Name: [redacted]

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ECE 2 Midterm
(120 points total)
5 November 2019

<u>Read the question and the possible answers</u> before attempting any calculations. Write your name on the first page (each page if you separate the pages for any reason).

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Section 1 (60 points) Multiple choice Circle the correct answer. (60 pts.) There is no penalty for guessing. **There is no partial credit on Section 1 (Multiple choice)..**

1. (4 pts) A sample of GaAs is donor doped ($N_d = 3.2\text{x}10^{16}$ /cm^3). What is the approximate equilibrium concentration of holes in the sample at 300 K?

   a. 6.3 x 10^3 /cm^3 b. 1.8 x 10^6 /cm^3 c. 10^{-4} /cm^3 d. $3.2\text{x}10^{16}$ /cm^3

2. (4 pts) A 1 μm long silicon resistor with 1 μm^2 cross section is doped with Phosphorous at $N_d = 10^{16}$. What is the approximate current in the sample with an applied voltage of 10V?

   a. 0.32 mA b. 3.2 mA c. 1.6 mA d. 0.016 mA

3. (4 pts) A semiconductor has a bandgap of 1 eV and effective densities of states in the conduction and valence band both equal to 3 x 10^{19}. It has donor doping $N_d = 7\text{x}10^{16}$ and acceptor doping $N_a = 4\text{x}10^{16}$. What is the position of the Fermi level *relative to the conduction band, ($E_c - E_F$),* at 300 K.

   a. 0.82 eV b. 0.18 eV c. 1.00 eV d. 0.5 eV

4. (4 pts) From the 4 choices below which is the longest wavelength that a pure sample of GaP (Eg = 2.25 eV) will strongly absorb.

   a. 0.20 μm b. 0.70 μm c. 1.5 μm d. 0.50 μm

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boron 5 **B** 10.811	carbon 6 **C** 12.011	nitrogen 7 **N** 14.007
aluminium 13 **Al** 26.982	silicon 14 **Si** 28.086	phosphorus 15 **P** 30.974
gallium 31 **Ga** 69.723	germanium 32 **Ge** 72.61	arsenic 33 **As** 74.922
indium 49 **In** 114.82	tin 50 **Sn** 118.71	antimony 51 **Sb** 121.76

5. (4 pts) A small section of the periodic table of elements is shown on the left. Which of the following elements *could* be an *acceptor* in silicon?

   a. phosphorus b. germanium c. aluminum d. antimony

6. (4 pts) Which of the following is NOT a possible compound semiconductor.

   a. InP b. AsSb c. GaSb d. AlP

7. (4 pts) Consider the finite square potential well. Are the following statements True or False:

   a) There are a finite number of bound states in the well
   b) The wave function of a bound state is zero everywhere outside the well

   a. a-True b-True b. a-True b-False c. a-False b-True d. a-False b-False

8. (5 pts) Given the following normalized wave function of a particle (mass = m) in an infinitely deep well extending from 0 to L, which of the expressions below represents the expected value of momentum squared $<p_x^2>$:

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right)$$

a. $\int_0^L \frac{2}{L} p_x^2 \sin^2\left(\frac{3\pi}{L}x\right)dx$ **b.** $\int_0^L \frac{2}{L} x^2 \sin^2\left(\frac{3\pi}{L}x\right)dx$

c. $\int_0^L \frac{-i\hbar \cdot 6\pi}{L^2} \sin\left(\frac{3\pi}{L}x\right)\cos\left(\frac{3\pi}{L}x\right)dx$ **d.** $\frac{9\pi^2\hbar^2}{L^2}$ **e.** None

9. (4 pts) What is the approximate ionization energy of donors in InSb?

   a. 13.6 eV b. 0.007 eV c. 0.025 eV d. 0.0007 eV

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10. (4 pts) A sample of an unknown semiconductor has an intrinsic concentration of $3\text{x}10^{13}$ /cm^3 at room temperature (300 K) and $3\text{x}10^{15}$ /cm^3 at 450K. What is the approximate band gap of the semiconductor, E_g?

a. 0.65 eV b. 1.0 eV c. 0.45 eV d. 1.4 eV

(This area is left intentionally blank, use it for scratch paper for the multiple choice section)

$$n_i = \sqrt{N_c \cdot N_v}\, e^{\left(\frac{-E_g}{2kT}\right)}$$

$$\frac{n_i(450)}{n_i(300)} = \left(\frac{450}{300}\right)^{3/2} \cdot e^{\left(-\frac{E_g}{2k \cdot 450}\right)} \Big/ e^{\left(-\frac{E_g}{2k \cdot 300}\right)}$$

$$\frac{n_i(450)}{n_i(300)} = \left(\frac{450}{300}\right)^{3/2} \cdot e^{\left(\frac{E_g}{2k}\left(\frac{1}{300} - \frac{1}{450}\right)\right)}$$

$$\ln\left[\frac{n_{450}}{n_{300}} \cdot \frac{1}{L}\right] = \frac{E_g}{0.026}\left(1 - \frac{300}{450}\right)$$

$$E_g = 0.4066 \text{ eV}$$

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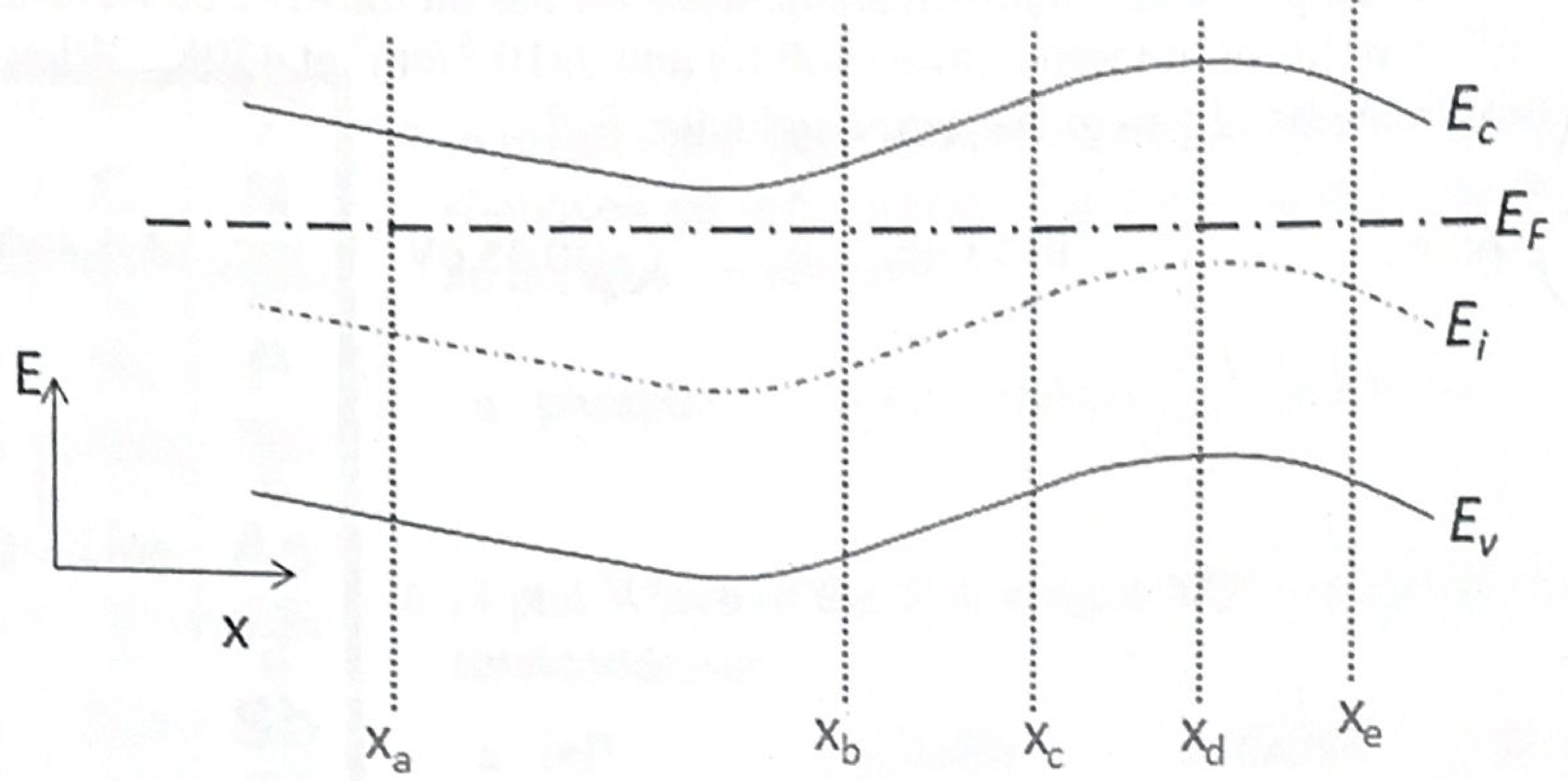

n-type

Answer the questions below based on the band diagram above. You may assume that the doping varies slowly throughout and that charge neutrality is maintained. The semiconductor is isolated. No calculations are necessary. Choose only one answer (there may be more than one correct answer… choose one only!)

11. (3 pts) Of the positions marked, where is the doping concentration lowest?

a. X_a b. X_b c. X_c d. X_d e. X_e

12. (3 pts) Of the positions marked, where is the electron concentration highest?

a. X_a b. X_b c. X_c d. X_d e. X_e

13. (3 pts) At which of these pairs of points do electrons diffuse to the right?

a. X_a, X_b b. X_b, X_c c. X_c, X_d d. X_c, X_e e. X_a, X_e

14. (3 pts) In which region is the built-in electric field constant throughout?

a. X_a to X_b b. X_b to X_c c. X_c to X_d d. X_a to X_e e. X_d to X_e

15. (3 pts) Is the semiconductor in equilibrium?

a. Yes b. No c. Can't tell

16. (3 pts) In which region is the doping described by $\cancel{N_a}$ N_d $(x) = A\exp(bx)$?

a. X_a to X_b b. X_b to X_c c. X_c to X_d d. X_a to X_e e. X_d to X_e

Section 2: Problems (60 points)

Show your work. Full credit for the correct answer with work shown. Sensible answers get partial credit. Generous partial credit for an incorrect answer with the correct ideas if clear and brief (without extraneous or irrelevant equations).

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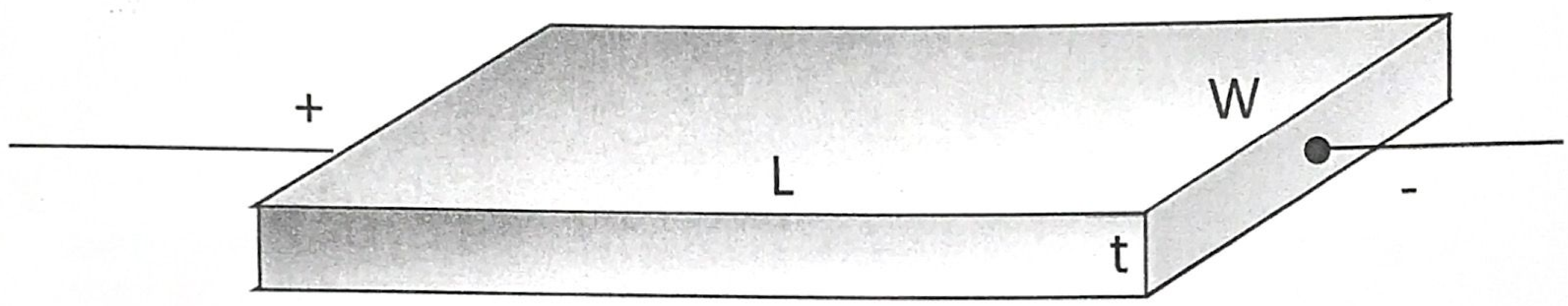
1. (25 pts) Resistance, resistivity, conductivity, current.



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a) (10 pt) Consider a semiconductor resistor at 300K as shown above. If the resistor is made of intrinsic indium antimonide, InSb, and L=W=100 μm, t=0.1μm, what is the resistance from end to end. Answers within +/- 5% get full credit.

$R = \frac{\rho}{t} \cdot \frac{L}{W}$

$\rho = \frac{1}{n \cdot e \cdot \mu_n + p \cdot e \cdot \mu_p}$

~~Since InSb is n-type semiconductor, we can neglect the affection from hole p~~

this is true because $\mu_n >> \mu_p$

~~Also, $n = n_i$ because it is intrinsic & at 300 K~~

$R = \frac{\rho}{t} \cdot \frac{L}{W} = \frac{1}{t(n_i \cdot e \cdot \mu_n + p_i \cdot e \cdot \mu_p)} \cdot \frac{L}{W} = \cancel{\frac{1}{(2.0\times10^{16}) \cdot (1.6\times10^{-19}C)(4.0\times10^{4}\,cm^2/V \cdot s) \cdot (0.1\times10^{-4}cm)} \cdot \left(\frac{100\,um}{100\,um}\right)}$

$= \frac{1 \cdot \left(\frac{100\,um}{100\,um}\right)}{[(2\times10^{16})(1.6\times10^{-19})(40000+800)] \cdot 10^{-5}cm}$

$= \boxed{77\ \Omega/square}$

this should have worked math

$\cancel{\boxed{= 780\ \Omega/square}}$

Correct

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b) (10 pt) Now consider another semiconductor resistor at 300K as shown above. If the resistor is made of p-type indium antimonide, InSb, doped at $N_a = 8\times10^{17}$ and L=W=100 μm, t=0.1μm, what is the resistance from end to end. Assume mobility doesn't change with doping. Answers within +/- 5% get full credit.

$p_o + N_d^+ = n_o + N_a^-$ $\Rightarrow$ $\left(\frac{n_i^2}{n_o}\right) = n_o + N_a$

~~There is no N_d^+ doping, and "n" can be neglect because it's p-type semiconductor~~

$n_i^2 = n_o^2 + n_o \cdot N_a$

$n_o = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 5.0\times10^{14}$

$p_o = \frac{n_i^2}{n_o} = \frac{(2\times10^{16})^2}{5.0\times10^{14}} = 8.0\times10^{17}$

~~$\therefore p = N_a^- = 8\times10^{17}$~~

$R = \frac{\rho}{t} \cdot \frac{L}{W} = \frac{1}{(n \cdot e \cdot \mu_n + p \cdot e \cdot \mu_p)t} \cdot \frac{L}{W} = \cancel{\frac{1}{(8\times10^{17})(1.6\times10^{-19}C)(800\,cm^2/V \cdot s) \cdot (0.1\times10^{-4}cm)} \cdot \left(\frac{100\,um}{100\,um}\right)}$

$\frac{1 \cdot \left(\frac{100\,um}{100\,um}\right)}{[(5.0\times10^{14})(4\times10^{4}) + (8.0\times10^{17})(800)] \cdot (1.6\times10^{-19})(0.1\times10^{-4})} = \boxed{941\ \Omega/square}$

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c) (5 pts) Find the doping level (either N_a or N_d) at which an InSb resistor like the one shown above would have a maximum value of resistance. Assume mobility doesn't change with doping. Answers within +/- 10% get full credit.

$$R = \frac{1}{(n\cdot e\cdot \mu_n + p\cdot e\cdot \mu_p)t}\cdot\frac{L}{W}$$

$$\cancel{(\frac{1}{R}) = (\frac{n_i^2}{p}\cdot\mu_n + p\cdot\mu_p)\cdot\frac{W\cdot e\cdot t}{L}}$$

$$= \frac{1}{(\frac{n_i^2}{p}\cdot\mu_n + p\cdot\mu_p)\cdot et}\cdot\frac{L}{W}$$

$$\cancel{\frac{d(\frac{1}{R})}{dp}}$$

$$= \frac{1}{\frac{n_i^2}{p}\cdot\mu_n + p\cdot\mu_p}\cdot\frac{L}{W\cdot e\cdot t}$$

$$\sigma = \frac{n_i^2}{p}e\cdot\mu_n + p\cdot e\cdot\mu_p$$

$$\frac{d\sigma}{dp} = -\frac{n_i^2}{p^2}e\cdot\mu_n + e\cdot\mu_p = 0$$

~~If the conductivity is the maximum then the resistance~~

$$\cancel{\frac{1}{R} = (\frac{n_i^2}{p}\cdot\mu_n + p\cdot\mu_p)\cdot\frac{W\cdot e\cdot t}{L}}$$

$$\mu_n\cdot\frac{n_i^2}{p^2} = \mu_p$$

$$\frac{dR}{dp} =$$

$$p = \sqrt{(\frac{\mu_p}{\mu_n})^{-1}}\cdot n_i$$

$$\boxed{p = 1.41\times10^{17}}$$

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d) (5 pts bonus) Is there a level of doping that would produce the same resistance as in the intrinsic case (choose one)?

a. Yes, there is a level of acceptor doping that would give the same resistance as in the intrinsic case.

b. Yes, there is a level of donor doping that would give the same resistance as in the intrinsic case.

c. No, there is no level of doping that would give the same resistance as in the intrinsic case.

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e) (5 pts bonus… possibly) What is this doping level?

$$R = \frac{\rho}{t}\cdot\frac{L}{W}$$

since t, L, W are the same, we only need to consider ρ

$$\frac{1}{p\cdot e\cdot\mu_p} = \frac{1}{n_0 e\mu_n + p_0 e\mu_p}$$

$$\cancel{\rho_0 = \frac{1}{n\cdot e\cdot\mu_n}} \quad \cancel{\rho_1 = \frac{1}{p\cdot e\cdot\mu_p}}$$

$$\cancel{\text{let } \rho_0 = \rho_1 \quad \frac{1}{n\cdot e\cdot\mu_n} = \frac{1}{p\cdot e\cdot\mu_p}}$$

$$\cancel{p = \frac{n\mu_n}{\mu_p} = \frac{\frac{n_i^2}{p}\mu_n}{\mu_p}}$$

$$\cancel{p = \sqrt{n_i^2\cdot\frac{\mu_n}{\mu_p}}}$$

$$\cancel{\boxed{p = 1.41\times10^{17}}}$$

$$p\cdot\mu_p = n_i(\mu_n + \mu_p)$$

$$\frac{n_i^2}{n} = n_i\frac{\mu_n + \mu_p}{\mu_p}$$

$$n = n_i\cdot\frac{\mu_p}{\mu_n + \mu_p}$$

$$n = 3.9\times10^{14}$$

$$\boxed{p = \frac{n_i^2}{n} = 1.02\times10^{18}}$$

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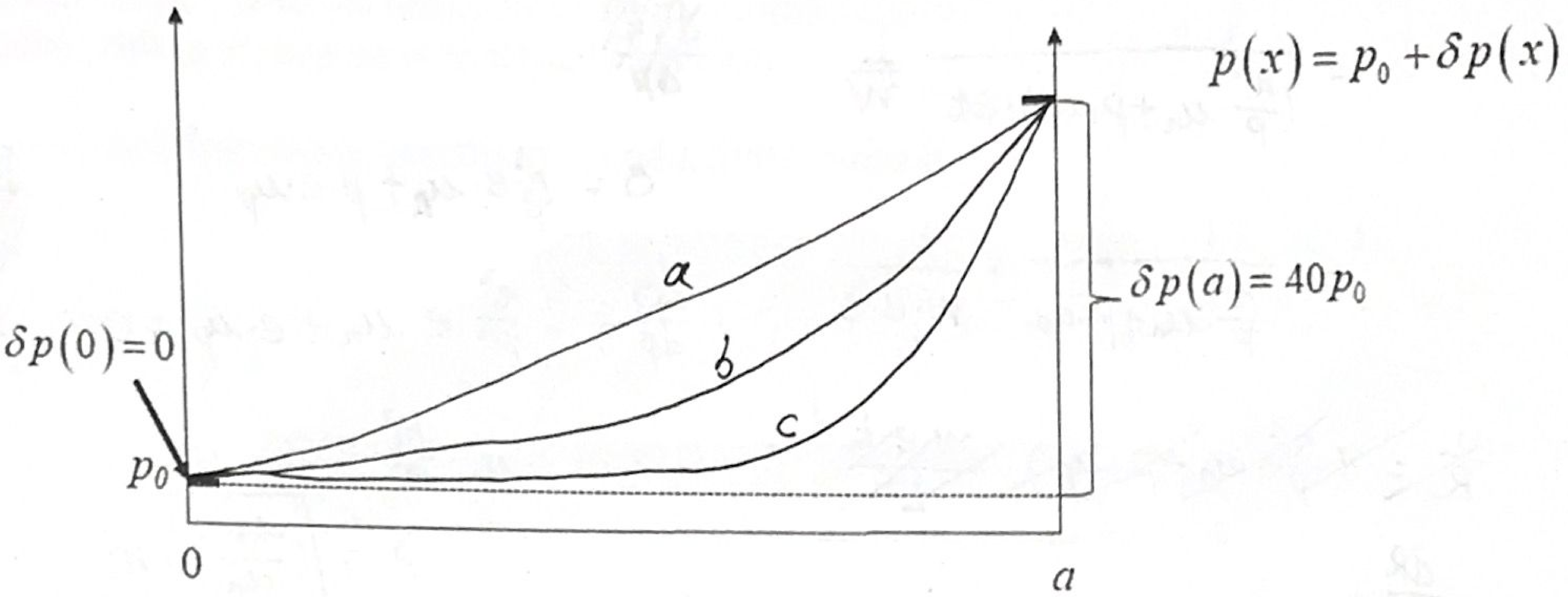
2. (35 pts) Diffusion in long and short base diodes.
A slice of n-type semiconductor has the boundary conditions shown. At the right end there is a steady supply of excess holes, $\delta p(a) = 40p_0$, and at the left end the hole concentration is at equilibrium, that is $\delta p(0) = 0$.



Please sketch the excess hole concentration from 0 to a for the following 3 cases with different diffusion lengths. Please be sure to identify the three traces with the proper letter a), b) or c). (Traces *not labeled* lose about half the credit)

$\delta p(t) = C_1 e^{x/L_p} + C_2 \cdot e^{-x/L_p}$

15/15

a) (5 pts) $L_p = 5a$
b) (5 pts) $L_p = a/2$
c) (5 pts) $L_p = a/5$

The traces do not need to be numerically precise but they must have the correct characteristics. I'm looking for details of each and relationships between the three.

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d) (10 pts) Solve for the excess holes for case b above only: Write down the appropriate differential equation. Write down the general solution. Apply the boundary conditions and simplify $\delta p(x)$ as much as possible in terms of given parameters.

$$\boxed{\delta p(x) = \left(\frac{40p_0}{e^2 - e^{-2}}\right)\left(e^{2x/a} - e^{-2x/a}\right)}$$

steady state

$$\frac{\partial \delta p(x,t)}{\partial t} = D_p \cdot \frac{\partial^2 \delta p(x)}{\partial x^2} - \frac{\delta p}{\tau_p} = 0$$

$$\therefore \frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p(x)}{D_p \cdot \tau_p} = \frac{\delta p(x)}{L_p^2}$$

General Solution

$$\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p}$$

Initial Condition

$\delta p(0) = 0$

$\delta p(a) = 40p_0$

$$\begin{cases} 0 = C_1 + C_2 \\ 40p_0 = C_1 e^{a/L_p} + C_2 e^{-a/L_p} \end{cases}$$

$$\therefore 40p_0 = C_1 \left(e^{a/L_p} - e^{-a/L_p}\right)$$

$$C_1 = \frac{40p_0}{e^{a/L_p} - e^{-a/L_p}} = \frac{40p_0}{e^2 - e^{-2}}$$

$$C_2 = -C_1 = \frac{-40p_0}{e^{a/L_p} - e^{-a/L_p}} = \frac{-40p_0}{e^2 - e^{-2}}$$

$$\cancel{\therefore \delta p(x) = \left(\frac{40p_0}{e^2 - e^{-2}}\right)\left(e^{x/L_p} - e^{-x/L_p}\right)}$$

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(more space on the next page and part e !!!)

e) (10 pts) For the case you solved in part d, find the ratio of diffusion currents at the two ends $J_p(0)/J_p(a)$ (be sure the ratio has the proper sign).

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$$J_p(x) = e \cdot D_p \cdot \frac{d(P_0+\delta P)}{dx} = e \cdot D_p \cdot \frac{d\delta p(x)}{dx}$$

$$\frac{J_p(0)}{J_p(a)} = \frac{e \cdot D_p \cdot \frac{d\delta p(0)}{dx}}{e \cdot D_p \cdot \frac{d\delta p(a)}{dx}} = \frac{\frac{40P_0}{e^2-e^{-2}} \cdot \frac{d}{dx}\left(e^{2x/a} - e^{-2x/a}\right)}{\frac{40P_0}{e^2-e^{-2}} \cdot \frac{d}{dx}\left(e^{2x/a} - e^{-2x/a}\right)} \quad \begin{matrix} x=0 \\ \\ x=a \end{matrix}$$

$$= \frac{\frac{2}{a} \cdot e^0 + \frac{2}{a} \cdot e^0}{\frac{2}{a} \cdot e^{2a/a} + \frac{2}{a} \cdot e^{-2a/a}}$$

$$= \boxed{\frac{2}{e^2+e^{-2}}}$$