EE2 Fall 2015

94 Name:

EE 2 Midterm
(120 points total)

2 November 2015

Read the question and the possible answers before attempting any calculations. Write your name on the first page (each page if you separate the pages for any reason).

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Section 1 (60 points) Multiple choice Circle the correct answer. (50 pts.) There is no penalty for guessing. An educated guess with at least 50/50 probability is possible on most problems. **There is no partial credit on Section 1 (Multiple choice).**

$n_i = 1.79 \cdot 10^6$

donor

1. (4 pts) A sample of GaAs is doped with Arsenic ($N_d = 2x10^{16}$ /cm^3). What is the approximate equilibrium concentration of holes in the sample at 300 K.

   a. 2 x 10^{16} /cm^3 b. 1 x 10^4 c. 1.45 x 10^{10} d. x 10^2

   $p_o = \frac{n_i^2}{n_o} = \frac{(1.79 \cdot 10^6)^2}{2 \cdot 10^{16}}$ $N_d \simeq n_o$

2. (4 pts) A sample of Germanium has an intrinsic concentration of 2.4 x 10^{13} /cm^3 at room temperature (300 K). What is the approximate intrinsic concentration at 450 K?

   $\frac{2}{3} \cdot \frac{1}{300} = \frac{1}{450}$

   a. 2 x 10^{11} /cm^3 b. 3 x 10^{15} c. 4 x 10^{13} d. 9 x 10^9

   $n_{i_{300}} = 2.4 \cdot 10^{13}$ $n_{i_{450}} = N_c e^{-\left(\frac{E_c - E_i}{kT}\right)} e^{\frac{+E_g}{2 \cdot 3kT}}$ $-1 + \frac{1}{3} = -\frac{2}{3}$

c

3. (4 pts) A sample of silicon is doped with Phosphorous at $N_d = 10^{15}$. Which of the following is closest to the equilibrium electron concentration at 25 K. close to OK

   a. 10^{15} /cm^3 b. 2 x 10^{17} c. 10^{13} d. 2.8 x 10^{19}

c

$N_c = 3E19$

4. (5 pts) A semiconductor has a bandgap of 1 eV and an effective density of states in the conduction band of 3 x 10^{19} . It has n-type doping $N_d = 10^{16}$ and p-type doping $N_a = 7$ x10^{15}. What is the position of the Fermi level relative to the conduction band at 300 K ($E_c - E_F$). $E_g = 1eV$

   a. 0.208 eV b. 1.00 eV c. 0.239 eV d. -0.208 eV

   $E_c - E_F = 0.0259 \cdot \ln\left(\frac{3E19}{10^{16}} - 1\right) = .207$

d

5. (5 pts) From the 4 choices below which is the longest wavelength that a pure sample of InP will strongly absorb.

a. 0.20 μm   b. 2.0 μm   c. 1.20 μm   d. 0.60 μm

6. (5 pts) The classical momentum p_x corresponds to which of these quantum operators.

a. $\frac{\partial}{\partial t}$ b. $\frac{\hbar}{j}\frac{\partial}{\partial x}$ c. $\frac{\hbar^2}{j}\frac{\partial^2}{\partial x^2}$ d. $\frac{1}{2}mv^2$

c

7. (5 pts) Electrons in a semiconductor when undoped have a mean time between collisions of $\tau = 10$ nsec. When doped *only* with substance A at a concentration of X, $\tau = 0.2$ nsec. When doped *only* with substance B at a concentration Y, $\tau = 0.3$ nsec. What is the approximate mean time between collisions when doped with both substances?

a. 0.5 nsec   b. 10.5 nsec   c. 0.12 nsec   d. 0.21 nsec

$\tau_n = \frac{1}{\alpha(n_0+p_0)}$ $\frac{1}{\tau} = \frac{1}{10ns} + \frac{1}{0.2} + \frac{1}{0.3}$

τ = a number lower than 10, .2, .3

8. (5 pts) The probability of occupancy of electron states with energy $E=E_F$ at 300 K is:

a. 1   b. 0.5   c. $\exp[-E_F/kT]$ d. 10^{-13}

9. (5 pts) Undoped silicon is uniformly illuminated to produce steady state excess carriers of 10^{14}/cm^3 (both electrons and holes). If the recombination lifetime is 10 nsec (10^{-8} seconds), approximately how long will it take for the excess charge to reduce to 10^{11}/cm^3 (making it nearly intrinsic again).

$\Delta p = 10^{14}$
$\Delta n = 10^{14}$

a. 70 nsec   b. 7 nsec   c. 10 nsec   d. 140 nsec

$\tau = 10ns$

$\delta p = \Delta p\, e^{-t/\tau_n}$ $10^{11} = 10^{14} e^{-t/10ns}$

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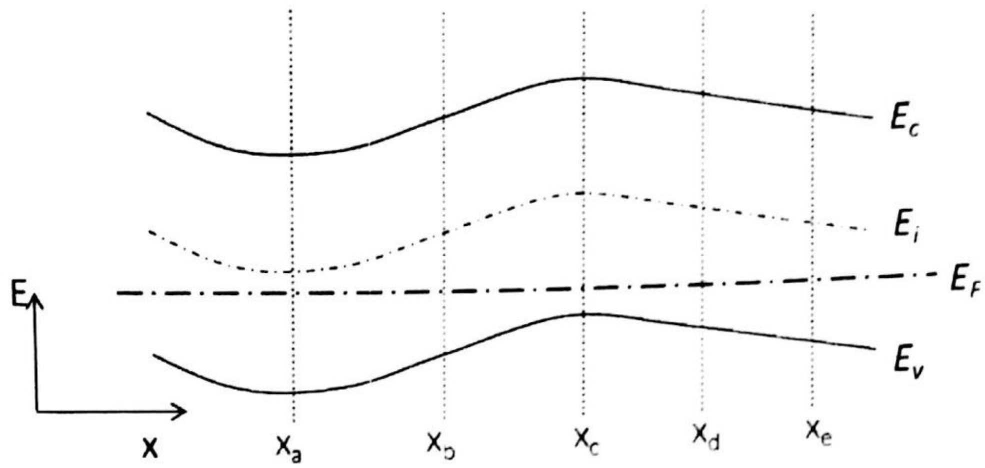

Referring to the band edge diagram above, answer the following questions. You may assume that the doping varies slowly throughout and that charge neutrality is maintained. The semiconductor is isolated. No calculations are necessary. Choose only one answer (there may be more than one correct answer… choose one only!)

10. (3 pts) Where is the electron concentration highest?

a. x_a b. x_b c. x_c d. x_d e. x_e

11. (3 pts) Where is the hole concentration highest?

a. x_a b. x_b c. x_c d. x_d e. x_e

12. (3 pts) At which of the following pairs of points is electric field equal in magnitude and sign?

a. x_a, x_b b. x_b, x_c c. x_c, x_d d. x_d, x_e e. x_b, x_e

13. (3 pts) Where is electric field nearly zero (choose one, there is more than one right answer)?

a. x_a b. x_b c. x_c d. x_d e. x_e

14. (3 pts) Is the semiconductor in equilibrium?

a. Yes b. No c. Can't tell

15. (3 pts) At which point do holes diffuse to the left?

a. x_a b. x_b c. x_c d. x_d e. x_e

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Section 2: Problems (60 points)

Show your work. Full credit for the correct answer with work shown. Sensible answers (correct order of magnitude) get partial credit. Generous partial credit for an incorrect answer with the correct ideas <u>if clear and brief</u>. Negative credit for irrelevant or incorrect equations.

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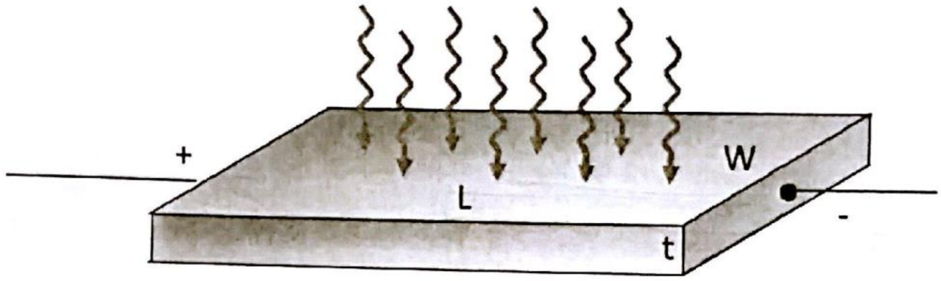
5. (30 pts) Optical Switch
Consider a thin slice of pure (intrinsic) silicon uniformly illuminated as shown below. The slab is thin enough so that carriers are generated uniformly throughout under steady-state illumination.



a) (10 pt) Write an expression for the resistance of the slab when there is no illumination. We want the end to end resistance in terms of charge, concentrations, mobilities and dimensions. Do not plug in any numbers yet!

10

intrinsic resistance $R = \frac{\rho L}{t w}$

$\boxed{R = \frac{L}{tw} \frac{1}{e n_i(\mu_n + \mu_p)}}$ ✓

$\sigma = e(n\mu_n + p\mu_p)$ for intrisic values so

$= 1.6E{-19}\ C\,(1.45E10(1350) + 1.45E10(480))$

$= 4.2456 \cdot 10^{-6}\ \frac{C\,cm^2}{Vs\ cm^3} \Rightarrow \frac{C}{V \cdot cm} \Rightarrow \frac{1}{\Omega\, cm}$ so $\rho =$

b) (5 pt) Calculate the resistance (without illumination) under the conditions that t = 1.0 µm, L = 100 µm, W = 25µm (additional values you may need can be found in the table on the formula sheet)

5

$\rho = 235538\ \Omega cm$

$L = 100\ \mu m = 100 \cdot 10^{-4}\ cm$

$w = 25\mu m = 25E{-4}\ cm$

$t = 1\mu m = 1E{-4}\ cm$

so $R = \frac{235538\ \Omega cm \cdot 100E{-4}\ cm}{1E{-4}cm \cdot 25E{-4}cm}$

$= \boxed{9.422 \cdot 10^{9}\ \Omega}$ ✓

$$\boxed{R = \frac{L}{wt} \frac{1}{e(\mu_n + \mu_p)(n_i + G_{op}\tau_n)}}$$

c) (10 pt) Write an expression for the resistance under illumination. Use G_{op} to signify the uniform optical generation rate of electron-hole pairs. Define any variables you introduce and indicate which parameters are known and which are unknown.

10

same answer

so $\delta n =$ steady amount quantity unknown

$\delta n = \delta p = G_{op} \cdot \tau_n$ for $\delta n \ll n_0 + p_0$ low injection

$G_{op} = \alpha_r((n_0 + p_0)\delta n + \delta n^2)$

$$R = \frac{L}{wt} \frac{1}{e(\mu_n n + \mu_p p)}$$

mean time for recombination $\tau_n = \frac{1}{\alpha(n_0 + p_0)}$ value unknown

where $n = n_0 + \delta n = n_0 + G_{op}\tau_n$

$= p = p_0 + \delta p = p_0 + G_{op}\tau_n$

so $\boxed{R = \frac{L}{wt} \frac{1}{e(\mu_n(n_0 + G_{op}\tau_n) + \mu_p(p_0 + G_{op}\tau_n))}}$

d) (5 pt) Now assume that the slab is under an illumination of $G_{op}=10^{15}$ EHP/cm^{3}sec. Suppose a resistance measurement is made under this condition resulting in a resistance 10^{-4} lower than the resistance in b). Calculate the carrier lifetime. [Hint: you can do this calculation without any of the information in a, b and c]

$n_0 = n_i = p_0$

5

$9.422 \cdot 10^5 =$

resistance 10^{-4} lower, then ρ is 10^{-4} lower the σ is 10^4 higher

$$10^4 = \frac{\cancel{e(\mu_n + \mu_p)}(n_i + G_{op}\tau_n)}{\cancel{e(\mu_n + \mu_p)}(n_i)}$$

$$10^4 = \frac{n_i + G_{op}\tau_n}{n_i}$$

$$10^4 = 1 + \frac{G_{op}\tau_n}{n_i}$$

$$\frac{G_{op}\tau_n}{n_i} = 10^4 - 1 \cong 10^4$$

so $\boxed{\tau_n \cong \frac{10^4 n_i}{G_{op}} = 0.145\text{s}}$

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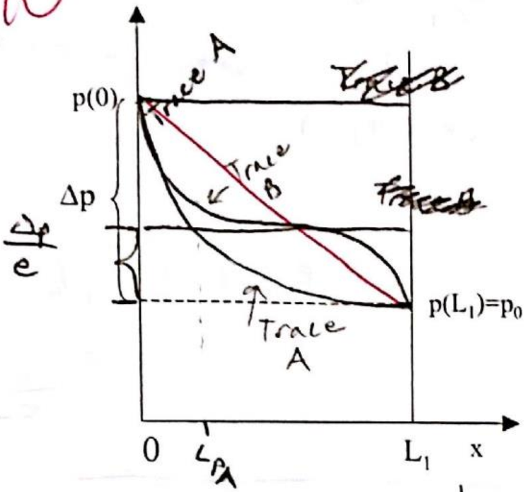
2. (30 pts) Diffusion in a short base diode.
A slice of n-type semiconductor has the boundary conditions shown. At one end there is a steady supply of excess holes, $\delta p(0)=\Delta p$, and at the other end the hole concentration is the equilibrium value, p_0.

a. (10 pts) Derive an expression for $\delta p(x) = p(x) - p_0$ in terms of the minority carrier diffusion length, L_p, the dimension, L_1, and the boundary value excess charge, Δp.

10



using general solution — diff eq.?

$$\delta p = C_1 e^{+x/L_p} + C_2 e^{-x/L_p}$$

input initial conditions

$$\delta p(0) = \Delta p = C_1 e^{x/L_p} + C_2 e^{-x/L_p}$$

$$\Delta p = C_1 + C_2$$

$$\delta p(L_1) = 0 = C_1 e^{L_1/L_p} + C_2 e^{-L_1/L_p}$$

system of equations, can solve for coefficients

$p(L_1) = p_0$

and $p(x) = p_0 + \delta p(x)$

so at $p(L_1) = p_0$

$\delta p(L_1) = 0$

$$C_1 e^{L_1/L_p} = -C_2 e^{-L_1/L_p}$$

$$-C_1 e^{L_1/L_p} e^{L_1/L_p} = C_2$$

$$\cancel{\Delta p = C_1 e^{x/L_p} + C_1 e}$$

$$\Delta p = C_1 - C_1 e^{2L_1/L_p}$$

$$\Delta p = C_1 (1 - e^{2L_1/L_p})$$

$$\boxed{C_1 = \frac{\Delta p}{(1 - e^{2L_1/L_p})}}$$

$$\frac{\Delta p}{(1 - e^{2L_1/L_p})} e^{L_1/L_p} = C_2 e^{-L_1/L_p}$$

$$C_2 = \frac{\Delta p\, e^{2L_1/L_p}}{(1 - e^{2L_1/L_p})}$$

$\boxed{\text{continues on scratch paper}}$

b. (10 pts) VERY IMPORTANT – Draw p(x) from 0 to L_1 **on a <u>linear</u> scale** for: two cases A) $L_1 = 10L_p$ ($L_1 >> L_p$) AND B) $L_1 = 0.1L_p$ ($L_1 << L_p$). (<u>Use the graph above and label the traces A and B</u>)

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(Next page for part c !!!)

Name:_

c. (5 pts) Simplify the expression you obtained in a) for the case where $L_p >> L_l$. [Hint: the Taylor expansion for $e^x = 1 + x$, for very small x.]

3

$e^x = 1 + x$ x very small

$x = e^x - 1$
$-x = 1 - e^x$

$$\delta p(x) = p(x) - p_0 = \frac{\Delta p}{1 - e^{-2}} e^{x/L_p} + \frac{\Delta p}{e^{-2} - 1} e^{-x/L_p}$$

-2 -2

$$= \frac{-\Delta p}{2}\left(e^{x/L_p} + e^{-x/L_p}\right)$$

$$= \frac{-\Delta p}{2}$$

seen on scratch paper

$100\,\mu m^2 = \frac{cm}{10000} \cdot \frac{}{10000}$

d. (5 pts.) Write an expression for the hole current density at x=0 for the $L_p >> L_l$ case in terms of known quantities (L_l, L_p, Δp and material constants)?

2

$$J_p = -e D_p \frac{dp(x)}{dx}$$

$$= -e D_p \left(-\frac{5\Delta p}{L_p} e^{x/L_p} - \frac{6\Delta p}{L_p} e^{-x/L_p} \right)$$

when $\boxed{J_p(0) = -e D_p \left(\frac{-11\Delta p}{L_p} \right)}$ $\quad D_p = \frac{kT\mu_p}{e}$

e. (5 pt bonus) If the semiconductor is n-type silicon with $N_d = 10^{16}$ and the dimension $L_l = 0.1\ \mu m$, $\Delta p = 6x10^{14}$, $L_p = 100\ \mu m$ and cross sectional Area $= 100\ \mu m^2$. What is the hole current at x=0?

$n_o = 10^{16}$

~~$\Delta p = 6 \cdot 10^{14}$~~

0

$$J_p(x) = J_p(0) = -e D_p \left(\frac{-11\Delta p}{L_p} \right) \rightarrow 7 \cdot 10^{18}$$

$p(0) = \Delta_p + p_o$
$= 6E14 + 1.45E10$
$= \Delta p$
$= 6E14$

$$D_p = \frac{kT\mu_p}{e} = \frac{0.0259 \cdot 410}{1.6E-19}$$

$$\boxed{I(0) = 7 \cdot 10^{18} \cdot A = 7 \cdot 10^{18} \cdot 100\,\mu m^2 = \boxed{7 \cdot 10^{12} A}}$$

$\cancel{\delta p}$

$$C_1 = \frac{\Delta p}{(1-e^{2L_1/L_p})}$$

$$\Delta p = C_1 + C_2$$

$$\Delta p = \frac{\Delta p}{1-e^{2L_1/L_p}} + C_2$$

$$C_2 = \Delta p - \frac{\Delta p}{1-e^{2L_1/L_p}}$$

$$C_2 = \Delta p \left(1 - \frac{1}{1-e^{2L_1/L_p}}\right)$$

SO

$\cancel{\delta p(x) = p_0 + \delta}$

$$\boxed{\delta p(x) = p(x) - p_0 = \frac{\Delta p}{(1-e^{2L_1/L_p})} e^{x/L_p} + \Delta p\left(1 - \frac{1}{1-e^{2L_1/L_p}}\right) e^{-x/L_p}}$$

2c) $L_p >> L_1$

$$\delta p(x) = \frac{\Delta p}{1-e^{0.2}} e^{x/L_p} + \Delta p\left(1 - \frac{1}{1-e^{0.2}}\right) e^{-x/L_p}$$

$$= \frac{\Delta p}{-0.2} e^{x/L_p} + \Delta p\left(1 - \frac{1}{-0.2}\right) e^{-x/L_p}$$

$$= \boxed{-5\Delta p \underbrace{e^{x/L_p}}_{= 1 + \frac{x}{L_p}} + 6\Delta p \underbrace{e^{-x/L_p}}_{= 1 - \frac{x}{L_p}}}$$