Name: SOLUTION

Student ID:

Problem	points	max
1		3
2		9
3		10
4		3

EE1 - Winter 11: QUIZ 1

Tuesday, January 25, 2011

Answer ALL 4 questions. Write your answers directly onto this handout. Show all your work. You are allowed to use your 3"x5" index card as cheat-sheet and a calculator.

Problem 1

(3/25)

Calculate the area of the section of Earth's surface defined by $\phi = [0^o, 45^o]$, and $\theta = [45^0, 90^o]$. Assume a constant Earth radius of r=6371 km.

$$S = \int_0^{45} d\phi \int_{45}^{90} d\theta \; r^2 \sin\theta \quad ①$$

$$= -r^2 \frac{\pi}{4} \cdot \underbrace{\cos\theta \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}}_{-\frac{\sqrt{2}}{2}} = r^2 \pi \frac{\sqrt{2}}{8} \quad ①$$

$$= 2.3 \cdot 10^7 \, km^2 \quad ①$$

Problem 2

(9/25)

Consider an infinite line charge of ρ_L on the z-axis. Find expressions for the work done by moving a test charge q as follows:

1. From ρ_1 to ρ_2 while keeping all other coordinates constant
2. From Φ_1 to Φ_2 while keeping all other coordinates constant
3. From z_1 to z_2 at y=5 cm and x=10 cm.
4. From x_1 to x_2 at y=0.

$$W = -q \cdot \int \underline{E} \, d\underline{l}$$

1. $$\underline{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho} \underline{a}_\rho \quad 1 \qquad d\underline{l} = d\rho \, \underline{a}_\rho \quad 1$$

$$W = -q \int_{\rho_1}^{\rho_2} \frac{\rho_L}{2\pi\varepsilon_0 \rho} \cancel{\underline{a}_\rho} \, d\rho \, \cancel{\underline{a}_\rho} \quad 1$$

$$= - q \frac{\rho_L}{2\pi\varepsilon_0} \ln\left(\frac{\rho_1}{\rho_2}\right) \quad 1$$

2. $$d\underline{l} = \rho \, d\phi \, \underline{a}_\phi \quad 1 \qquad \perp \qquad \underline{E} \Rightarrow W = 0 \quad 1$$

3. $$d\underline{l} = dz \cdot \underline{a}_z \qquad \perp \qquad \underline{E} \Rightarrow W = 0 \quad 1$$

4. 1 since $y = 0$ $$d\underline{l} = dx \cdot \underline{a}_x = d\rho \cdot \underline{a}_\rho \quad 1$$

$\Rightarrow$ same result as 1) 1

Problem 3 (10/25)

(a) A solid sphere with radius R and charge $+Q$ has a uniform volume charge density ρ_V. Use Gauss's law to find an expression of the vector electric field $\vec{E}(r)$ inside ($0 \leq r \leq R$) and outside ($r > R$) the sphere.

(b) Assume you add a spherical, concentric shell of charge $-Q$ outside the solid sphere with $R_{shell} = 2R$. How does the electric field between the two spheres ($R < r < R_{shell}$ and outside the shell ($r > R_{shell}$) change ?

(c) Make a drawing of E as a function of r for both cases (a and b).

$\oint \underline{D}\, d\underline{A} = Q_{encl}$, $\quad \underline{D} = D_r \cdot \underline{a}_r$ from symmetrie (1)

assume spherical Gaussian surface with radius r , $\quad d\underline{A} = dA\, \underline{a}_r$ (1)

$$\Rightarrow \oint \underline{D}\, d\underline{A} = \oint D\, dA = D \cdot \oint dA = D \cdot A = D \cdot 4\pi r^2$$

(1) ↑ because D const. everywhere on sphere

a) $0 \leq r \leq R$: $\quad Q_{encl} = \dfrac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} Q$ (1)

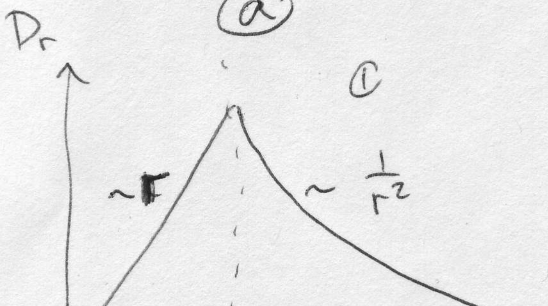
$$\Rightarrow D = \frac{r^3 Q}{4\pi r^2 R^3} = \frac{Q}{4\pi R^3} \cdot r \quad (1)$$

$r > R$: $\quad Q_{encl} = Q =$ fixed $\quad \Rightarrow D = \dfrac{Q}{4\pi r^2}$ (1) (i.e. like point charge)

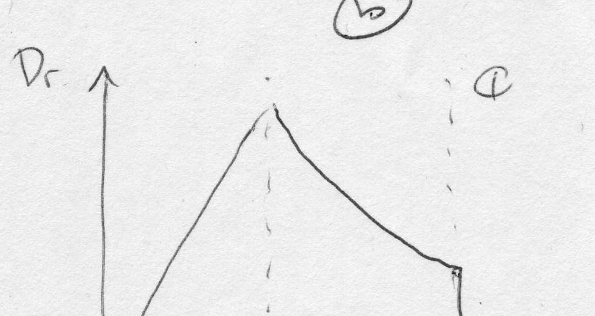
b.) inside shell no change, (1)

D outside shell is $\underline{\text{zero}}$ since $Q_{encl} = 0$ (1)

c.) (a) (1)



(b) (1)



Problem 4

(3/25)

The electric flux densityis given as

$$\vec{D} = 2\rho \cdot cos\left(\frac{\phi}{2}\right) \cdot \vec{a_\rho} + 6\rho \cdot sin\left(\frac{\phi}{2}\right) \cdot \vec{a_\phi} + \rho^2 \cdot \vec{a_z} \qquad \frac{C}{m^2}$$

Find an expression for the charge density ρ_V at any point in space.

$$\underline{\nabla} \cdot \underline{D} = \rho_V = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z} \qquad 1$$

$$D_\rho = 2\rho \cdot \cos\left(\frac{\phi}{2}\right)$$

$$D_\phi = 6\rho \sin\frac{\phi}{2}$$

$$D_z = \rho^2$$

$$\Rightarrow \rho_V = \frac{1}{\rho} 4\rho \cos\left(\frac{\phi}{2}\right) + \frac{1}{\rho} 3\rho \cos\left(\frac{\phi}{2}\right) + 0 \qquad \frac{C}{m^3} \quad 1$$

$$= 7 \cos\left(\frac{\phi}{2}\right) \qquad \frac{C}{m^3} \qquad 1$$