbf{k}\cdot\mathbf{r}}\ ,\quad E=\frac{\hbar^2k^2}{2m}\quad k_x=n_x\frac{2\pi}{L},k_y=n_y\frac{2\pi}{L},k_z=n_z\frac{2\pi}{L},\quad n_x,n_y,n_z=\ldots-2,-1,0,1,2,\ldots$$

Ohm's law: $\mathbf{J}=\sigma\boldsymbol{\mathcal{E}}$

Conductivity of free electron gas (i.e. metal): $\sigma=\dfrac{ne^2\bar{t}}{m}$ $\qquad \sigma=\rho^{-1}$

Conductivity of semiconductor: $\sigma=ne\mu_n+pe\mu_p$

Semiconductor electron/hole mobility $\mu_n=\dfrac{e\bar{t}_n}{m_n^*}$ $\qquad \mu_p=\dfrac{e\bar{t}_p}{m_p^*}$

3D free electron Density of States $N(E)=\dfrac{1}{2\pi^2}\left(\dfrac{2m^*}{\hbar^2}\right)^{3/2}E^{1/2}$

Electron Effective mass $m^*=\dfrac{\hbar^2}{d^2E/dk^2}$

Fermi-Dirac distribution for electrons: $f(E)=\dfrac{1}{e^{(E-E_F)/kT}+1}$, for holes: $f(E)=1-\dfrac{1}{e^{(E-E_F)/kT}+1}$,

Equilibrium Carrier concentrations $n_0=\displaystyle\int_{E_C}^{\infty}f(E)N(E)dE$

Equilibrium Carrier concentrations in non-degenerate limit (E_C-$E_F \gg kT$ and E_F-$E_V \gg kT$).

$$n_0=N_Ce^{-(E_C-E_F)/kT}\ ,\quad p_0=N_Ve^{-(E_F-E_V)/kT}$$

$$N_C=2\left(\frac{2\pi m_n^*kT}{h^2}\right)^{3/2}\ ,\quad N_V=2\left(\frac{2\pi m_p^*kT}{h^2}\right)^{3/2}$$

$$n_i=\sqrt{n_0p_0}=\sqrt{N_CN_V}e^{-E_g/2kT}$$

Intrinsic Fermi Level $E_i=\dfrac{kT}{2}\ln\left(\dfrac{N_V}{N_C}\right)+\dfrac{E_V+E_C}{2}$

Einstein relation for diffusion coeff: $D=\dfrac{kT}{e}\mu$ $\qquad$ Diffusion length $L=\sqrt{D\tau}$

Debye screening length (for n-type): $L_D=\sqrt{\dfrac{\epsilon kT}{e^2n_0}}$

Dielectric relaxation time $\tau_D=\dfrac{\epsilon}{\sigma}=\dfrac{\epsilon}{n_0e\mu_n}$ (n-type) $=\dfrac{\epsilon}{p_0e\mu_p}$ (p-type)

Minority carrier rate equations for excess optical uniform carrier generation (low-level injection)

$$\frac{d(\delta n)}{dt}=g_{op}-\frac{\delta n}{\tau_n}\ \text{(p-type)}\ ,\qquad \frac{d(\delta p)}{dt}=g_{op}-\frac{\delta p}{\tau_p}\ \text{(n-type)}$$

Continuity equations for excess minority carriers (low-level injection, no E-fields, no optical generation)

$$\frac{d(\delta n)}{dt}=D_n\frac{d^2(\delta n)}{dx^2}-\frac{\delta n}{\tau_n}\ \text{(p-type)}\ ,\qquad \frac{d(\delta p)}{dt}=D_n\frac{d^2(\delta p)}{dx^2}-\frac{\delta p}{\tau_p}\ \text{(n-type)}$$

Fundamental constants

Planck's constant:	h=6.63×10^{-34} J s	h=4.14×10^{-15} eV s
	$\hbar$=1.06×10^{-34} J s	$\hbar$=6.58×10^{-16} eV s
Permittivity of free space	ϵ_0=8.85×10^{-12} F/m	ϵ_0=8.85×10^{-14} F/cm
Permeability of free space	μ_0=4π × 10^{-7} Ns^2/C^2	
Conversion from eV to J	1 eV=1.60×10^{-19} J	
Boltzmann's constant	k=1.38×10^{-23} J/K	k=8.62×10^{-5} eV/K
Bare electron mass	m_0=9.11×10^{-31} kg	m_0=5.69×10^{-16} eV s^2 cm^{-2}
Speed of light	c=2.998x10^8 m/s	c=2.998x10^{10} cm/s
Fundamental charge	e = 1.602×10^{-19} C	

1Å=10^{-10} m, 1 nm=10^{-9} m, 1 μm=10^{-6} m.

Material properties

All parameters at room temp	Silicon	GaAs
Crystal Structure	Diamond	Zincblende
a	5.43 Å	5.65 Å
Mass density	2.33 g/cm^3	5.31 g/cm^3
ϵ_r	11.8	13.2
E_g	1.11 eV	1.43 eV
μ_n (for intrinsic/low doping)	1350 cm^2/V s	8500 cm^2/V s
μ_p (for intrinsic/low doping)	480 cm^2/V s	400 cm^2/V s
m^*_n	0.26m_0	0.067m_0
m^*_p	0.49m_0	0.5m_0
Effective DOS N_c	2.8×10^{19} cm^{-3}	4.7×10^{17} cm^{-3}
Effective DOS N_v	1.0×10^{19} cm^{-3}	7.0×10^{18} cm^{-3}
n_i	1.5×10^{10} cm^{-3}	2×10^6 cm^{-3}
D_n (for intrinsic/ low doping)	35 cm^2 s^{-1}	220 cm^2 s^{-1}
D_p (for intrinsic/ low doping)	12.5 cm^2 s^{-1}	10 cm^2 s^{-1}

Useful equations

Electron momentum: $p = mv = \hbar k = h/\lambda$ Planck relation: $E = hf = \hbar\omega$

Time independent Schrodinger's Equation (1D): $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$

Expectation value for position $\langle x\rangle = \int_{-\infty}^{\infty} xP(x)dx = \int_{-\infty}^{\infty} x|\psi(x)|^2\,dx$, $\langle x^2\rangle = \int_{-\infty}^{\infty} x^2|\psi(x)|^2\,dx$.

Expectation value for momentum $\langle p\rangle = -j\hbar\int_{-\infty}^{\infty}\psi^*(x)\dfrac{\partial}{\partial x}\psi(x)dx = \hbar\int_{-\infty}^{\infty} k|\psi(k)|^2\,dk$,

Uncertainty $\Delta x = \sqrt{\langle (x-\langle x\rangle)^2\rangle} = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$ $\Delta p_x = \sqrt{\langle (p_x-\langle p_x\rangle)^2\rangle} = \sqrt{\langle p_x^2\rangle - \langle p_x\rangle^2}$

Heisenberg uncertainty principle: $(\Delta x)(\Delta p_x) \geq \dfrac{\hbar}{2}$

Solution to 1D particle in box (infinite quantum well of width L):

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right) \text{ for } 0 < x < L,\quad \psi(x) = 0 \text{ otherwise} \quad, E = \frac{n^2\hbar^2\pi^2}{2mL^2},\quad n = 1,2,...$$

Drift/Diffusion current densities:

$$J_n(x) = e\mu_n n(x)\mathcal{E}(x) + eD_n \frac{dn(x)}{dx},$$

$$J_p(x) = e\mu_p p(x)\mathcal{E}(x) - eD_p \frac{dp(x)}{dx}$$

p-n diode (assumes abrupt junction)

Contact potential $V_0 = \frac{kT}{e}\ln\frac{p_p}{p_n} = \frac{kT}{e}\ln\frac{n_n}{n_p} = \frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right)$, $\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{eV_0/kT}$

Depletion (transition) region width $W = \sqrt{\frac{2\epsilon(V_0 - V)}{e}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$, $x_{n0} = W\frac{N_A}{N_A + N_D}$

Maximum Electric field within depletion region $\mathcal{E}_{\max} = \sqrt{\frac{2e(V_0 - V)}{\epsilon}\left(\frac{N_A N_D}{N_A + N_D}\right)}$

Ideal diode I-V:

$$I = eA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)\left(e^{eV/kT} - 1\right) = \left(\frac{Q_p}{\tau_p} + \frac{Q_n}{\tau_n}\right)\left(e^{eV/kT} - 1\right) = I_0\left(e^{eV/kT} - 1\right)$$

Ideal diode: excess minority carriers

$$\Delta p_n = \delta p(x_{n0}) = p(x_{n0}) - p_n = p_n\left(e^{eV/kT} - 1\right)$$

$$\Delta n_p = \delta n(-x_{p0}) = n(-x_{p0}) - n_p = n_p\left(e^{eV/kT} - 1\right),$$

Ideal diode: excess stored minority charge $Q_p = eAL_p\Delta p_n$ $\quad Q_n = eAL_n\Delta n_p$,

Capacitance $C = \left|\frac{dQ}{dV}\right|$

Depletion (junction) Capacitance $C_j = \epsilon A\sqrt{\frac{e}{2\epsilon(V_0 - V)}\frac{N_A N_D}{N_A + N_D}} = \frac{\epsilon A}{W}$

Diffusion Capacitance

$$C_d = C_{d,n} + C_{d,p} = \tau_p g_p + \tau_n g_n = \frac{e^2 A}{kT}\left(n_p L_n + p_n L_p\right)e^{eV/kT} \quad \text{(applies when } V \gg kT/e\text{)},$$

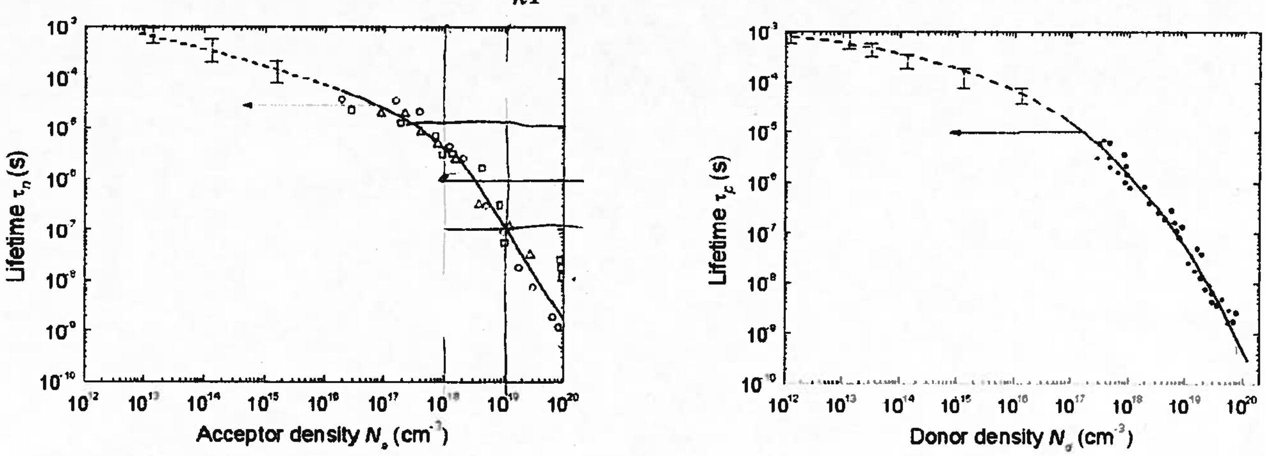

Figure. Minority carrier lifetime for electrons (left) and holes (right) as a function of doping density in silicon at room temperature.

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