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## EE 2 Midterm
### (120 points total)

### 2 November 2015

**Read the question and the possible answers** before attempting any calculations. Write your name on the first page (each page if you separate the pages for any reason).

(42) **Section 1 (60 points) Multiple choice** Circle the correct answer. (50 pts.) There is no penalty for guessing. An educated guess with at least 50/50 probability is possible on most problems. **There is no partial credit on Section 1 (Multiple choice)..**

1. (4 pts) A sample of GaAs is doped with Arsenic ($N_d = 2 \times 10^{16}$ /cm$^3$). What is the approximate equilibrium concentration of holes in the sample at 300 K.

   a. $2 \times 10^{16}$ /cm$^3$    b. $1 \times 10^4$    c. $1.45 \times 10^{10}$    d. $\approx 10^2$

   $n_o \approx N_d \Rightarrow$   $p_o = \dfrac{n_i^2}{N_d}$

2. (4 pts) A sample of Germanium has an intrinsic concentration of $2.4 \times 10^{13}$ /cm$^3$ at room temperature (300 K). What is the approximate intrinsic concentration at 450 K?

   a. $2 \times 10^{11}$ /cm$^3$    b. $3 \times 10^{15}$    c. $4 \times 10^{13}$    d. $9 \times 10^9$

   $n_i = 2 \left( \dfrac{2\pi m_n^* kT}{h^2} \right)^{3/4} \left( \dfrac{2\pi m_p^* kT}{h^2} \right)^{3/4} exp\left( \dfrac{-E_g}{kT} \right)$

3. (4 pts) A sample of silicon is doped with Phosphorous at $N_d = 10^{15}$. Which of the following is closest to the equilibrium electron concentration at 25 K.

   a. $10^{15}$ /cm$^3$    b. $10^{13}$    c. $2 \times 10^{17}$    d. $2.8 \times 10^{19}$
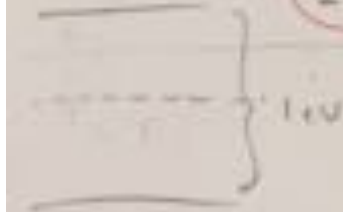
   $N_c = 3 \times 10^{19}$   $E_g = 1, V$   $p_o + N_A = n_o + N_d$   $\dfrac{n_i^2}{n_i}$

4. (5 pts) A semiconductor has a bandgap of 1 eV and an effective density of states in the conduction band of $3 \times 10^{19}$. It has n-type doping $N_d = 10^{16}$ and p-type doping $N_a = 7 \times 10^{15}$. What is the position of the Fermi level relative to the conduction band at 300 K ($E_c - E_f$).

   $E_c - E_f = -kT \ln\left( \dfrac{n_o}{N_c} \right)$

   a. 0.239 eV    b. 1.00 eV    c. 0.208 eV    d. -0.208 eV

   $\sqrt{N_c N_v} exp\left( \dfrac{-E_g}{2kT} \right) = N_c exp\left( \dfrac{-E_c}{kT} \right) \dfrac{E_{i}}{2kT}$

5. (5 pts) From the 4 choices below which is the longest wavelength that a pure sample of InP will strongly absorb.

~ 06 eV

(a.) 0.60 μm        b. 2.0 μm        c. 1.20 μm        d. 0.20 μm

$E_g \approx 1.4 \, eV$

6. (5 pts) The classical momentum $p_x$ corresponds to which of these quantum operators.

a. $\frac{\partial}{\partial r}$        b. $\frac{\hbar^2}{j}\frac{\partial^2}{\partial x^2}$        (c.) $\frac{\hbar}{j}\frac{\partial}{\partial x}$        d. $\frac{1}{2}mv^2$

7. (5 pts) Electrons in a semiconductor when undoped have a mean time between collisions of $\tau = 10$ nsec. When doped only with substance A at a concentration of X, $\tau = 0.2$ nsec. When doped only with substance B at a concentration Y, $\tau = 0.3$ nsec. What is the approximate mean time between collisions when doped with both substances?

a. 0.5 nsec        b. 10.5 nsec        c. 0.21 nsec        (d.) 0.12 nsec

$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} = \left(\frac{1}{.2} + \frac{1}{.3}\right)^{-1} = .12$$

8. (5 pts) The probability of occupancy of electron states with energy $E=E_f$ at 300 K is:

a. 1        b. $10^{13}$        c. $\exp[-E_f/kT]$        (d.) 0.5

9. (5 pts) Undoped silicon is uniformly illuminated to produce steady state excess carriers of $10^{14}$/cm³ (both electrons and holes). If the recombination lifetime is 10 nsec ($10^{-8}$ seconds), approximately how long will it take for the excess charge to reduce to $10^{11}$/cm³ (making it nearly intrinsic again).
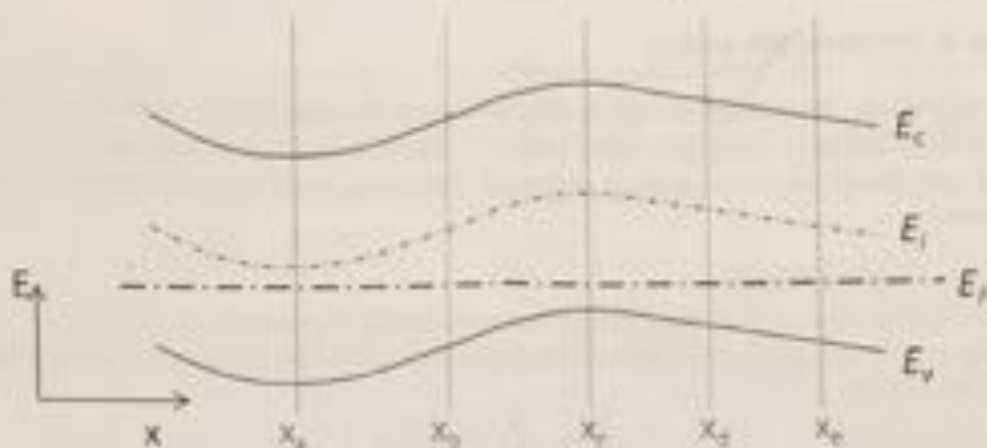
a. 10 nsec        b. 7 nsec        (c.) 70 nsec        d. 140 nsec

$$\Delta n_{exp}\left(\frac{-t}{\tau_r}\right) = \Delta n_1 \implies t = \tau_r \ln\left(\frac{\Delta n_o}{\Delta n_1}\right) \sim 69.0 \, ns.$$

Name_____



Referring to the band edge diagram above, answer the following questions. You may assume that the doping varies slowly throughout and that charge neutrality is maintained. The semiconductor is isolated. No calculations are necessary. Choose only one answer (there may be more than one correct answer... choose one only!)

10. (3 pts) Where is the hole concentration highest?

   a. $X_a$       b. $X_b$       c. $X_c$       d. $X_d$       e. $X_e$

11. (3 pts) Where is the electron concentration highest?

   a. $X_a$       b. $X_b$       c. $X_c$       d. $X_d$       e. $X_e$

12. (3 pts) At which of the following pairs of points is electric field equal in magnitude and sign?

   a. $X_a, X_b$     b. $X_b, X_c$     c. $X_c, X_d$     d. $X_d, X_e$     e. $X_b, X_e$

13. (3 pts) At which point do holes diffuse to the left?

   a. $X_a$       b. $X_b$       c. $X_c$       d. $X_d$       e. $X_e$

14. (3 pts) Is the semiconductor in equilibrium?

   a. Yes       b. No       c. Can't tell

15. (3 pts) Where is electric field nearly zero (choose one, there is more than one right answer)?

   a. $X_a$       b. $X_b$       c. $X_c$       d. $X_d$       e. $X_e$

**Section 2: Problems (60 points)**

Show your work. Full credit for the correct answer with work shown. Sensible answers (correct order of magnitude) get partial credit. Generous partial credit for an incorrect answer with the correct ideas _if clear and brief_. Negative credit for irrelevant or incorrect equations.

5. (30 pts)  Optical Switch

Consider a thin slice of pure (intrinsic) silicon uniformly illuminated as shown below. The slab is thin enough so that carriers are generated uniformly throughout under steady-state illumination.

(25)



a) (10 pt) Write an expression for the resistance of the slab when there is _no illumination_. We want the end to end resistance in terms of charge, concentrations, mobilities and dimensions. Do not plug in any numbers yet!

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$$R = \frac{\rho}{t}\frac{L}{W} = \frac{L}{\sigma\, t\, W} = \frac{L}{q\left(n_o \mu_n + p_o \mu_p\right) t\, W}$$

$$n_o = p_o = n_i$$

b) (5 pt) Calculate the resistance (without illumination) under the conditions that t = 1.0 μm, L = 100 μm, W = 25μm (additional values you may need can be found in the table on the formula sheet)

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for intrinsic Si at 300 K :  $n_o = p_o = n_i = 1.45 \times 10^{10}$ cm$^{-3}$.

$\mu_n = 1350$

$\mu_p = 460$

$R = 9.42 \times 10^{13}$.     $10^{10}$    _did you convert units to cm?  1μm = 10$^{-4}$ cm_

c) (10 pt) Write an expression for the resistance under illumination. Use $G_{op}$ to signify the uniform optical generation rate of electron-hole pairs. Define any variables you introduce and indicate which parameters are known and which are unknown.

**10**

$\delta_n$ - excess carrier concentration          $\delta_n = G_{op}\tau$

$\tau_n$ - carrier lifetime

$$n = n_o + \delta_n = n_o + G_{op}\tau_n$$
$$p = p_o + \delta_p = n_i + G_{op}\tau_n$$

$$\boxed{R = \frac{1}{\sigma t w} = \frac{L}{q(n_o + G_{op}\tau_n)(\mu_n + \mu_p)}} \checkmark$$

$\tau_n$ is unknown

$$\sigma = q(n\mu_n + p\mu_p) = q\left[(n_o + G_{op}\tau_n)\mu_n + (n_o + G_{op}\tau_n)\mu_p\right]$$
$$= q(n_o + G_{op}\tau_n)(\mu_n + \mu_p)$$

d) (5 pt) Now assume that the slab is under an illumination of $G_{op} = 10^{13}$ EHP/cm$^3$sec. Suppose a resistance measurement is made under this condition resulting in a resistance $10^{-4}$ lower than the resistance in b). Calculate the carrier lifetime. [Hint: you can do this calculation without any of the information in a, b and c]

**2.**

$$\frac{R_1}{R_o} = 10^{-4} = \frac{n_i}{(n_i + G_{op}\tau)} \approx \frac{n_i}{G_{op}\tau}$$

$$\frac{\dfrac{L}{\sigma_1 t w}}{\dfrac{L}{\sigma_o t w}} = K$$

$$\boxed{\tau = \frac{n_i(K-1)}{G_{op}} = 14.5 \text{ ns}}$$

$$\frac{1}{\sigma_1} = \frac{K}{\sigma_o} \Rightarrow \sigma_1 = \frac{\sigma_o}{K}$$

$$n_i = \frac{1}{K}(n + G_{op}\tau)$$

(30)

2. (30 pts)  Diffusion in a short base diode.
A slice of n-type semiconductor has the boundary conditions shown. At one end there is a steady supply of excess holes, $\delta p(0) = \Delta p$, and at the other end the hole concentration is the equilibrium value, $p_0$.

a. (10 pts) Derive an expression for $\delta p(x) = p(x) - p_0$ in terms of the minority carrier diffusion length, $L_p$, the dimension, $L_1$, and the boundary value excess charge, $\Delta p$.

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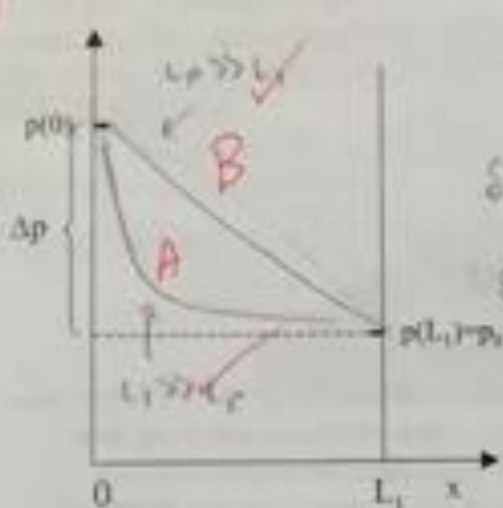


$$\frac{d^2 \delta p}{dx^2} = \frac{\delta_p}{L_p^2}$$

$$\delta p = C_1 \exp\left(\frac{x}{L_p}\right) + C_2 \exp\left(\frac{-x}{L_p}\right)$$

$$\delta_p(0) = \Delta p \qquad \delta_p(L_1) = 0$$

$$\Delta p = C_1 + C_2$$

$$C_1 \exp\left(\frac{L_1}{L_p}\right) + C_2 \exp\left(\frac{-L_1}{L_p}\right) = 0$$

$$C_1 \exp\left(\frac{L_1}{L_p}\right) = -C_2 \exp\left(\frac{-L_1}{L_p}\right)$$

$$-C_1 \exp\left(\frac{2L_1}{L_p}\right) = C_2$$

$$\Delta p = C_1\left(1 - \exp\left(\frac{2L_1}{L_p}\right)\right)$$

$$C_1 = \frac{\Delta p}{1 - \exp\left(\frac{2L_1}{L_p}\right)}$$

$$C_2 = \frac{\Delta p \exp\left(\frac{2L_1}{L_p}\right)}{\exp\left(\frac{2L_1}{L_p}\right) - 1}$$

$$\boxed{\delta_p(x) = \frac{\Delta p \exp\left(\frac{x}{L_p}\right)}{1 - \exp\left(\frac{2L_1}{L_p}\right)} + \frac{\Delta p \exp\left(\frac{2L_1 - x}{L_p}\right)}{\exp\left(\frac{2L_1}{L_p}\right) - 1}}$$

b. (10 pts) VERY IMPORTANT – Draw p(x) from 0 to $L_1$ on a **linear** scale for: two cases  A) $L_1 = 10 L_p$ ($L_1 \gg L_p$) AND  B) $L_1 = 0.1 L_p$ ($L_1 \ll L_p$). (Use the graph above and **label** the traces A and B).
(Next page for part c !!!)

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**5**

c. (5 pts) Simplify the expression you obtained in a) for the case where $L_p \gg L_1$. [Hint: the Taylor expansion for $e^x = 1 + x$, for very small x.]

$$\delta_p(x) = \frac{\Delta p}{1 - \exp\left(\frac{2L_1}{L_p}\right)} \left[\exp\left(\frac{x}{L_p}\right) - \exp\left(\frac{2L_1 - x}{L_p}\right)\right]$$

$$\approx \frac{\Delta p}{1 - \left(1 + \frac{2L_1}{L_p}\right)} \left[\left(1 + \frac{x}{L_p}\right) - \left(1 + \frac{2L_1}{L_p} - \frac{x}{L_p}\right)\right] = \frac{\Delta p}{-\frac{2L_1}{L_p}}\left(\frac{2x}{L_p} - \frac{2L_1}{L_p}\right)$$

$$\approx -\Delta p \left(\frac{2x}{L_p}\left(\frac{L_p}{2L_1}\right) - \frac{2L_1}{L_p}\left(\frac{L_p}{2L_1}\right)\right) = \boxed{-\frac{\Delta p x}{L_1} + \Delta p}$$  ✓  !! Corr

d. (5 pts.) Write an expression for the hole current density at x=0 for the $L_p \gg L_1$ case in terms of known quantities ($L_1$, $\Delta p$ and material constants)?

**5**

$$J_p(x) = q\mu_p p(x) E(x) - q D_p \frac{dp(x)}{dx}$$

$$= -q D_p \frac{d\delta_p}{dx}$$

$$= -q D_p \left(\frac{-\Delta p}{L_1}\right) = \frac{q D_p \Delta p}{L_1}$$

e. (5 pt bonus) If the semiconductor is n-type silicon with $N_d = 10^{18}$ and the dimension $L_1 = 0.1$ μm, $\Delta p = 6\times10^{14}$, $L_p = 100$ μm and cross sectional Area = 100 μm². What is the <u>hole</u> current at x=0?

◯

$$D = \frac{kT}{q} \mu_p = \quad 12.43?$$

$$J_p = \frac{q D_p \Delta p}{2} = 6\times 10^{+}$$

$$I = J_p A = 5.96 \times 10^{-16} \quad ✗$$