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Physics for Electrical Engineers

Prof. B. Jalali **EE2**

Midterm Exam **Thursday 7, 2013**

1. Which of the following semiconductors are transparent, partially transparent, non-transparent for visible light (λ=0.4-0.7μm): Si, GaAs, GaP, and GaN?

2. Electron mobility in Si is 1400 $cm^2V^{-1}s^{-1}$. Calculate the momentum relaxation time of electrons. Effective mass $m_e^*/m_0 = 0.33$

3. Calculate:
   a. Calculate the dielectric relaxation time in p-type Ge at room temperature. Assume all acceptors are ionized. $N_a=10^{15}cm^{-3}$, ε_R=16, μ_p=1900 $cm^2V^{-1}s^{-1}$. — mult by ε_0 $\varepsilon = \varepsilon_0 \varepsilon_R$
   b. Calculate the dielectric relaxation time in intrinsic Si at 300K. ε=12, μ_n=1400 $cm^2V^{-1}s^{-1}$, $\mu_n=3.1\mu_p$.
   c. Find the Debye length in p-type Ge at 300K if $N_a=10^{14}cm^{-3}$. Assume all acceptors are ionized. ε=16.

4. Explain how the Heisenber uncertainty principle can help conserve momentum during indirect recombination.

5. Assuming that $m_e^*/m_0 = 1.08$ and $m_p^*/m_0 = 0.591$, a silicon sample is doped with 10^{16} donor atoms/cm^3. Draw an energy level diagram showing the location of the Fermi level with respect to the middle of the band gap for:
   a. T=77K(liguid nitrogen).
   b. T=300K.
   c. T=600.

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Semiconductor	E_g, eV	Band	Effective mass,[a] m_0		Mobility, cm^2/V sec		ϵ
			m_e^*	m_h^*	μ_e	μ_h	
Ge	0.66	I	0.57	0.37	3900	1900	16.0
Si	1.12	I	1.08	0.59	1400	450	11.9
GaAs	1.42	D	0.063	0.53	8800	400	12.9
GaP	2.26	I	0.8	0.83	250	150	11.4
GaN	3.44	D	0.22	0.61	8500	400	10.4

SEMICONDUCTOR PHYSICS

Electron Momentum: $p = mv = \hbar \mathbf{k} = \frac{h}{\lambda}$ Planck: $E = h\nu = \hbar\omega$

Kinetic: $E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{p^2}{m} = \frac{\hbar^2}{2m^*}\mathbf{k}^2$ (3–4) Effective mass: $m^* = \frac{\hbar^2}{d^2E/d\mathbf{k}^2}$ (3–3)

Total electron energy $= P.E. + K.E. = E_c + E(\mathbf{k})$

Fermi-Dirac e^- distribution: $f(E) = \frac{1}{e^{(E-E_F)/kT}+1} \cong e^{(E_F-E)/kT}$ for $E \gg E_F$ (3–10)

Equilibrium: $n_0 = \int_{E_c}^{\infty} f(E)N(E)dE = N_c f(E_c) = N_c e^{-(E_c-E_F)/kT}$ (3–15)

$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$ $N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$ (3–16), (3–20)

$p_0 = N_v[1 - f(E_v)] = N_v e^{-(E_F-E_v)/kT}$ (3–19)

$n_i = N_c e^{-(E_c-E_i)/kT}$, $p_i = N_v e^{-(E_i-E_v)/kT}$ (3–21)

$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT} = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_n^* m_p^*)^{3/4} e^{-E_g/2kT}$ (3–23), (3–26)

Equilibrium: $\begin{matrix} n_0 = n_i e^{(E_F-E_i)/kT} \\ p_0 = n_i e^{(E_i-E_F)/kT} \end{matrix}$ (3–25) $n_0 p_0 = n_i^2$ (3–24)

Steady state: $\begin{matrix} n = N_c e^{-(E_c-F_n)/kT} = n_i e^{(F_n-E_i)/kT} \\ p = N_v e^{-(F_p-E_v)/kT} = n_i e^{(E_i-F_p)/kT} \end{matrix}$ (4–15) $np = n_i^2 e^{(F_n-F_p)/kT}$ (5–38)

Here, F_n is E_F in an n-type material, and F_p is E_F in a p-type material

$\mathscr{E}(x) = -\frac{dV(x)}{dx} = \frac{1}{q}\frac{dE_i}{dx}$ (4–26)

Poisson: $\frac{d\mathscr{E}(x)}{dx} = -\frac{d^2V(x)}{dx^2} = \frac{\rho(x)}{\epsilon} = \frac{q}{\epsilon}(p - n + N_d^+ - N_a^-)$ (5–14)

$\mu \equiv \frac{q\bar{t}}{m^*}$ (3–40a) Drift: $v_d \cong \frac{\mu\mathscr{E}}{1 + \mu\mathscr{E}/v_s}\begin{cases} = \mu\mathscr{E} \text{ (low fields, ohmic)} \\ = v_s \text{ (high fields, saturated vel.)} \end{cases}$ (Fig. 6–9)

Drift current density: $\frac{I_x}{A} = J_x = q(n\mu_n + p\mu_p)\mathscr{E}_x = \sigma\mathscr{E}_x$ (3–43)

Resistivity $\rho = \dfrac{1}{q(\mu_n n + \mu_p p)}$ Uncertainty Relationship: $\Delta x \cdot \Delta p_x \geq \hbar / 2$

Group Velocity $\upsilon_g = \dfrac{1}{\hbar}\left(\dfrac{\partial E}{\partial k}\right) = \dfrac{\partial \omega}{\partial k}$ Phase Velocity $\upsilon_p = \dfrac{\omega}{k} = f\lambda = \dfrac{\upsilon_g}{2}$

Density of states $Z(E) = \dfrac{4\pi V (2m)^{3/2}}{h^3} E^{1/2}$ Intrinsic Fermi energy $E_i = \dfrac{kT}{2}\ln\left(\dfrac{N_V}{N_C}\right) + \dfrac{E_C + E_V}{2}$

Conduction Current:

$$J_n(x) = q\mu_n n(x)\mathscr{E}(x) + qD_n \frac{dn(x)}{dx}$$

drift diffusion (4–23)

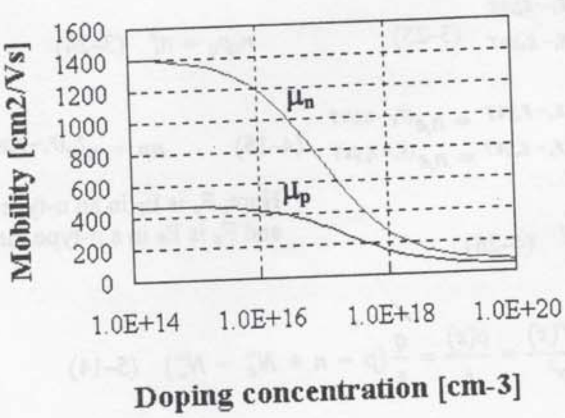
$$J_p(x) = q\mu_p p(x)\mathscr{E}(x) - qD_p \frac{dp(x)}{dx}$$

$$J_{\text{total}} = J_{\text{conduction}} + J_{\text{displacement}} = J_n + J_p + C\frac{dV}{dt}$$

Continuity: $\dfrac{\partial p(x,t)}{\partial t} = \dfrac{\partial \delta p}{\partial t} = -\dfrac{1}{q}\dfrac{\partial J_p}{\partial x} - \dfrac{\delta p}{\tau_p}$ $\quad \dfrac{\partial \delta n}{\partial t} = \dfrac{1}{q}\dfrac{\partial J_n}{\partial x} - \dfrac{\delta n}{\tau_n}$ (4–31)

For steady state diffusion: $\dfrac{d^2 \delta n}{dx^2} = \dfrac{\delta n}{D_n \tau_n} \equiv \dfrac{\delta n}{L_n^2}$ $\quad \dfrac{d^2 \delta p}{dx^2} = \dfrac{\delta p}{L_p^2}$ (4–34)

Diffusion length: $L \equiv \sqrt{D\tau}$ Einstein relation: $\dfrac{D}{\mu} = \dfrac{kT}{q}$ (4–29)



$\tau_D = \varepsilon / \sigma$

$$L_D = \sqrt{\frac{\epsilon kT}{q^2 N_d}}$$

p-n JUNCTIONS

Equilibrium: $V_0 = \frac{kT}{q}\ln\frac{p_p}{p_n} = \frac{\mathrm{kT}}{\mathrm{q}}\ln\frac{N_a}{n_i^2/N_d} = \frac{\mathrm{kT}}{\mathrm{q}}\ln\frac{N_aN_d}{n_i^2}$ (5–8)

$\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{qV_0/kT}$ (5–10) $W = \left[\frac{2\epsilon(V_0 - V)}{q}\left(\frac{N_a + N_d}{N_aN_d}\right)\right]^{1/2}$ (5–57)

One-sided abrupt p^+-n: $x_{n0} = \frac{WN_a}{N_a + N_d} \simeq W$ (5–23) $V_0 = \frac{qN_dW^2}{2\epsilon}$

$\Delta p_n = p(x_{n0}) - p_n = p_n(e^{qV/kT} - 1)$ (5–29) $E_{\max} = \frac{qN_DW_n}{\varepsilon}$ For P^+N diode

$\delta p(x_n) = \Delta p_n e^{-x_n/L_p} = p_n(e^{qV/kT} - 1)e^{-x_n/L_p}$ (5–31b)

Ideal diode: $I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)(e^{qV/kT} - 1) = I_0(e^{qV/kT} - 1)$ (5–36)

$or\ I = qA\left(\frac{D_p}{W_n}p_n + \frac{D_n}{W_p}n_p\right)\left(e^{qV/kT} - 1\right)$

Non – ideal: $I = I_{srec}\left(e^{qV/2kT} - 1\right)$

With light: $I_{\mathrm{op}} = qAg_{\mathrm{op}}(L_p + L_n + W)$ (8–1)

Responsivity $R = \frac{I_{op}}{P_{op}} = \frac{\eta q\lambda}{hc}$, $\eta \equiv$ quantum efficiency $E_{photon} = h\nu = \frac{hc}{\lambda}$

Capacitance: $C = \left|\frac{dQ}{dV}\right|$ (5–55)

Junction Depletion: $C_j = \epsilon A\left[\frac{q}{2\epsilon(V_0 - V)}\frac{N_dN_a}{N_d + N_a}\right]^{1/2} = \frac{\epsilon A}{W}$ (5–62)

Stored charge
exp. hole dist.: $Q_p = qA\int_0^\infty \delta p(x_n)dx_n = qA\Delta p_n\int_0^\infty e^{-x_n/L_p}dx_n = qAL_p\Delta p_n$ (5–39)

$I_p(x_n = 0) = \frac{Q_p}{\tau_p} = qA\frac{L_p}{\tau_p}\Delta p_n = qA\frac{D_p}{L_p}p_n(e^{qV/kT} - 1)$ (5–40)

$g = \frac{dI}{dV} = \frac{qAL_pp_n}{\tau_p}\frac{d}{dV}(e^{qV/kT}) = \frac{q}{kT}I$ (5–67c) diffusion capacitance $C_d = g \cdot \tau$

Long p^+-n: $i(t) = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p(t)}{dt}$ (5–47)

Non-Idealities:

$$I_{recombination} = \frac{qAx_{deff} n_i \exp(\frac{V_F}{2V_T})}{\tau_{p0} + \tau_{n0}},\ I_{generation} = \frac{qAx_d n_i}{\tau_{p0} + \tau_{n0}} = \frac{qAx_d n_i}{2\tau_g}$$

Junction Breakdown $V_{BR} = \frac{\varepsilon E_{cr}{}^2}{2q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right) - V_{bi}$

Noise:

Shot noise: $< i^2 >= 2qIf$

Thermal noise: $< i^2 >= \frac{4kTf}{R}$

Noise power: $P_n =< i^2 > R$

CONSTANTS

Permittivity of Silicon: ε_{Si}=11.8
Bandgap of Silicon at T=300 K: E_g=1.12 eV
n_i (for Si at room temperature T=300 K) = 1×10^{10} cm^{-3}
N_C (for Si at room temperature T=300 K) = 2.8×10^{19} cm^{-3}
N_V (for Si at room temperature T=300 K) = 1.04×10^{19} cm^{-3}

Avogadro's number	$N_A = 6.02 \times 10^{23}$ molecules/mole
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K
	$= 8.62 \times 10^{-5}$ eV/K
Electronic charge (magnitude)	$q = 1.60 \times 10^{-19}$ C
Electronic rest mass	$m_0 = 9.11 \times 10^{-31}$ kg
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-14}$ F/cm
	$= 8.85 \times 10^{-12}$ F/m
Planck's constant	$h = 6.63 \times 10^{-34}$ J-s
	$= 4.14 \times 10^{-15}$ eV-s
Room temperature value of kT	$kT = 0.0259$ eV
Speed of light	$c = 2.998 \times 10^{10}$ cm/s

	Prefixes:		
1 Å (angstrom) = 10^{-8} cm	milli-,	m-	$= 10^{-3}$
1 μm (micron) = 10^{-4} cm	micro-,	μ-	$= 10^{-6}$
1 nm = 10Å = 10^{-7} cm	nano-,	n-	$= 10^{-9}$
2.54 cm = 1 in.	pico-,	p-	$= 10^{-12}$
1 eV = 1.6×10^{-19} J	kilo-,	k-	$= 10^{3}$
	mega-,	M-	$= 10^{6}$
	giga-,	G-	$= 10^{9}$

A wavelength λ of 1 μm corresponds to a photon energy of 1.24 eV.

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1) Energy of a photon of visible light:

$$E = hf = \frac{hc}{\lambda} = \frac{(6.64\times10^{-34})(3\times10^{8}\,m/s)}{(.4\times10^{-6}\,m \text{ to } .7\times10^{-6}\,m)}$$

$E_{photon} = 2.84\times10^{-19}\,J$ to $4.98\times10^{-19}\,J$

$E_{photon} = 1.77\,eV$ to $3.11\,eV$ ✓ 20/20

E_g for Si: 1.12 eV transparent ✗
E_g for GaAs: 1.42 eV partially transparent ✗
E_g for GaP: 2.26 eV non transparent ✗
E_g for GaN: 3.49 eV. non transparent ✗

2) $\mu_{n_{Si}} = 1400\,cm^2 V^{-1} s^{-1} = \frac{q\,\bar{t}}{m^*}$ $\quad m^* = .33\,m_0$

$1400\,cm^2 V^{-1} s^{-1} = \frac{(1.602 \times 10^{-19} C)(\bar{t})}{.33\,m_0}$

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$1400\,cm^2 V^{-1} s^{-1} = \frac{(1.602 \times 10^{-19} C)(\bar{t})}{.33\,(9.11 \times 10^{-31} kg)}$

$.14\,m^2 V^{-1} s^{-1} = 5.32 \times 10^{11}\,(\bar{t})$

$\boxed{\bar{t} = 2.63 \times 10^{-13}\ s}$ ✓

3) a) $\tau_D = \frac{\varepsilon}{\sigma}$ $\quad * \varepsilon = \varepsilon_0 \varepsilon_R = 16\,\varepsilon_0$

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$\sigma = q(\mu_n n + \mu_p p) \approx q(\mu_p p)$ $\quad * \mu_p p >> \mu_n n$

$\sigma \approx (1.602 \times 10^{-19} C)(1900\,cm^2 V^{-1} s^{-1})(10^{15}\,cm^{-3})$ $\quad * p \approx 10^{15}$

$\sigma \approx .304$

$\tau_D = \frac{16\,\varepsilon_0}{.304} = \frac{16\,(8.85 \times 10^{-14})}{.304} = \boxed{4.66 \times 10^{-12}\ s}$ ✓

b) $\tau_D = \frac{\varepsilon}{\sigma_{Si}}$ $\quad * \varepsilon = \varepsilon_0 \varepsilon_R = 12\,\varepsilon_0$

$\sigma = q(\mu_n n + \mu_p p)$

$\sigma = (1.602 \times 10^{-19} C)\left(1400\,cm^2 V^{-1} s^{-1} \cdot 1 \times 10^{10}\,cm^{-3} + \frac{1}{3.1}(1400\,cm^2 V^{-1} s^{-1}) \cdot 1 \times 10^{10}\,cm^{-3}\right)$

$\sigma = 2.966 \times 10^{-6}$

$$\boxed{\tau_{o_{si}} = \frac{12\varepsilon_0}{2.966\times10^{-6}} = 3.58\times10^{-7}\,s}$$ ✓

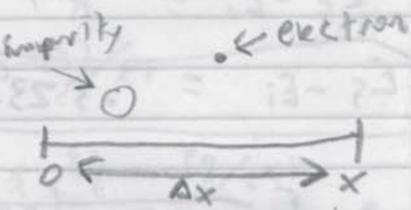
c) $L_D = \sqrt{\frac{\varepsilon K T}{q^2 N_d}}$ $\varepsilon = 16\,\varepsilon_0,\ T = 300K,\ N_d = 10^{14}\,cm^{-3}$

$N_d = 10^{14}\,cm^{-3} = 10^{20}\,m^{-3}$

$$L_D = \sqrt{\frac{16\varepsilon_0\, K\,(300K)}{(1.602\times10^{-19})^2(1\times10^{20}\,m^{-3})}}$$

$$L_D = \sqrt{\frac{16\cdot 8.85\times10^{-12}\cdot 1.38\times10^{-23}\cdot 300}{(1.602\times10^{-19})^2(1\times10^{20}\,m^{-3})}} = \boxed{4.78\times10^{-7}\,m}$$ ✓

4) $\Delta x\,\Delta p_x \geq \hbar/2$ ✓



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The heisenberg uncertainty principle helps conserve momentum in indirect recombination ? because we cannot say for sure what momentum a specific impurity site/electron has or else we lose all positional information. So, the impurity-electron system at which recombination occurs has a range of momentums and this is where the momentum is conserved during recombination. This is because the electron is confined to a region.

5) a) $n_0 = n_i e^{(E_f - E_i)/kT}$ ✓

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at 77K: $n_i = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} (m^*_n m^*_p)^{3/4} e^{-E_g/2kT}$ ✓

$n_i = 2.32 \times 10^{24} e^{-E_g/2kT}$

$n_i = 2.32 \times 10^{24} e^{(-1.12\,eV/2kT)} = 5.32 \times 10^{-13} m^{-3}$

$n_0 = 5.32 \times 10^{-13} m^{-3} e^{(E_f - E_i)/2kT}$

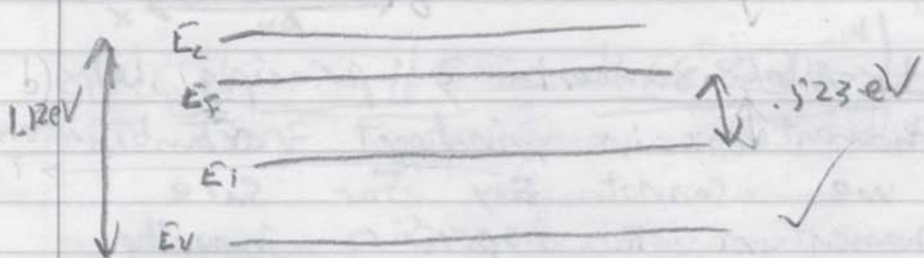
$\ln \frac{n_0}{5.32 \times 10^{-13}} = \ln e^{(E_f - E_i)/2kT}$

$\ln\left(\frac{n_0}{5.32 \times 10^{-13}}\right) = \frac{E_f - E_i}{kT}$

$n_0 = 10^{16} cm^{-3} = 10^{22} m^{-3}$ ✓

$E_f - E_i = kT \ln\left(\frac{n_0}{5.32 \times 10^{-13}}\right)$

$\boxed{E_f - E_i = .523 eV}$ ✓



✓

b)

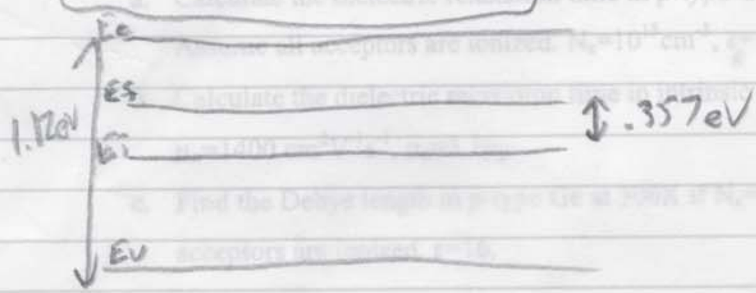
$T = 300K$: $n_0 = n_i e^{(E_f - E_i)/kT}$

$n_i = 1\times 10^{10}\ cm^{-3}$, $n_0 = 10^{16}\ cm^{-3}$

$$\frac{10^{16}\ cm^{-3}}{10^{10}\ cm^{-3}} = e^{(E_f - E_i)/kT}$$

$$13.81 = \frac{(E_f - E_i)}{kT}$$

$$\boxed{.357\,eV = E_f - E_i}$$



c) $T = 600K$ $\quad n_0 = n_i e^{(E_f - E_i)/kT}$

$$n_i = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_n^* m_p^*)^{3/4} e^{-E_g/2kT}$$

$n_i = 1.0034 \times 10^{21}\ m^{-3}$

$n_0 = 10^{16}\ cm^{-3} = 1\times 10^{22}\ m^{-3}$

$$\frac{n_0}{n_i} = e^{(E_f - E_i)/kT}$$

$$9.97 = e^{(E_f - E_i)/kT}$$

$$\boxed{E_f - E_i = .119\ eV}$$