

EE 2 Midterm
(120 points total)

31 October 2016

Read the question and the possible answers before attempting to write your name on the first page (each page if you separate the pages)

48

Section 1 (60 points) Multiple choice Circle the correct answer. ... There is no penalty for guessing. An educated guess without calculation and with at least 50/50 probability is possible on most problems.

1. (4 pts) A sample of GaAs has an intrinsic concentration of $1.8 \times 10^6 / \text{cm}^3$ at room temperature (300 K). What is the approximate intrinsic concentration at 450 K?

- a. $2 \times 10^8 / \text{cm}^3$ b. 2×10^{10} c. 2×10^3 d. 2×10^6

$n_i = 1.8 \times 10^6 \text{ cm}^{-3}$
 $n_i = \left(\frac{T}{T_0}\right)^{3/2} \sqrt{N_v(T_0) N_c(T_0)} \exp\left(-\frac{E_g}{2kT}\right)$

2. (4 pts) A sample of silicon is doped with Phosphorous at $N_d = 10^{15}$. Which of the following is closest to the equilibrium electron concentration at 25 K.

- a. $8 \times 10^{14} / \text{cm}^3$ b. 10^{12} c. 2×10^{17} d. 2.8×10^{19}

$N_d = 10^{15}$

$n_i = 10^{10} \text{ cm}^{-3} \left(\frac{m_0}{2.6 \times 10^1}\right)$ (3.6) (4)

3. (5 pts) In the finite square potential well are the following statements True or False:



- a) The wave function of a bound state can be non-zero outside the well
 b) There are an infinite number of bound states in the well

- a. a-True b-True b. a-True b-False c. a-False b-True d. a-False b-False

$\frac{dE}{dk} = (3.6)$

4. (5 pts) Band structure calculation of a new semiconductor results in the following relationship between energy and k-vector at the conduction band minimum:

$E = 3.6 \frac{\hbar^2 k^2}{m_0} + 0.17$

What is the effective mass of electrons (expressed as a multiple of the free electron mass m_0) in this new semiconductor?

- a. $3.6m_0$ b. $0.28m_0$ c. $7.2m_0$ d. $0.14m_0$

$\frac{\hbar^2 k^2}{m_0} = \frac{m_0}{4(3.6)} \frac{d^2 E}{dk^2}$ (4)(3.6)

$m^* = \frac{\hbar^2}{d^2 E / dk^2}$

$\frac{d^2 E}{dk^2} = (3.6 \frac{\hbar^2}{m_0})(2)$
 $\frac{d^2 E}{dk^2} = (4)(3.6) \frac{\hbar^2}{m_0}$

$\frac{1}{3.6}$

5. (5 pts) From the 4 choices below which is the longest wavelength that a pure sample of GaSb will strongly absorb.

- a. 1.20 μm b. 2.0 μm ~~c. 0.20 μm~~ d. 0.60 μm

6. (4 pts) Given the following wave function of a particle (mass = m) in an infinite well, determine the energy of the state:

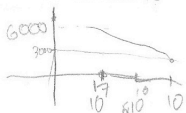
$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi}{L}x\right)$ $\psi(x) > 0$
 $\psi(L) = 0$

- a. $E = \frac{\pi^2 \hbar^2}{2mL^2}$ b. $E = \frac{9\sqrt{2}\pi^2 \hbar^2}{2mL^2 \sqrt{L}}$ c. $E = \frac{9\pi^2 \hbar^2}{2mL^2}$ d. $E = \frac{4\pi^2 \hbar^2}{2mL^2}$

$k = \sqrt{\frac{2mE}{\hbar^2}}$ $\sqrt{\frac{2}{L}} \sin(3\pi x)$

7. (5 pts) An unknown semiconductor has a mobility of 3000 cm^2/Vs when doped at $N_d = 10^{17}/\text{cm}^3$ and a mobility of 6000 cm^2/Vs when doped at $5 \times 10^{16}/\text{cm}^3$. What is the best estimate of the mobility when doped at 1.5×10^{17} ?

- a. 4500 cm^2/Vs b. 3000 cm^2/Vs c. 700 cm^2/Vs d. 2000 cm^2/Vs



$k = \frac{2mE}{\hbar^2}$ $E = (k^2) \frac{\hbar^2}{2m}$
 $E = \left(\frac{9\pi^2}{L^2}\right) \frac{\hbar^2}{2m}$

8. (3 pts) In any system in equilibrium the percentage of occupied electron states with energy $E = E_F$ at 300 K is:

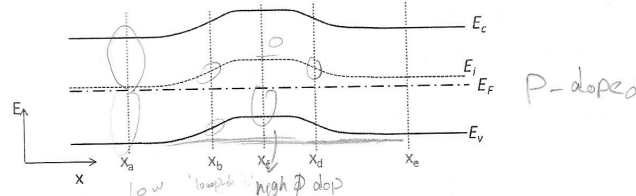
- a. 50% b. <1% c. $\exp[-E_F/kT]$ d. 100%

$E_n \text{ (scaly)} = 1.45 \times 10^{14} \text{ cm}^{-3}$

9. (5 pts) Undoped silicon is uniformly illuminated to produce steady state excess carriers of $1.45 \times 10^{14}/\text{cm}^3$ (both electrons and holes). When the light is turned off, the electron concentration reaches $2n_i$ in 400 nsec. What is the approximate value of the recombination lifetime?

- a. 13 nsec b. 400 nsec c. 43 nsec d. 800 nsec

$\tau_s = \frac{-(400 \text{ nsec})}{\ln\left(\frac{2 \times 10^{10}}{1.4 \times 10^{14}}\right)}$ $n(0) = 1.4 \times 10^{14} \text{ cm}^{-3}$
 $\frac{2 \times 10^{10}}{1.4 \times 10^{14}} = \frac{1}{70000}$ $\ln\left(\frac{1}{70000}\right) = -\frac{1}{\tau}$



Referring to the band edge diagram above, answer the following questions. You may assume that the doping varies slowly throughout and that charge neutrality is maintained. The semiconductor is isolated. No calculations are necessary. Choose only one answer (there may be more than one correct answer... choose one only!)

10. (3 pts) Where is the doping concentration lowest?

- a. X_a b. X_b c. X_c d. X_d e. X_e

11. (3 pts) Where is the electron concentration highest?

- a. X_a b. X_b c. X_c d. X_d e. X_e

12. (3 pts) At which of the following pairs of points is built-in electric field highest in magnitude?

- a. X_a, X_e b. X_b, X_d c. X_c, X_d d. X_d, X_e ~~e. X_b, X_c~~

13. (3 pts) Where is the hole concentration approximately equal to n_i ?

- a. X_a b. X_b c. X_c d. X_d e. X_e

14. (3 pts) Is the semiconductor in equilibrium? \rightarrow Fermi level constant

- a. Yes b. No c. Can't tell

15. (3 pts) At which point do holes diffuse to the left?

- a. X_a b. X_b c. X_c d. X_d e. X_e

16. (2 pts) Where is the product of hole and electron concentration largest?

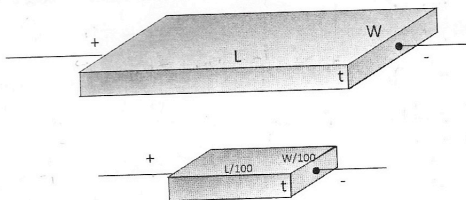
- a. X_a b. X_c c. X_e d. equal everywhere

Section 2: Problems (60 points)

Show your work. Full credit for the correct answer with work shown. Sensible answers (correct order of magnitude) get partial credit. Generous partial credit for an incorrect answer with the correct ideas if clear and brief. Positive credit may be negated if irrelevant or incorrect equations are included (keep clutter to a minimum!).

+29

1. (30 pts) **Scaled Resistors** [Read all 4 parts, a-b-c-d before starting]
 Consider two thin slices of n-type silicon (doping $N_d \gg n_i$). The slice above is the "long" slice and the slice below is the "short" slice with dimensions as indicated (L , W and t have the same meaning in both illustrations, that is, the length and width of the short slice have been reduced as shown but t is unchanged. The slices are not drawn to scale).



- a) (5 pt) Write expressions for the resistances (R_{long} , R_{short}) of the two slices in the low voltage limit. We want the end to end resistance in terms of charge, electron concentration (n), mobility (μ) and dimensions (L , W , t). Do not plug in any numbers yet!

+5

$$R = \frac{\rho L}{t \cdot W}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{e \mu (n_i + N_d)}$$

Since $N_d \gg n_i \Rightarrow \rho = \frac{1}{e \mu N_d}$

$$n_0 = N_d$$

$$R_{long} = \frac{1}{e \mu N_d} \left[\frac{L}{t \cdot W} \right]$$

$$R_{short} = \frac{1}{e \mu N_d} \left[\frac{L/100}{t \cdot (W/100)} \right]$$

$R_{short} = R_{long}$

- b) (5 pt) Calculate the resistances (R_{long} , R_{short}) under the conditions that $N_d = 10^{17}$, $t = 0.1 \mu m$, $L = 100 \mu m$, $W = 25 \mu m$, maximum applied voltage = 0.1 V (additional values you may need can be found on the formula sheet)

+5

$$E_{short} = \frac{V}{L} = \frac{0.1V}{10^{-4}cm}$$

$$E_{short} = 10^3 \frac{V}{cm}$$

$N_d \gg n_i$

$N_d = 10^{17} cm^{-3}$

$\mu_n = 10^4 cm^2/Vs$ (From graph)

$n = 10^{17} cm^{-3}$

$t = 0.1 \mu m = 10^{-5} cm$

$L = 100 \mu m = 10^{-2} cm$

$W = 25 \mu m = 2.5 \times 10^{-3} cm$

$$R_{long} = \frac{1}{(1.6 \times 10^{-19} C)(750)(10^{17})} \cdot (10^{-2} cm) \cdot (2.5 \times 10^{-3} cm)$$

$R_{long} = 3333 \Omega = R_{short}$

R_{short} For both short and long resistors, field is low

$E = \frac{V}{L} = \frac{0.1V}{100 \times 10^{-4} cm} = \frac{0.1V}{10^{-2} cm}$

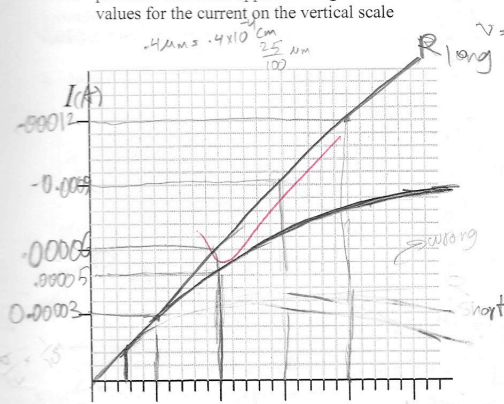
$E = 10 \frac{V}{cm}$ inside \rightarrow low field

$$I_{short} = (10^7) \left(\frac{10^7 \text{ cm}}{5} \right) (1.6 \times 10^{-19}) \cdot (4 \times 10^4 \text{ cm}) (10^5 \text{ cm}) = 0.00064 \text{ A}$$

EE2 Fall 2016

Name: Milad Nazari

- c) (10 pt) Sketch the current-voltage (I-V) characteristics of both resistors on the plot provided below for applied voltages from 0 to 5V (marked on scale). Provide values for the current on the vertical scale



$v = IR$
 $\langle v_x \rangle = -\mu E_x$
 $J = q n v_x$
 $R_{long} = \frac{L}{A \sigma} = \frac{L}{q n \mu A}$
 $\frac{1}{\rho} = \frac{q n \mu}{A}$
 $E = \frac{V}{L}$
 $E = \frac{5V}{10 \mu m} = \frac{5V}{10^{-5} m} = 5 \times 10^5 \frac{V}{m}$
 $\frac{5}{10 \mu m} = 5 \times 10^5$
 $R_{long} = \frac{L}{A \sigma} = \frac{L}{q n \mu A}$
 $I = \frac{V}{R} = \frac{1V}{R}$

- d) (10 pt) Write a general expression for the current as a function of voltage and other material constants (given quantities: N_d , L , W , t , V , AND parameters you know for Silicon). The expression must be valid for the entire range of voltage (0 to 5V) for both resistors. Calculate this value at 5V in both resistors.

$R = \rho \frac{L}{W \cdot t} = \left(\frac{V}{J} \right) \left(\frac{L}{n \cdot v_d \cdot c} \right) \left(\frac{L}{W \cdot t} \right)$
 $R = \frac{V}{J} \frac{L^2}{n \cdot v_d \cdot c \cdot W \cdot t}$
 $I = \frac{V}{R} = \frac{J \cdot W \cdot t \cdot n \cdot v_d \cdot c}{L^2}$
 $I = \frac{q n \mu V W t}{L^2}$
 As electric field increases, velocity gets saturated.
 OHM LAW: $I = \frac{V}{R} = \frac{q n \mu V W t}{L^2}$
 If low field (voltage) and velocity not saturated: $I = N_d \cdot \mu_n \cdot \frac{V}{L} \cdot e \cdot W \cdot t$
 If High field (voltage) $E_{cr} > 10^4 \frac{V}{cm} \Rightarrow I = (N_d) \left(2 \times 10^4 \frac{cm}{s} \right) \cdot e \cdot W \cdot t$

23

2. (30 pts) **Unknown Semiconductor** - A semiconductor (not silicon) at 300 K has $N_c = N_v = 1.73 \times 10^{19} / \text{cm}^3$, $n_i = 1.73 \times 10^{15} / \text{cm}^3$.

a. (15) A sample of the semiconductor is doped with n-type dopant $N_d = 3 \times 10^{15}$. Calculate the equilibrium concentrations n_0 and p_0 .

Since n_0 is closed to N_d , we use equations

$$n_0 p_0 = n_i^2$$

$$p_0 + N_d = n_0 + N_a$$

$$N_0 = p_0 + N_d$$

$$n_0 = \frac{n_i^2}{p_0} + N_d$$

$$n_0^2 = (N_d^2 n_0) + n_i^2$$

$$n_0^2 - N_d n_0 - n_i^2 = 0 \rightarrow \text{Quadratic}$$

$$n_0 = \frac{N_d \pm \sqrt{N_d^2 + 4(1)(n_i^2)}}{2} = 3 \times 10^{15} \text{ cm}^{-3}$$

$$p_0 = \frac{n_i^2}{n_0} = 9.97 \times 10^{-3} \text{ cm}^{-3}$$

$n_0 p_0 = n_i^2$
 $p_0 = \frac{n_i^2}{n_0}$
 $p_0 = 9.97 \times 10^{-3} \text{ cm}^{-3}$

b. (10) Another sample of the semiconductor is doped with n-type dopant $N_d = 8 \times 10^{18}$. Calculate the position of the Fermi level referenced to the nearest band edge (not E).

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$\left(\frac{n_0}{N_c}\right) = \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$\ln\left(\frac{n_0}{N_c}\right) = \frac{E_F - E_c}{kT} \Rightarrow \ln\left(\frac{n_0}{N_c}\right)(kT) = E_F - E_c$$

$$\ln\left(\frac{8 \times 10^{18}}{1.73 \times 10^{19}}\right)(0.026 \text{ eV}) = E_F - E_c$$

Since $N_d \gg n_i$
 $n_0 = N_d$

c. (5) Calculate the bandgap of this semiconductor

$$n_i = \sqrt{N_c N_v} e^{-\frac{E_g}{2kT}} \Rightarrow |E_c - E_F| = 0.02 \text{ eV}$$

$$\left(\frac{n_i}{\sqrt{N_c N_v}}\right) = \frac{-E_g}{2kT} \Rightarrow E_g = (2kT) \ln\left(\frac{n_i}{\sqrt{N_c N_v}}\right)$$

$$E_g = 2 \times (0.026 \text{ eV}) \left(\ln\left(\frac{1.73 \times 10^{15}}{\sqrt{1.73 \times 10^{19} \times 1.73 \times 10^{19}}}\right)\right) = 0.239 \text{ eV}$$

Great!

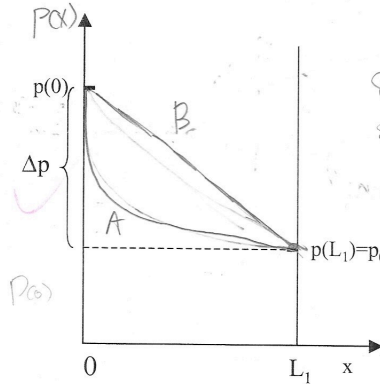
Name: Milad Mourim

5

3. (5 pts Bonus) Diffusion in a short base diode.

A slice of n-type semiconductor has the boundary conditions shown. At one end there is a steady supply of excess holes, $\delta p(0) = \Delta p$, and at the other end the hole concentration is the equilibrium value, p_0 .

- a. Sketch $p(x)$ from 0 to L_1 for: A) $L_1 = 10L_p$ ($L_1 \gg L_p$) AND B) $L_1 = 0.1L_p$ ($L_1 \ll L_p$). (Use the graph below and label the traces A and B). Only the sketch counts, you may use the space for any calculations but they won't be graded.



$$\begin{aligned}
 \delta p(x) &= c_1 e^{\frac{x}{L_p}} + c_2 e^{-\frac{x}{L_p}} \\
 \delta p(L_1) &= c_1 e^{\frac{10L_p}{L_p}} + c_2 e^{-\frac{10L_p}{L_p}} \\
 \delta p(L_1) &= c_1 e^{10} + c_2 e^{-10} = p_0 \\
 \delta p(0) &= c_1 + c_2 = \Delta p \\
 \delta p \quad c_1 &= \Delta p - c_2 \\
 c_1 (\Delta p - c_2) e^{10} + c_2 e^{-10} &= p_0 \\
 \Delta p e^{10} - c_2 (e^{10} - e^{-10}) &= p_0 \\
 c_2 &= \frac{p_0 - \Delta p e^{10}}{e^{10} - e^{-10}}
 \end{aligned}$$

Please answer the following survey questions prior to handing in your test (The survey questions have absolutely no impact on your grade):

Q1 - Which of the following TA sections do you attend:

- 9-10am 1-2pm 3-4pm 4-5pm*

Q2 - How many section meetings have you attended:

- 0/5 1/5 2/5 3/5 4/5 5/5