EE2 Midterm Examination Prof. H. R. Fetterman
11/9/04 1 and 1/3 hours Closed Book.

Please do all work on separate sheets

1a.For the indicated planes find the Miller indices. Please show your work.

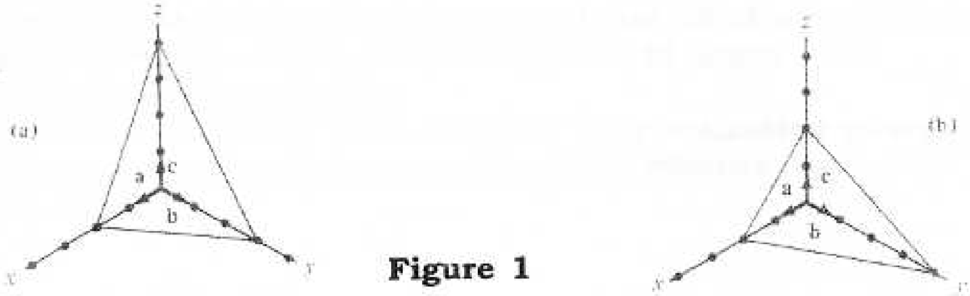


Figure 1

1b. In the Figure 2 below an electron is in a two dimensional box with the **Ψ_x wavefunction** corresponding to an energy of **4 eV.**

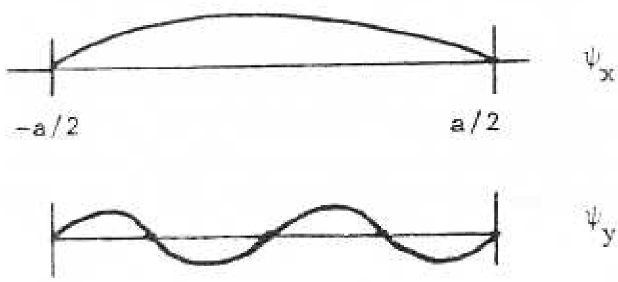


Figure 2

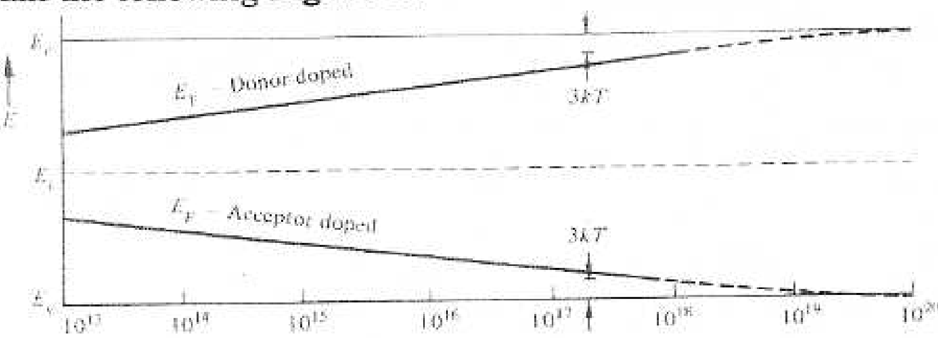
What is the total energy of this 2D system? If the **Ψ_x wavefunction was raised to an n = 3 level** what would be the total energy?

1c. An energy band is approximated by the expression:

$$\mathbf{E(k) = (\hbar^2 k^2/2m_o) - Ak^4}$$

Using the condition that **v_g = 0 at k = π/a** calculate **A.** Also calculate **m*** **when k = 0** and **k = π/a** . What is the value of k **for which v_g is maximum?**

2a. Using the equations for n and n_i write an expression for the difference between the Fermi level **(E_F)** and the intrinsic Fermi energy **(E_i)** in n type material. Explain the following **Figure 3**.



2b. Write the expression for charge neutrality keeping both terms $\mathbf{N_D}$ and $\mathbf{N_A}$. Now solve the resulting quadratic equation to find an expression for **n**. Show that in a compensated n type (large N_D and N_A) the minority hole concentration : $\mathbf{p_{no} = n_i^2 / n_{no} = n_i^2/(N_D - N_A)}$

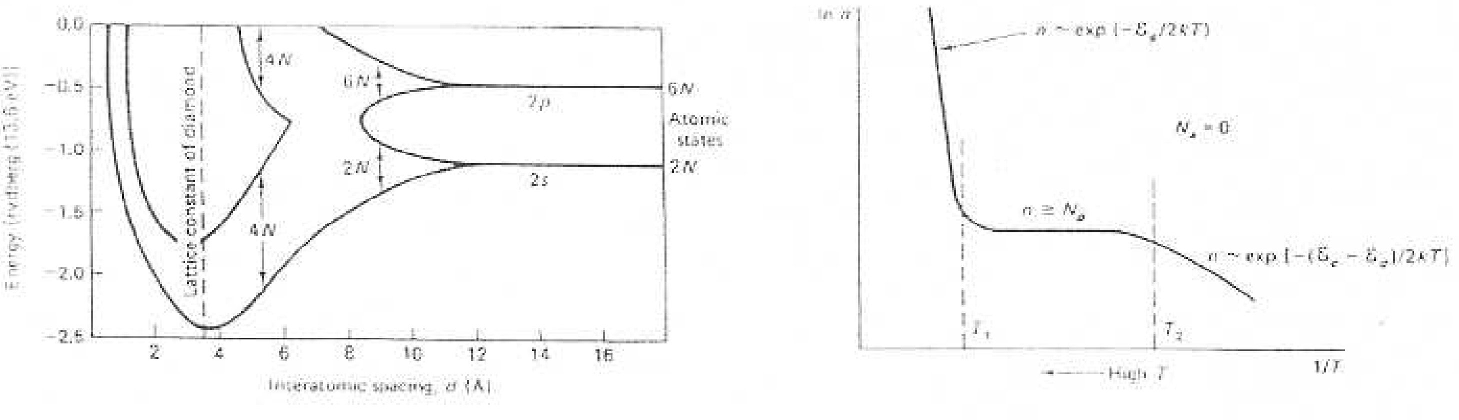
2c. Explain why **As** is a donor in **Si material** and why **Ga** is a acceptor in **Si material.** Now explain what would happen if **Si** atoms were introduced into **GaAs** material.

In terms of the donor atoms estimate their **ionization energy** if the dielectric constant of the material is **10** and the effective mass of an electron is $\mathbf{0.5m_o}$.

3a. Using the density of state function **Z(p)** find the average **p at T = 0** in terms of $\mathbf{p_F}$ **.** Now derive an expression for the average value of $\mathbf{p^2}$. What is the **average kinetic energy** based upon this value?

3b. Convert the density of electrons, per unit volume, **Z(p)dp** to a function of energy **Z(E)dE**. Now derive the total number of electrons **(n),** per unit volume, in a metal as a **function of the Fermi energy at T = 0.** Write an expression for the **Fermi energy as a function of n.** How would the Fermi Energy change if the effective mass was **reduced by a factor of 2?**

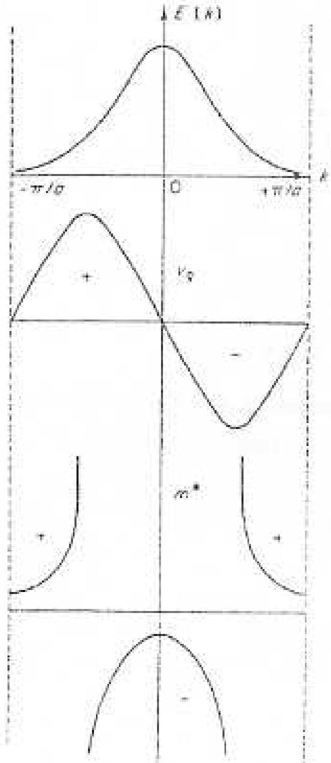
3c. Finally, please explain the following diagrams in **Figure 4**, and the equations, in no more than **three sentences each.**



(a) (b)

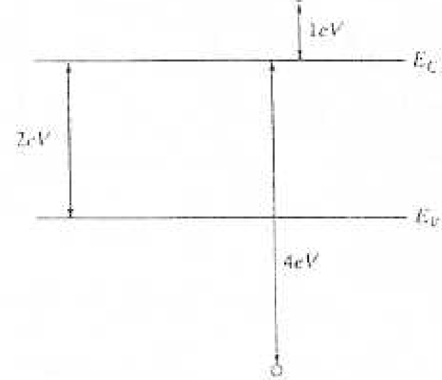
(c)

Explain the motion of an electron starting at k = 0 and moving to π/a



(d)

Which Carrier has larger Kinetic energy



$$Z(E) = \frac{4\pi V(2m)^{\frac{3}{2}}}{h^3} E^{\frac{1}{2}}$$

$$dN_E = f(E)\, Z(E)\, dE$$

$$\phi_{av} = \int \frac{\phi dN_E}{N}$$

$$Z(p)\, dp = 8\pi p^2\, dp\, V/h^3$$

$$N = \frac{2Vm^3}{h^3} \int_{v_x}\int_{v_y}\int_{v_z} f_0(v_x, v_y, v_z)\, dv_x dv_y dv_z$$

$$dN_p = Z(p)\, dp\, f(E) = \frac{8\pi V}{h^3} \frac{1}{e^{\frac{E-E_F}{kT}} + 1} p^2 dp$$

$$p_x = -i\hbar \frac{\partial}{\partial x}$$

$$E = \hbar\omega$$

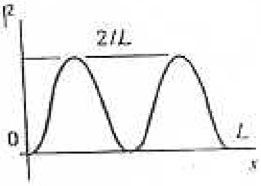
probability of finding a particle is $\Psi^*\Psi\, dx\, dy$

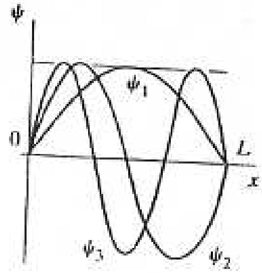
$$J_x = -ne\langle v_x \rangle$$

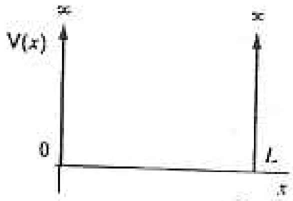
$$F = ma = \hbar \frac{\partial k}{\partial t}$$

$$\upsilon = \frac{c}{\lambda}$$

$$a = \frac{\partial v}{\partial k}$$







$$v_g = \frac{\partial \omega}{\partial k}$$

$$\Delta k = \frac{\text{Force}}{\hbar} \tau_c = \frac{-q\mathcal{E}}{\hbar}$$

$$E_H = -\frac{m_0 q^4}{2(4\pi\varepsilon_0 \hbar n)^2} = -\frac{13.6}{n^2}\text{eV}.$$

$$E_B = -\frac{m_n^* q^4}{2(4\pi K_S \varepsilon_0 \hbar)^2}$$

$$p_0 = \frac{n_i^2}{n_0}$$

$$n_0 + N_a = \frac{n_i^2}{n_0} + N_d$$

$$\sigma = q(n\mu_n + p\mu_p)$$

$$P(x,t)\, dx = \Psi^*(x,t)\Psi(x,t)\, dx$$

mobility has the dimension $cm^2 V^{-1} sec^{-1}$

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{1}{\hbar}\frac{dE}{dk}\right)$$

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x) = E\psi(x)$$

$$\frac{p_x^2}{2m} = \frac{\hbar^2 k_x^2}{2m}$$

$$f(E_c) = \frac{1}{e^{\frac{E_c - E_F}{kT}} + 1} \approx e^{-\left(\frac{E_c - E_F}{kT}\right)}$$

$$E = \frac{p^2}{2m}$$

$$\frac{1}{\hbar}\frac{d^2E}{dt\, dk} = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt}$$

$$\mathcal{E}(x) = -\frac{dV(x)}{dx}$$

$$dn_E = \frac{4\pi}{h^3}\frac{(2m_c)^{\frac{3}{2}}(E - E_c)^{\frac{1}{2}}\, dE}{\left(e^{\frac{E-E_F}{kT}} + 1\right)}$$

$$n = \int_{E_c}^{E_{top}} dn_E$$

$$p = \sqrt{2m_c(E - E_c)}$$

$$J_n = q\mu_n n\mathcal{E} + qD_n\frac{dn}{dx}$$

$$N_c = 2\left(\frac{2\pi m_c kT}{h^2}\right)^{3/2}$$

$$N_v = 2\left(\frac{2\pi m_v kT}{h^2}\right)^{3/2}$$

$$\sigma = \frac{J}{\mathcal{E}}$$

$$N = \int_0^\infty Z(E)\frac{dE}{e^{(E-E_F)/kT} + 1}$$

$$n = 2\left(\frac{2\pi m_c kT}{h^2}\right)^{\frac{3}{2}} e^{-\left(\frac{E_c - E_F}{kT}\right)}$$

$$n_0 + N_a = p_0 + N_d$$

$$\langle E\rangle = \frac{1}{N}\int_0^\infty Z(E)\frac{E\, dE}{e^{(E-E_F)/kT} + 1}$$

$\frac{kT}{q}$ dimension of volt

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$\frac{\sqrt{2mE_n}}{\hbar} = \frac{n\pi}{L}$$

$$n = N_c e^{-\left(\frac{E_c - EF}{kT}\right)}$$

$$N_c = 2\left(\frac{2\pi m_c kT}{h^2}\right)^{\frac{3}{2}}$$

$$k = n\pi/L. \qquad n = 1, 2, \ldots$$

$$E = E_x + E_y + E_z$$

$$p = 2\left(\frac{2\pi m_v kT}{h^2}\right)^{\frac{3}{2}} e^{-\left(\frac{E_F - E_v}{kT}\right)}$$

$$p = n + N_A^-$$

$$E_n = n^2\hbar^2\pi^2/2mL^2 = n^2h^2/8mL^2$$

$$n_0 p_0 = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

$$E_{n_x n_y n_z} = (\hbar^2\pi^2/2mL^2)(n_x^2 + n_y^2 + n_z^2)$$

$$N_D^+ = N_D\left(1 - \frac{1}{e^{\left(\frac{E_D - E_F}{kT}\right)} + 1}\right)$$

$$p_i = N_v e^{-(E_i - E_v)/kT}$$

$$N_A^- = \frac{N_A}{1 + e^{\frac{E_A - E_F}{kT}}}$$

$$n_i = N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right]$$

$$n_i^2 = n_i \times p_i$$

$$\psi_x(x) = \psi_x(x + L_x)$$

$$\psi_x = A\sin kx, \quad k = \frac{\sqrt{2mE}}{\hbar} \qquad k = \frac{n\pi}{L},$$

$$n_i = \sqrt{N_c N_v}\, e^{\frac{-E_g}{2kT}}$$

$$\mathcal{E} = -\frac{d\psi}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

$$\psi(x, y, z) = \psi_x(x)\,\psi_y(y)\,\psi_z(z)$$