EE2 Midterm Exam Spring 2014 Prof. B. Jalali
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Please use kT = 26 meV (T = 300 K) for the following problems unless otherwise specified.

1. An electron is located in a one-dimensional potential energy well having width of 5Å. Determine
   (a) The kinetic energy of the electron in the ground state in units of eV.
   (b) The frequency and wavelength of the spectral radiation of an electron that drops from the next higher state to the ground state.

2. According to statistical physics, the average kinetic energy of an electron in a medium of free electrons at thermal equilibrium is 3kT/2, where k is Boltzmann's constant and T is in degrees Kelvin. Use the electron rest mass in the cheat sheet, and at T = 450K:
   (a) Find the RMS group velocity of the electron.
   (b) Determine the RMS de Broglie wavelength for the electron.
   (c) If the uncertainty in the value of momentum is 4 part per million, what is the uncertainty in electron's position?
   (d) Find the RMS frequency of the electron, assuming that the phase velocity v_p is equal to half the group velocity v_g that you found in part (b): $v_p = v_g/2$

3. A sample of intrinsic silicon is doped with 10^{15} atoms/cm^{-3} of phosphorus. Assuming complete ionization at 300 K and 600 K, calculate: (E_g = 1.12 eV at 300 K and E_g = 1.032 eV at 600 K)
   (a) Electron and hole concentration
   (b) Fermi level with respect to the midgap energy level $(E_C+E_V)/2$ in electron volt

4. Consider a Si bar 200 μm long and 0.01 cm^2 in cross-sectional area doped with 10^{17}cm^{-3} boron. Given the diffusion coefficients D_n = 20 cm^2/s and D_p = 10 cm^2/s.
   (a) Find the resistivity ρ and conductivity σ;
   (b) Find the resistance R along the length of the bar.

5. If a steady light is shone on a Si sample with boron doping concentration $N_A = 10^{15}$ cm^{-3}, given the generation rate G = 10^{16} cm^{-3}/s, and the carrier lifetime $\tau_n = \tau_p$ = 0.7 ms, calculate and draw the position of quasi Fermi levels relative to E_i, E_c, and E_v at room temperature (T = 300K) in electron volt.

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1. (a)

$$E = \frac{\hbar^2 k^2}{2m}$$

$$k = \frac{n\pi}{L} \Rightarrow E = \frac{n^2 h^2}{8mL^2}$$

$$n = 1 \Rightarrow E_1 = 2.4 \times 10^{-19}\ \text{J} = 1.5\ \text{eV}$$

(b)

$$\Delta E = E_2 - E_1 = (4-1)E_1 = 3E_1 = 7.23 \times 10^{-19}\ \text{J} = 4.51\ \text{eV}$$

$$\Delta E = h\nu \Rightarrow \nu = \frac{\Delta E}{h} = 1.09 \times 10^{15}\ \text{Hz}$$

$$\lambda = \frac{c}{\nu} = 2.75 \times 10^{-7}\ \text{m} = 275\ \text{nm}$$

2.

(a)

$$\mathrm{E} = \frac{3}{2}\mathrm{k_B T} = \frac{1}{2}\mathrm{mv^2} = \frac{\mathrm{p^2}}{2\mathrm{m}}$$

$$\mathrm{k_B T} = 38.8\ \mathrm{meV},\quad \mathrm{m_e} = 9.1 \times 10^{-31}\mathrm{kg}$$

$$\mathrm{p} = \sqrt{3\mathrm{mk_B T}} = 1.303 \times 10^{-25}\ \mathrm{kg \cdot m \cdot s^{-1}}$$

$$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{6.626 \times 10^{-34}\mathrm{J \cdot s}}{1.303 \times 10^{-25}\ \mathrm{kg \cdot m \cdot s^{-1}}} = 5.085\ \mathrm{nm}$$

(b)

$$\mathrm{v_g} = \frac{\mathrm{p}}{\mathrm{m_e}} = \frac{1.064 \times 10^{-25}}{9.11 \times 10^{-31}} = 1.43 \times 10^5\ \mathrm{m \cdot s^{-1}}$$

(c)

$$\Delta\mathrm{x} = \frac{\hbar/2}{\Delta\mathrm{p}} = \frac{6.63 \times 10^{-34}/4\pi}{1.303 \times 10^{-25} \times 4 \times 10^{-6}} = 101.2\ \mu\mathrm{m}$$

(d)

$$\mathrm{f} = \frac{\mathrm{v_p}}{\lambda} = \frac{\mathrm{v_g}}{2\lambda} = 1.4065 \times 10^{13}\ \mathrm{Hz}$$

3.

(a) $T = 300$ K:

$$n_i = 10^{10}\ \text{cm}^{-1}$$

$$n = N_D = 10^{15}\ \text{cm}^{-1}$$

$$p = \frac{n_i^2}{n} = 10^5\ \text{cm}^{-1}$$

$T = 600$ K:

$$n_i = 10^{10}\ \text{cm}^{-1} \times \sqrt{\left(\frac{600}{300}\right)^3 \times \left(\frac{\exp(-1.032/0.052)}{\exp(-1.12/0.026)}\right)} = 3.134 \times 10^{15}\ \text{cm}^{-1}$$

$$n = \frac{N_D}{2} + \sqrt{\left(\frac{N_D}{2}\right)^2 + n_i^2} = 3.67 \times 10^{15}\ \text{cm}^{-1}$$

$$p = \frac{n_i^2}{n} = 2.67 \times 10^{15}\ \text{cm}^{-1}$$

(b) $E_F - E_{midgap} = k_B T \ln\left(\frac{n}{n_i}\right) + \frac{k_B T}{2} \ln\left(\frac{N_V}{N_C}\right)$

$T = 300$ K:

$$E_F - E_{midgap} = k_B T \ln\left(\frac{n}{n_i}\right) + \frac{k_B T}{2} \ln\left(\frac{N_V}{N_C}\right) =$$

$$= 4.5635 \times 10^{-20}\ \text{J} = 0.2848\ \text{eV}$$

$T = 600$ K:

$$E_F - E_{midgap} = k_B T \ln\left(\frac{n}{n_i}\right) + \frac{k_B T}{2} \ln\left(\frac{N_V}{N_C}\right) =$$

$$= -2.7943 \times 10^{-21}\ \text{J} = -0.0174\ \text{eV}$$

4.

(a) The conductivity is given by $\sigma = \frac{1}{\rho} = nq\mu_n + pq\mu_p$. According to the Einstein relationship $\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q}$, the electron and hole mobilities can be found to be μ_n=772 cm^2/s and μ_p=386 cm^2/s. In a n type semiconductor with n=N_D=10^{17}cm^{-3}, p<<n, μ_n and μ_p are usually on the same order, therefore,

$$\sigma = nq\mu_n + pq\mu_p \approx nq\mu_n = 12.35\,(\Omega.cm)^{-1},\quad \rho = \frac{1}{\sigma} = 0.081\,\Omega.cm$$

(b) The resistance of a Si bar with L=200μm =2*10^{-2} cm, A=10^{-2} cm^2 is given by

$$R = \rho\frac{L}{A} = 0.162\Omega.$$

5.

$$\Delta p = \Delta n = G\tau = 7 \times 10^{12}\ \text{cm}^{-3} \Rightarrow n = 7 \times 10^{12}\ \text{cm}^{-3}, p = 10^{15}\ \text{cm}^{-3}$$

$$E_{F_n} - E_i = kT \ln\left(\frac{n}{n_i}\right) = 2.71 \times 10^{-20}\ \text{J} = 0.169\ \text{eV}$$

$$E_{F_n} - E_C = kT \ln\left(\frac{n}{N_C}\right) = -6.297 \times 10^{-20}\ \text{J} = -0.393\ \text{eV}$$

$$E_{F_n} - E_V = E_{F_n} - (E_C - E_G) = 1.165 \times 10^{-19}\ \text{J} = 0.727\ \text{eV}$$

$$E_{F_p} - E_i = kT \ln\left(\frac{n_i}{p}\right) = -4.769 \times 10^{-20}\ \text{J} = -0.298\ \text{eV}$$

$$E_{F_p} - E_V = kT \ln\left(\frac{N_V}{p}\right) = 3.831 \times 10^{-20}\ \text{J} = 0.239\ \text{eV}$$

$$E_{F_p} - E_C = E_{F_p} - (E_V + E_G) = -1.411 \times 10^{-19}\ \text{J} = -0.88\ \text{eV}$$