EE 2 Midterm
(120 points total)
8 November 2017

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Read the question and the possible answers before attempting any calculations. Write your name on the first page (each page if you separate the pages for any reason).

50

Section 1 (60 points) Multiple choice Circle the correct answer. (50 pts.) There is no penalty for guessing. An educated guess with at least 50/50 probability is possible on most problems. **There is no partial credit on Section 1 (Multiple choice)…**

1. (4 pts) A sample of InSb is donor doped ($N_d = 2x10^{16}$ /cm^3). What is the approximate equilibrium concentration of holes in the sample at 300 K.

   a. 1.3 x 10^{16} /cm^3 b. 3.2 x 10^{16} /cm^3 c. ~~1.3~~ x 10^{10} /cm^3 d. ~~10^4~~ /cm^3

   $n_o p_o = n_i^2$

   $p_o = \frac{n_i^2}{N_d} = \frac{(2\times10^{16})^2}{2\times10^{16}} = 2\times10^{16}$

2. (4 pts) A sample of Germanium has an intrinsic concentration of 2.4 x 10^{13} /cm^3 at room temperature (300 K). What is the approximate intrinsic concentration at 400 K?

   a. 2.7 x ~~10^{11}~~ /cm^3 b. 8.8 x 10^{14} /cm^3 c. 3.6 ~~x~~ 10^{13} /cm^3 d. 9 ~~x~~ 10^9 /cm^3

3. (4 pts) A sample of silicon is doped with Phosphorous at $N_d = 10^{17}$. What is the approximate resistivity of the sample.

   a. 0.8 Ωcm b. 0.08 Ωcm c. 8 Ωcm d. 0.008 Ωcm

   $\rho = \frac{1}{\sigma} = \frac{1}{n e \mu_n} = \frac{1}{10^{17}\times1.6\times10^{-19}\times800} = 0.08$

4. (5 pts) A semiconductor has a bandgap of 1 eV and effective densities of states in the conduction and valence band both equal to 3 x 10^{19} . It has donor doping $N_d = 10^{16}$ and acceptor doping $N_a = 7 x10^{16}$. What is the position of the Fermi level *relative to the conduction band, ($E_c - E_F$)*, at 300 K.

   a. ~~-0.16 eV~~ b. ~~1.00 eV~~ c. 0.16 eV d. 0.84 eV

   $E_c - E_F = 0.026 \ln\left(\frac{3\times10^{19}}{10^{16} - 7\times10^{16}}\right) = 0.16$ eV

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5. (5 pts) From the 4 choices below which is the longest wavelength that a pure sample of InP (Eg = 1.35 eV) will strongly absorb.

a. 0.20 μm   b. ~~2.0 μm~~   c. 0.80 μm   ~~d. 1.1 μm~~

6. (5 pts) In the finite square potential well are the following statements True or False:

a) The wave function of a bound state is zero everywhere outside the well
b) There are a finite number of bound states in the well yes

a. a-True b-True   ~~b. a-True b-False~~   c. a-False b-True   d. ~~a-False~~ b-False

7. (5 pts) Given the following wave function of a particle (mass = m) in an infinitely deep well extending from 0 to L, which of the expressions below represents the expected value of momentum <p$_x$>:

$\langle Q \rangle$

$$\psi(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{3\pi}{L}x\right)$$

$$\frac{\int_{-\infty}^{\infty}\sqrt{\frac{2}{L}}\sin\left(\frac{3\pi}{L}x\right) P_x \cdot \sqrt{\frac{2}{L}}\sin\left(\frac{3\pi}{L}x\right) dx}{\int_{-\infty}^{\infty}\left(\sqrt{\frac{2}{L}}\sin\left(\frac{3\pi}{L}x\right)\right)^2 dx}$$

a. $\int_0^L \frac{2}{L} p_x \sin^2\left(\frac{3\pi}{L}x\right)dx$ ~~b.~~ $\cancel{\int_0^L \frac{-i\hbar\cdot 6\pi}{L^2}\sin\left(\frac{3\pi}{L}x\right)\cos\left(\frac{3\pi}{L}x\right)dx}$

~~c.~~ $\cancel{\int_0^L \frac{2}{L}x\sin^2\left(\frac{3\pi}{L}x\right)dx}$ ~~d.~~ $\cancel{\hbar k = \frac{3\pi\hbar}{L}}$ ~~e. None~~ ← Must do operator 1st

8. (5 pts) A doped layer of thickness 0.1μm in a semiconductor has a sheet resistance of 50 Ω/square. The layer is not necessarily uniformly doped. A resistor is made of this layer of length 100 μm and width 5 μm. What is the value of the resistance?

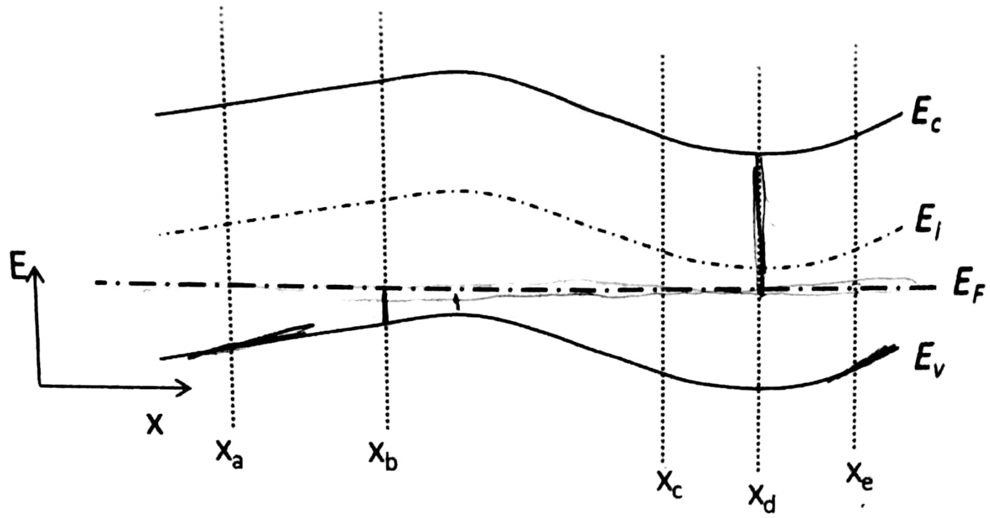
L

a. 1000 Ω   b. 50 Ω   c. 0.05 Ω   d. 10 kΩ

$$R = \frac{\rho L}{tW} = \frac{5\times 10^{-6}\cdot 100\times 10^{-6}}{5\times 10^{-6}\times 0.1\times 10^{-6}}$$

9. (5 pts) The probability of occupancy of electron states with energy E=E$_F$ at 300 K is:

a. 1   b. 10^{-13} c. exp[-E$_F$/0.026]   d. 0.5



Answer the questions below based on the band diagram above. You may assume that the doping varies slowly throughout and that charge neutrality is maintained. The semiconductor is isolated. No calculations are necessary. Choose only one answer (there may be more than one correct answer… choose one only!)

10. (3 pts) Of the positions marked, where is the electron concentration highest?

a. X_a b. X_b c. X_c d. X_d e. X_e

11. (3 pts) Of the positions marked, where is the hole concentration highest?

a. X_a b. X_b c. X_c d. X_d e. X_e

12. (3 pts) At which of these pairs of points is electric field equal?
slope.

a. X_a, X_c b. X_b, X_c c. X_a, X_b d. X_c, X_e e. X_b, X_d

13. (3 pts) At which of these pairs of points is doping equal?

a. X_a, X_b b. X_c, X_e c. X_c, X_d d. X_a, X_c e. X_b, X_d

14. (3 pts) Is the semiconductor in equilibrium?

a. Yes b. No c. Can't tell

15. (3 pts) At which point does electric field have its highest value?
Steepest slope

a. X_a b. X_b c. X_c d. X_d e. X_e

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Section 2: Problems (60 points)

Show your work. Full credit for the correct answer with work shown. Sensible answers get partial credit. Generous partial credit for an incorrect answer with the correct ideas if clear and brief (without extraneous or irrelevant equations).

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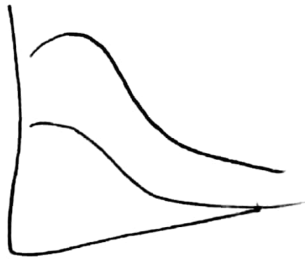
1) (30 pts) Scattering

We saw in class that there are two types of scattering mechanisms in semiconductors: ionized impurity scattering (which involves dopant atoms) and lattice scattering (which exists regardless of whether there are dopant atoms or not).

The following table shows some experimental data for the electron mobility of a new semiconductor for two doping conditions. The effective mass of the semiconductor has been previously determined to be $m_n^* = 0.1m_0$. $N_c = N_v = 5\times10^{18}$ and $E_g = 1.5$ eV.

Doping		Mobility	Mean time
No intentional doping ($\sim10^{14}$ /cm^3)	μ_0	10,000 cm^2/Vsec	5.69×10^{-9} s
10^{17}	μ_1	3,333	3.12×10^{-7} s
2×10^{17}	μ_2	2499.8	7.77×10^{-6} s
10^{18}	μ_3	1428.448	2.09×10^{-4} s



a) (10 pt) Write a *symbolic expression* for μ_2 the mobility at a doping of 2×10^{17} in terms of the two given mobilities, μ_0, μ_1, given quantities and physical constants. Calculate this mobility and *write the number in the table.* Define any intermediate variables you introduce and indicate which parameters are known and which are unknown.

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$$\frac{1}{\mu_2} = \frac{1}{\mu_0} + \frac{1}{\mu_1} \Rightarrow \boxed{\mu_2 = \frac{\mu_0\mu_1}{\mu_1+\mu_0}} = \frac{(10{,}000)(3333)}{10{,}000+3333} = \boxed{2499.8 \text{ cm}^2/\text{Vsec}}$$

It makes sense that $\mu_2 < \mu_1 < \mu_0$ because as the Doping concentration increases, mobility decreases.

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b) (10 pt) Write an expression for μ_3, the mobility at a doping of 10^{18}, in terms of the given mobilities (and possibly your result in part a). Calculate this mobility and *write the number in the table.*

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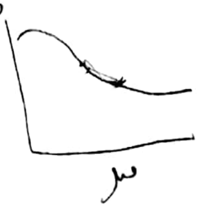
$$\frac{1}{\mu_3} = \frac{1}{\mu_1} + \frac{1}{\mu_2} \Rightarrow \frac{\mu_1 \cdot \mu_2}{\mu_1 + \mu_2} = \frac{(3333)(2499.8)}{3333 + 2499.8} = \mu_3$$

$$\boxed{\mu_3 = 1428.45 \ V/cm^2/usec}$$

It makes sense that $\mu_3 < \mu_2 < \mu_1 < \mu_0$ because like in part a, mobility decreases as doping increases like illustrated in the chart.



$$\frac{\tau_1}{\mu_1} = \frac{\tau_0}{\mu_0}$$

c) (10 pt) Calculate the mean time between scattering events (collisions) for the four cases and *fill in the table under the "Mean time" heading. Be sure to include the units!*

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units & scaling

- $\mu = \frac{e\tau}{M^*} \rightarrow \tau_0 = \frac{\mu_0 m^*}{e} = \frac{(10{,}000)(0.1 M_0)}{1.6 \times 10^{-19}} = \underline{5.69 \times 10^{\cancel{-9}\ 13} \ s}$

- $\tau_1 = \tau_0 \ln\left(\frac{\Delta n}{n_i}\right) = 5.69 \times 10^{-9} \ln\left(\frac{10^7 - 10^4}{1.5 \times 10^6}\right) = \underline{3.12 \times 10^{-7} \ s}$

where $n_i = \sqrt{N_V N_C} \ e^{-E_g/2kT} = 5 \times 10^{18} \ e^{-1.5/2(0.026)} = 1.5 \times 10^6/cm$

- $\tau_2 = \tau_1 \ln\left(\frac{\Delta n}{n_i}\right) = (3.12 \times 10^{-7}) \ln\left(\frac{2 \times 10^{17} - 10^{17}}{1.5 \times 10^6}\right) = \underline{\cancel{7.77} \times 10^{-6} \ s}$

why is τ_2 μsec when τ_1 is nsec

- $\tau_3 = \tau_2 \ln\left(\frac{\Delta n}{n_i}\right) = (7.77 \times 10^{-9}) \ln\left(\frac{10^{18} - 2 \times 10^{17}}{1.5 \times 10^6}\right) = \underline{2.09 \times 10^{-4} \ S}$

We should see an increase in the mean time, which we do

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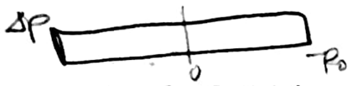
2. (30 pts) Diffusion in long and short base diodes.
A slice of n-type semiconductor has the boundary conditions shown. At one end there is a steady supply of excess holes, $\delta p(-a)=\Delta p$, and at the other end the hole concentration is maintained at zero, that is $\delta p(a)= -p_0$.



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a. (10 pts) Write down the applicable diffusion equation for $\delta p(x)$ in terms of the minority carrier diffusion length, L_p. Write down the general solution. Write down the two equations obtained by applying the boundary conditions in terms of a and the two boundary values. **You do not need to solve the system of equations for the general case.**

$$\frac{d^2\delta p(x)}{dx^2} = \frac{\delta p(x)}{L_p^2} \quad (1)$$

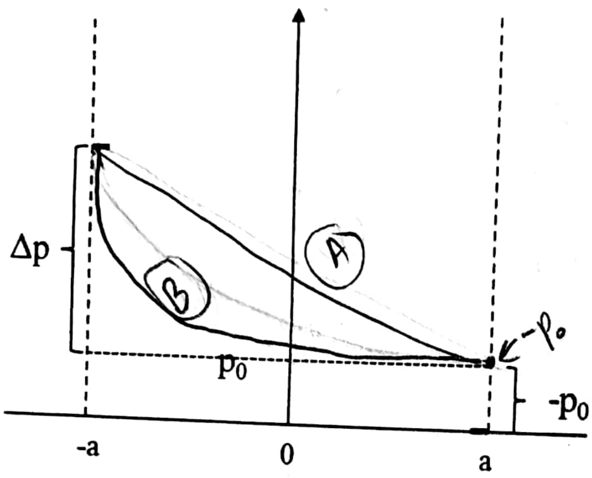
$$\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p} \quad (2)$$

- $\Delta p = C_1 e^{-a/L_p} + C_2 e^{a/L_p} \quad (3)$
- $-p_0 = C_1 e^{a/L_p} + C_2 e^{-a/L_p} \quad (4)$

applying the boundary conditions

$\delta p(-a) = \Delta p$

$\delta p(a) = -p_0$



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b. (10 pts) VERY IMPORTANT – *Sketch* p(x) from –a to a **on a linear scale** for: two cases **A)** a =0.1L_p (a << L_p) AND **B)** a = 10L_p (a >> L_p). (Use the graph above and **label** the traces **"A"** and **"B" unambiguously**). You will get credit for getting the endpoints correct, the shapes and other details which you can deduce from part a) above… there will be partial credit but the better the sketch the higher your score.

(Next page for part c !!!)

c. (5 pts) Simplify the equation for $\delta p(x)$ that you obtained in a) for the case where $L_p >> a$. [Hint: the Taylor expansion for $e^x = 1 + x$, for $x << 1$]. Apply the boundary conditions and solve for the unknown constants.

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$$\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p}$$

$\star \ \frac{x}{L_p} << 1$ small $L_p >> a$

$$\delta p(x) = C_1\left(1+\frac{x}{L_p}\right) + C_2\left(1-\frac{x}{L_p}\right)$$

$$\delta p(x) = C_1 + C_1\frac{x}{L_p} + C_2 - C_2\frac{x}{L_p} = (C_1+C_2) + \frac{x}{L_p}(C_1-C_2)$$

$$\Delta p = (C_1+C_2) - \frac{a}{L_p}(C_1-C_2) \ ; \ -p_0 = (C_1+C_2) + \frac{a}{L_p}(C_1-C_2)$$

$$\Delta p = C_1 + C_2 - \frac{a}{L_p}C_1 + \frac{a}{L_p}C_2 \Rightarrow \Delta p = C_1\left(1-\frac{a}{L_p}\right) + C_2\left(1+\frac{a}{L_p}\right)$$

See scratch paper for more work

$$C_1 = C_2 = \frac{\Delta p - p_0}{4} - \frac{\Delta p L_p}{4a}$$

$$\text{so } \boxed{\delta p(x) = \frac{\Delta p - p_0}{2} + x\left(\frac{\Delta p + p_0}{2a}\right)}$$

d. (5 pts.) Write an expression for the hole current density at x=0 for the $L_p >> a$ case in terms of known quantities (a, L_p, Δp and material constants)?

5

$$J_p(x) = e\mu_p \cancel{p(x)E(x)} - eD_p\frac{dp(x)}{dx} = -eD_p\frac{dp(x)}{dx}$$

$$L_p = \sqrt{D_p \tau}$$

$$D_p = \frac{L^2}{\tau}$$

$$\boxed{J_p(x) = -eD_p \cdot \frac{(\Delta p + p_0)}{2a}}$$

where D_p is the diffusion constant

$$\delta p(x) = \frac{\Delta p - p_0}{2} + \frac{x}{L_p}\left(\frac{\Delta p L_p}{2a}\right)$$

$$\frac{d\,\delta p(x)}{dx} = \frac{\Delta p}{2a}$$

doped $\mu = 1200$

e. (5 pt **bonus**) If the semiconductor is n-type silicon with $N_d = 10^{16}$ and the dimension $a = 0.5$ µm, $\Delta p = 6\text{x}10^{14}$, $L_p = 100$ µm and cross sectional Area = 10^4 µm^2. What is the total hole current at x=0? Is the current positive or negative?

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$$I_{total} = \cancel{A n e V_s} - \frac{eD_p\Delta p}{2aA}$$

$$\frac{D}{\mu} = \frac{kT}{e} \rightarrow D = \frac{kT\mu}{e} = 1.95\ \cancel{\times 10^{20}}$$

$$I_{total} = (10^4\times10^{-6})(10^{16})(1.6\times10^{-19})(2\times10^7) - \frac{(1.9\times10^{-19})(1.95\times10^{20})(6\times10^{14})}{(10^4\times10^{-6})\times 2(0.5\times10^{-6})}$$

$$I_{total} = 3.2\times10^{-14} - 2.223\times10^{24} = \boxed{-2.22\times10^{24}\ A}$$

Current is Negative

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c) $-P_0 = C_1 + C_2 + \frac{a}{4\rho} C_1 - \frac{a}{L_p} C_2 = C_1\left(1 + \frac{a}{2\rho}\right) + C_2\left(1 - \frac{a}{L_p}\right)$

$\Delta p = C_1\left(1 - \frac{a}{L_p}\right) + C_2\left(1 + \frac{a}{L_p}\right)$

$C_1 = \frac{\Delta p + C_2\left(1 + \frac{a}{L_p}\right)}{\left(1 - \frac{a}{L_p}\right)} = \frac{\Delta p}{\left(1 - \frac{a}{4}\right)} + C_2\left(1 + \frac{a}{L_p}\right)\left(1 - \frac{a}{4}\right)^{-1}$

$\Delta p = C_1 + C_2 - \frac{a}{L_p}\cancel{(C_1 - C_2)} \longrightarrow$ $\quad \frac{\Delta p L_p}{4a} + \cancel{\frac{\Delta p - P_0}{4}} - \frac{\Delta p - P_0}{4} + \frac{\Delta p L_p}{4a}$

$-P_0 = C_1 + C_2 + \frac{a}{L_p}\cancel{(C_1 - C_2)}$

$\Delta p - P_0 = 2C_1 + 2C_2$

$\Delta p - P_0 = 2(C_1 + C_2) \rightarrow C_1 + C_2 = \frac{\Delta p - P_0}{2} \rightarrow C_2 = \frac{\Delta p - P_0}{2} - C_1$

So $\Delta p = \frac{\Delta p - P_0}{2} - \frac{a}{L_p}(C_1 - C_2)$ $\quad -2P_0$

$-P_0 = \frac{\Delta p - P_0}{2} + \frac{a}{L_p}(C_1 - C_2)$ $\quad +2P_0$

$0 = \Delta p + \frac{2a}{L_p}(C_1 - C_2)$

$\Delta p = \frac{2a}{L_p} C_1 - \frac{2a}{L_p} C_2 \rightarrow C_1 = \left(\Delta p + \cancel{\frac{2a}{L_p}} C_2\right)\cancel{\frac{L_p}{2a}}$

$C_1 = \frac{\Delta p L_p}{2a} + C_2$

$C_1 = \frac{\Delta p L_p}{2a} + \frac{\Delta p - P_0}{2} - C_1 \rightarrow C_1 = \frac{\Delta p L_p}{4a} + \frac{\Delta p - P_0}{4}$

$C_2 = \frac{\Delta p - P_0}{2} - \frac{\Delta p L_p}{4a} - \frac{\Delta p - P_0}{4} = \frac{2\Delta p - 2P_0 - \Delta p + P_0}{4} - \frac{\Delta p L_p}{4a}$

$C_2 = \frac{\Delta p - P_0}{4} - \frac{\Delta p L_p}{4a}$ $\qquad C_1 = C_2$

[redacted]