UCLA Department of Electrical Engineering
EE2 – Physics for Electrical Engineers
Final Exam 2012
June 11 2012, (3 hours)

Name ______________________________ Student number ______________________________

This is a closed book exam – you are allowed 2 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	p-n band diagrams	20	
Problem 2	p-n diode electrostatics	20	
Problem 3	Generation/recombination dynamics	30	
Problem 4	p-n diode trends	20	
Problem 5	Equilibrium carrier concentrations	10	
Total		100	

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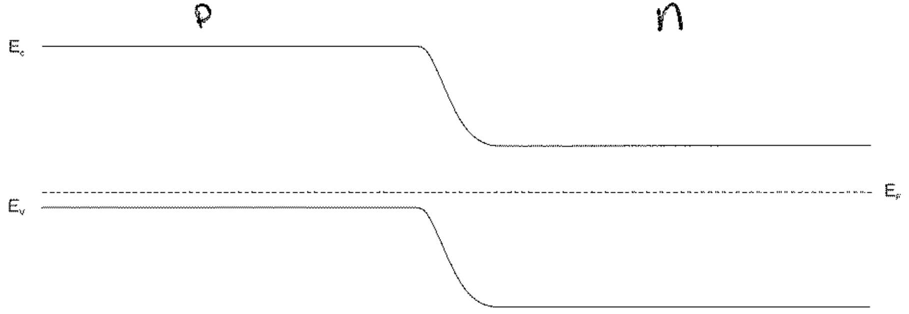
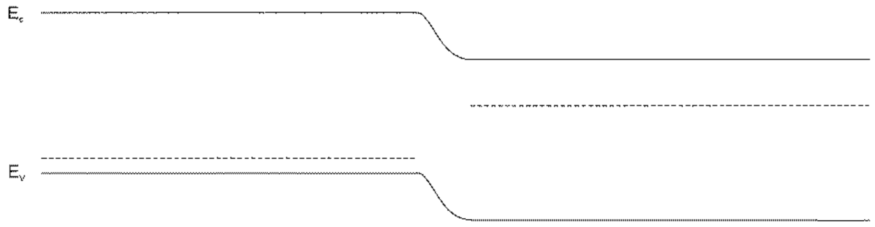
1. *p-n* band diagrams (20 points)

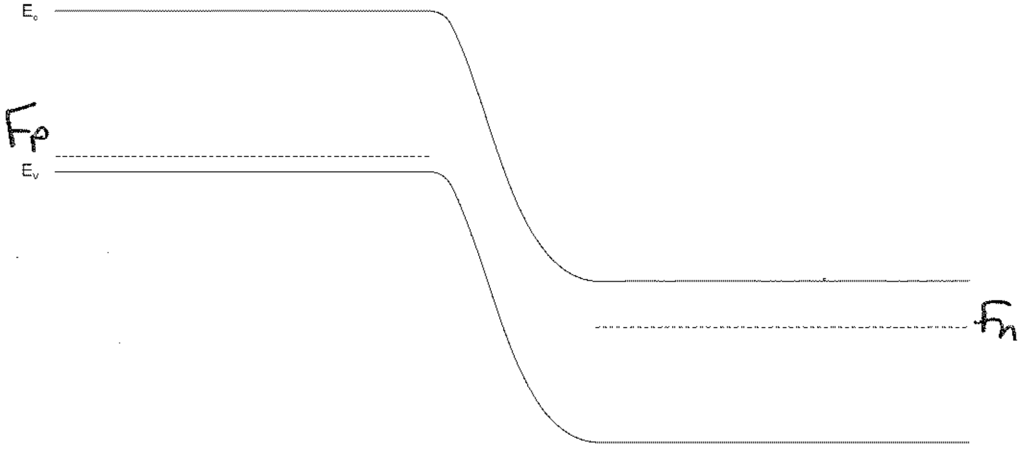
(a) (5 points) Label next to each of the 3 diagrams, whether this shows a *p-n* diode in forward, reverse, or zero bias.



Zero bias



Forward bias



Reverse bias

(b) (5 points) Which doping is larger, N_A or N_D?

N_A is larger as can be seen since $\underset{(p\text{-side})}{E_F - E_V} > \underset{(n\text{side})}{E_C - E_F}$.
(ie E_F is closer to valence band edge on p-side than E_F is to conduction band edge on n-side.)

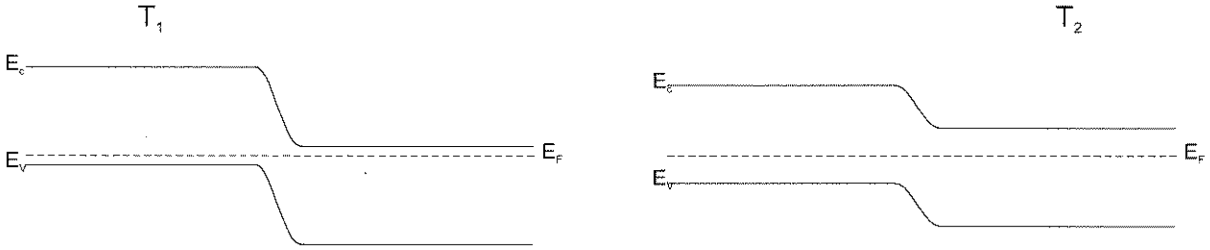
(c) (5 points) Assume this p-n diode is GaAs. Estimate the voltage bias *V* (in Volts) for the last diagram above.

$$\frac{F_p - F_n}{e} = V \approx -1.4 - -1.5\ V$$

since $F_p - F_n \approx E_g$

and $E_g \approx 1.43$ eV for GaAs.

(d) (5 points) The two bandstructure diagrams represent the same *p-n* diode at different temperatures T_1 and T_2. Which temperature is higher? Explain briefly why.



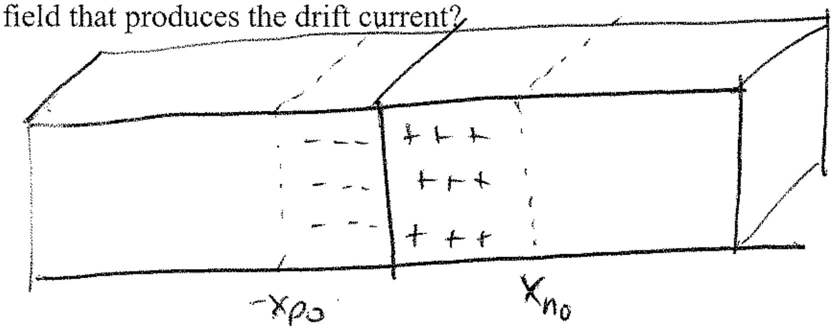
$T_2 > T_1$. As temperature increases the Fermi level moves further away from the band edge. This is required by the equations

$$n = \cancel{n} N_c e^{-(E_c - E_F)/kT}$$
$$p = N_v e^{-(E_F - E_v)/kT}$$

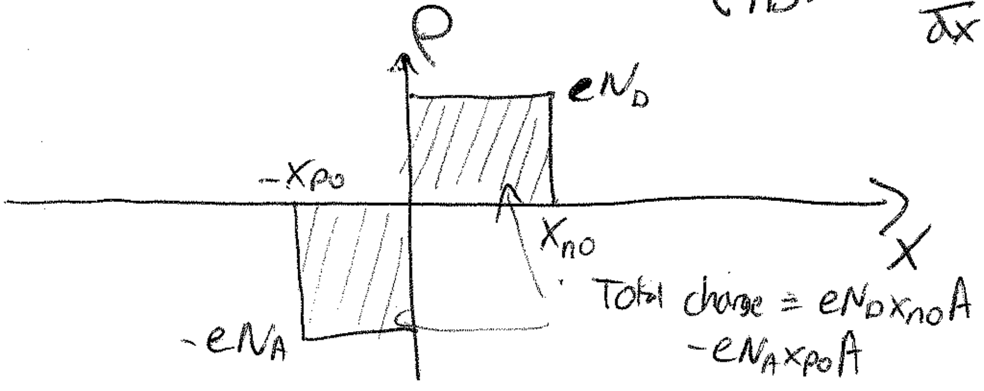
When we require $n = N_D$ on n-side and $p = N_A$ on p-side for extrinsic semiconductors.

2. p-n diode electrostatics (20 points)
   For a p-n junction in equilibrium, the diffusion current across the transition (depletion) region is balanced exactly by the drift current across the transition region so that no net current flows.

   (a) Drift current requires an electric field. Explain qualitatively: what is the source of the electric field that produces the drift current?



The source of the electric field is the space charge that results from the ionized donor/acceptor impurities. As electrons & holes diffuse from regions of high to low concentration, this leaves behind uncompensated charged dopants. The electric field is oriented from the n-side to p-side, as is determined by Gauss's Law. (1D: $\frac{d\mathcal{E}}{dx} = \rho/\epsilon$)



Total charge = $eN_D x_{no} A - eN_A x_{po} A$

This $\mathcal{E}$-field produces a drift current that cancels the diffusion current in equilibrium.

(b) Explain qualitatively: Why does the depletion region get thicker as the device is put into reverse bias?

As the reverse bias is increased, a greater electrostatic potential difference $(V_0 - V)$ must be dropped across the depletion region.
This is according to Poisson's Eq $\nabla^2 V = -\rho/\epsilon$.

To drop the larger potential drop, the depletion region becomes wider, exposing more space charge on each side. This leads to a larger $\mathcal{E}$-field or a wider region so that $V_0 - V = -\int_{-x_{p0}}^{x_{n0}} \mathcal{E} \cdot dx$.

3. Generation/recombination dynamics (30 points)

Consider a uniform piece of *p*-type GaAs doped with $N_A=10^{16}$ cm^{-3}. Optical excitation is used to generate electron-hole pairs at the generation rate of $g_{op}=10^{23}$ cm^{-3} s^{-1} uniformly throughout the sample. In this problem you should assume a minority carrier recombination time $\tau_n = \tau_p = 1$ ns.

(a) (10 points) Assume steady illumination of the sample with g_{op}. What is the steady-state excess electron and hole δn and δp, and total electron and hole concentration n and p ? Give numbers.

Steady state: $\frac{d}{dt} \to 0$

$$\frac{d(\delta n)}{dt} = g_{op} - \frac{\delta n}{\tau_n}$$

(SS): $\tau_n g_{op} = \delta n$

$\delta n = g_{op} \tau_n = 10^{14}\ cm^{-3}$

$\delta p = \delta n = 10^{14}\ cm^{-3}$ due to quasi neutrality requirements, + generation of EHPs.

$n = n_0 + \delta n \approx \delta n = 10^{14} cm^{-3}$ (since $\delta n >> n_0$) ($n_0 \approx \frac{n_i^2}{N_A} \approx \frac{4\times10^{12} cm^{-3}}{10^{16} cm^{-3}}$, $n_0 \approx 4\times10^{-4} cm^{-3}$)

$p = p_0 + \delta p = 1.01\times10^{16} cm^{-3}$

(b) (10 points) At what rate of optical generation g_{op} would we violate the assumption of "low-level injection"?

When $g_{op} = 10^{25} cm^{-3} s^{-1}$ $\delta n = 10^{16} cm^{-3} \approx p_0$.

This would be a clear violation of low-level injection.

However, even when $\delta n > \frac{p_0}{10}$ you could consider it a strict violation, so $g_{op}\ 10^{24} cm^{-3} s^{-1}$ would also be an okay answer.

(c) (10 points) At time t=0 the previously steady optical excitation is switched off. Derive expressions for the time dependent total carrier concentrations $n(t)$ and $p(t)$.

Initial condition $t < 0$ $\delta n = 10^{14}\,\text{cm}^{-3} = g_{op}\tau_n$

After $t \geq 0$ $g_{op} = 0$

$$\frac{d\delta n}{dt} = \frac{\delta n}{\tau_n}$$

$$\delta n = \delta n(t=0)\,e^{-t/\tau_n} = g_{op}\tau_n e^{-t/\tau_n}$$

$$\boxed{n(t) \simeq \delta n = (10^{14}\,\text{cm}^{3})\,e^{-t/\tau_n}} \quad \text{for } t > 0$$

$$\boxed{p(t) = 10^{16}\,\text{cm}^{-3} + 10^{14}\,\text{cm}^{-3}\,e^{-t/\tau_n}} \quad \text{for } t > 0$$

4. P-n diode trends (20 points)

Consider a *p-n* diode with an abrupt junction where $N_A = 10^{18}$ cm^{-3} and $N_D = 10^{15}$ cm^{-3}. For this device:

(a) Which parameter has a greater effect over the reverse saturation current I_0, τ_p or τ_n?

This is a p^+/n junction. $p_p \approx 10^{18}\,\text{cm}^{-3}$ $n_p \approx \frac{n_i^2}{10^{18}\,\text{cm}^{-3}} \approx 10^{2}\,\text{cm}^{-3}$

$n_n = 10^{15}\,\text{cm}^{-3}$ $p_n \approx 10^{5}\,\text{cm}^{-3}$

(Take $n_i^2 \approx 10^{20}\,\text{cm}^{-3}$)

D_p, D_n, L_p, L_n are of similar order of magnitude for electrons & holes.

So $\frac{D_p}{L_p} p_n >> \frac{D_n}{L_n} n_p$ and $I_0 \approx eA \frac{D_p p_n}{L_p}$

$L_p = \sqrt{D_p \tau_p}$ hole current dominates.

$$\boxed{\text{Therefore, } \tau_p \text{ is more important to determine } I_0.}$$

(b) Will the reverse saturation current I_0 increase or decrease as temperature increases?

As temperature increases I_0 will increase.

$$I_0 = \frac{eAD_p p_n}{L_p} \qquad p_n \approx \frac{n_i^2}{n_n} \approx \frac{N_c N_v e^{-E_g/kT}}{n_n}$$

As long as semiconductor is extrinsic, $n_n \approx N_D$ relatively independent of temperature.

However $p_n \propto n_i^2 \propto e^{-E_g/kT}$

So p_n increases exponentially with temperature.

(c) Which parameter has more control over the junction (depletion) capacitance C_j, N_A or N_D?

$\boxed{N_D}$

For a p^+/n junction, $X_{no} >> X_{po}$

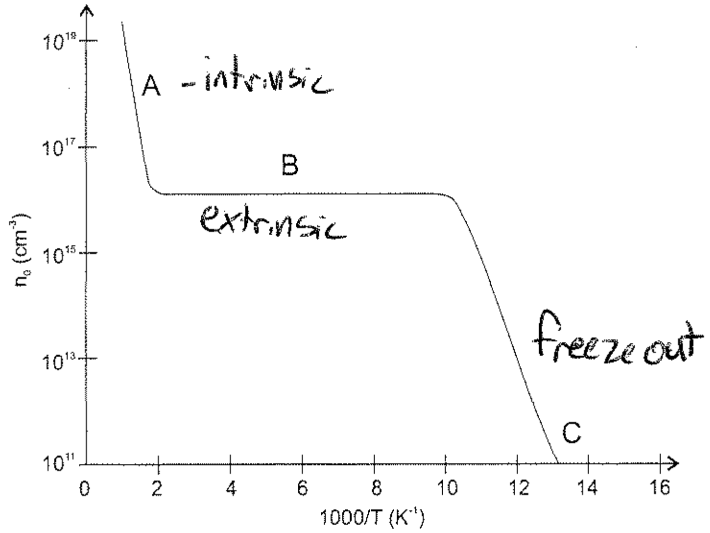
and $W \approx \sqrt{\frac{2\epsilon(V_0-V)}{e N_D}}$.

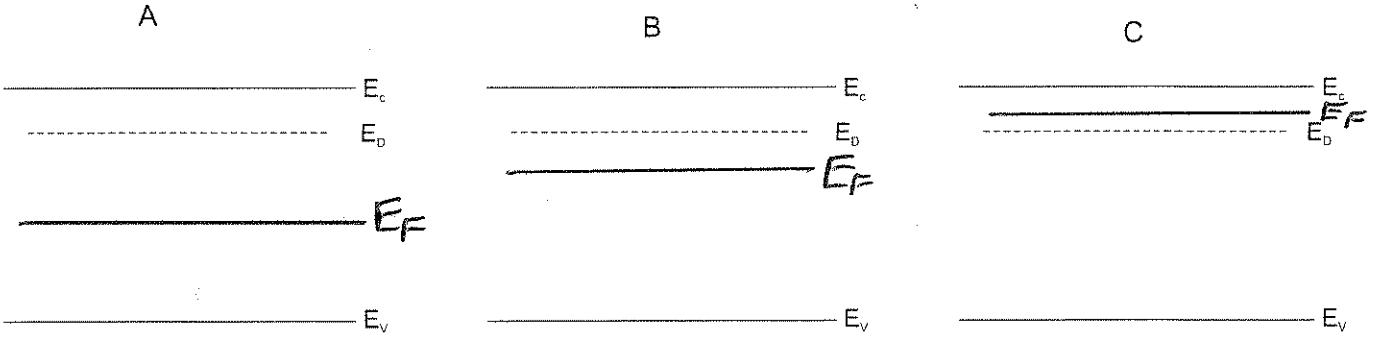
(d) Which parameter has a greater control over the reverse breakdown voltage V_{br}, N_A or N_D?

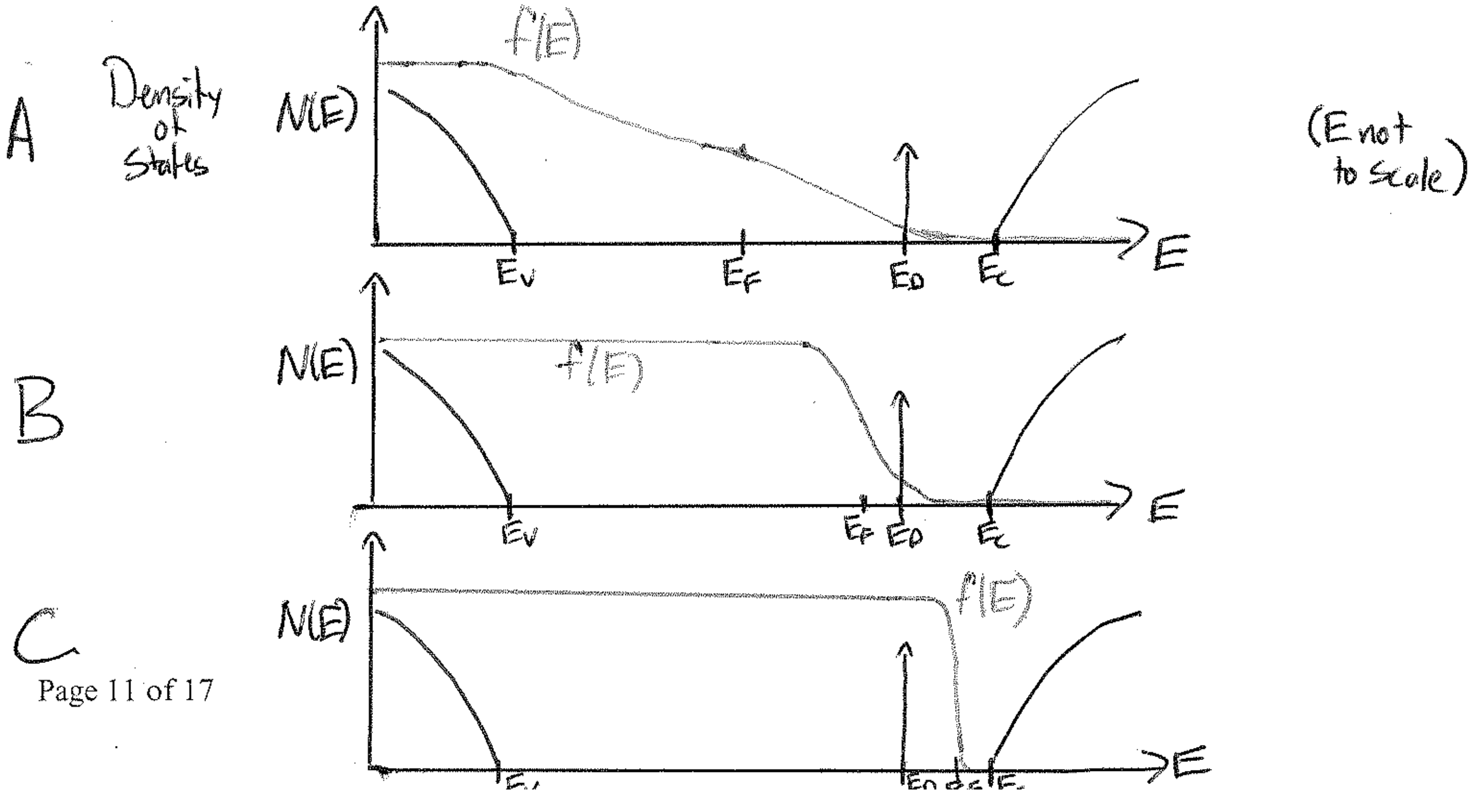
The reverse breakdown voltage mostly depends on the electric field in the depletion region. Since $N_A >> N_D$, most of the voltage drop takes place on the n-side, and $W \approx X_{no}$. Thus $\boxed{N_D}$ primarily will determine the magnitude of $\mathcal{E}$ in the depletion region.

5. Equilibrium carrier concentrations (10 points)

Consider the given plot of the equilibrium electron concentration n_0 versus temperature in Si (plotted logarithmically in terms of inverse temperature 1000/T). For each region of the plot A, B, C, sketch the position of the Fermi level E_F on the band diagram below.







Fundamental constants

Planck's constant:	h=6.63×10^{-34} J s	h=4.14×10^{-15} eV s
	$\hbar$=1.06×10^{-34} J s	$\hbar$=6.58×10^{-16} eV s
Permittivity of free space	ϵ_0=8.85×10^{-12} F/m	ϵ_0=8.85×10^{-14} F/cm
Permeability of free space	μ_0=4π × 10^{-7} Ns^2/C^2	
Conversion from eV to J	1 eV=1.60×10^{-19} J	
Boltzmann's constant	k=1.38×10^{-23} J/K	k=8.62×10^{-5} eV/K
Bare electron mass	m_0=9.11×10^{-31} kg	m_0=5.69×10^{-16} eV s^2 cm^{-2}
Speed of light	c=2.998x10^8 m/s	c=2.998x10^{10} cm/s
Fundamental charge	e = 1.602×10^{-19} C	

1Å=10^{-10} m, 1 nm=10^{-9} m, 1 μm=10^{-6} m.

Material properties

Silicon

All parameters at room temp	Silicon	GaAs
Crystal Structure	Diamond	Zincblende
a	5.43 Å	5.65 Å
Mass density	2.33 g/cm^3	5.31 g/cm^3
ϵ_r	11.8	13.2
E_g	1.11 eV	1.43 eV
μ_n (for intrinsic/low doping)	1350 cm^2/V s	8500 cm^2/V s
μ_p (for intrinsic/low doping)	480 cm^2/V s	400 cm^2/V s
m^*_n	0.26m_0	0.067m_0
m^*_p	0.49m_0	0.5m_0
Effective DOS N_c	2.8×10^{19} cm^{-3}	4.7×10^{17} cm^{-3}
Effective DOS N_v	1.0×10^{19} cm^{-3}	7.0×10^{18} cm^{-3}
n_i	1.5×10^{10} cm^{-3}	2×10^6 cm^{-3}
D_n (for intrinsic/ low doping)	35 cm^2 s^{-1}	220 cm^2 s^{-1}
D_p (for intrinsic/ low doping)	12.5 cm^2 s^{-1}	10 cm^2 s^{-1}

Useful equations

Electron momentum: $p = mv = \hbar k = h/\lambda$ Planck relation: $E = hf = \hbar\omega$

Time independent Schrodinger's Equation (1D): $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$

Heisenberg uncertainty principle: $(\Delta x)(\Delta p_x) = \frac{\hbar}{2}$

Solution to 1D particle in box (infinite quantum well of width L):

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right) \quad , \; E = \frac{n^2\hbar^2\pi^2}{2mL^2}, \; n = 0,1,2,...$$

Solution to 3D free particle in volume L^3:

$$\psi(x,y,z) = \frac{1}{L^{3/2}}e^{j\mathbf{k}\bullet\mathbf{r}} \;, \; E = \frac{\hbar^2k^2}{2m} \quad k_x = n_x\frac{2\pi}{L}, k_y = n_y\frac{2\pi}{L}, k_z = n_z\frac{2\pi}{L}, \; n_x, n_y, n_z = ...-2,-1,0,1,2,...$$

Ohm's law: $\mathbf{J} = \sigma \boldsymbol{\mathcal{E}}$

Conductivity of free electron gas (i.e. metal): $\sigma = \dfrac{ne^2\overline{t}}{m}$ $\qquad \sigma = \rho^{-1}$

Conductivity of semiconductor: $\sigma = ne\mu_n + pe\mu_p$

Semiconductor electron/hole mobility $\mu_n = \dfrac{ne\overline{t}_n}{m_n^*}$ $\qquad \mu_p = \dfrac{pe\overline{t}_p}{m_p^*}$

3D free electron Density of States $N(E) = \dfrac{1}{2\pi^2}\left(\dfrac{2m^*}{\hbar^2}\right)^{3/2} E^{1/2}$

Electron Effective mass $m^* = \dfrac{\hbar^2}{d^2E/dk^2}$

Fermi-Dirac distribution for electrons: $f(E) = \dfrac{1}{e^{(E-E_F)/kT}+1}$, for holes: $f(E) = 1 - \dfrac{1}{e^{(E-E_F)/kT}+1}$,

Equilibrium Carrier concentrations $n_0 = \displaystyle\int_{E_C}^{\infty} f(E)N(E)dE$

Equilibrium Carrier concentrations in non-degenerate limit (E_C-$E_F \gg kT$ and E_F-$E_V \gg kT$).

$$n_0 = N_C e^{-(E_C-E_F)/kT} \quad , \; p_0 = N_V e^{-(E_F-E_V)/kT}$$

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \quad , \; N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

$$n_i = \sqrt{n_0 p_0} = \sqrt{N_C N_V}\, e^{-E_g/2kT}$$

Intrinsic Fermi Level $E_i = \dfrac{kT}{2}\ln\left(\dfrac{N_V}{N_C}\right) + \dfrac{E_V + E_C}{2}$

Einstein relation for diffusion coeff: $D = \dfrac{kT}{e}\mu$ $\qquad$ Diffusion length $\qquad L = \sqrt{D\tau}$

Debye screening length (for n-type): $L_D = \sqrt{\dfrac{\epsilon kT}{e^2 n_0}}$

Dielectric relaxation time $\tau_D = \dfrac{\epsilon}{\sigma} = \dfrac{\epsilon}{n_0 e\mu_n}$ (n-type) $= \dfrac{\epsilon}{p_0 e\mu_p}$ (p-type)

Minority carrier rate equations for excess optical uniform carrier generation (low-level injection)

$$\frac{d(\delta n)}{dt} = g_{op} - \frac{\delta n}{\tau_n} \quad \text{(p-type)}\ , \qquad \frac{d(\delta p)}{dt} = g_{op} - \frac{\delta p}{\tau_p} \quad \text{(n-type)}$$

Continuity equations for excess minority carriers (low-level injection, no E-fields, no optical generation)

$$\frac{d(\delta n)}{dt} = D_n\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_n} \quad \text{(p-type)}\ , \qquad \frac{d(\delta p)}{dt} = D_n\frac{d^2(\delta p)}{dx^2} - \frac{\delta p}{\tau_p} \quad \text{(n-type)}$$

Drift/Diffusion current densities:

$$J_n(x) = e\mu_n n(x)\mathcal{E}(x) + eD_n\frac{dn(x)}{dx}$$

$$J_p(x) = e\mu_p p(x)\mathcal{E}(x) - eD_p\frac{dp(x)}{dx} \quad ,$$

p-n diode

Contact potential $V_0 = \frac{kT}{e}\ln\frac{p_p}{p_n} = \frac{kT}{e}\ln\frac{n_n}{n_p} = \frac{kT}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right)$, $\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{eV_0/kT}$

Depletion (transition) region width $W = \sqrt{\frac{2\epsilon\left(V_0 - V\right)}{e}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$, $x_{n0} = W\frac{N_A}{N_A + N_D}$

Ideal diode I-V:

$$I = eA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)\left(e^{eV/kT} - 1\right) = \left(\frac{Q_p}{\tau_p} + \frac{Q_n}{\tau_n}\right)\left(e^{eV/kT} - 1\right) = I_0\left(e^{eV/kT} - 1\right),$$

Ideal diode: excess minority carriers

$$\Delta p_n = \delta p(x_{n0}) = p(x_{n0}) - p_n = p_n\left(e^{eV/kT} - 1\right)$$
$$\Delta n_p = \delta n(-x_{p0}) = n(-x_{p0}) - n_p = n_p\left(e^{eV/kT} - 1\right),$$

Ideal diode: excess stored minority charge $Q_p = eAL_p\Delta p_n$ $Q_n = eAL_n\Delta n_p$,

Capacitance $C = \left|\frac{dQ}{dV}\right|$

Depletion (junction) Capacitance $C_j = \epsilon A\sqrt{\frac{e}{2\epsilon\left(V_0 - V\right)}\frac{N_A N_D}{N_A + N_D}} = \frac{\epsilon A}{W}$

Diffusion Capacitance

$$C_d = C_{d,n} + C_{d,p} = \tau_p g_p + \tau_n g_n = \frac{e^2 A}{kT}\left(n_p L_n + p_n L_p\right)e^{eV/kT} \quad \text{(applies when V} \gg kT/e\text{)} \quad ,$$

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