Name: ....

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EE1 - Winter 10: FINAL

Wednesday, March 17, 2010

Answer ALL 6 questions. Write your answers directly onto this handout. Show all your work. You are allowed to use up to two 3"x5" index cards as cheat-sheet and a calculator.

Problem 1 (Concept questions) (15/45)

[**1**] Imagine a closed surface that is not symmetric. The net enclosed charge is zero. What can you say about the electric field at a given point at this surface ?

(a) it is also zero.

(b) it is not zero

(c) can't tell

[**2**] If you move a charge a distance d against a static electric field and then back, how much work do you perform ?

(a) no work at all

(b) a small amount of work due to friction

(c) the field performs work

(d) can't tell

[**3**] A solid spherical conductor is given a net negative charge. The electrostatic potential of the conductor is ?

(a) smallest at the center

(b) smallest on the surface

(c) constant throughout the volume

(d) largest somewhere between center and surface

[**4**] Which of the following charge distributions creates a potential that increases with distance ?

(a) magnetic dipole

(b) spherical charge

(c) infinite sheet of charge

(d) conducting cylinder (inside)

[**5**] You connect a battery to a conducting wire with cylindrical cross-section. What can you say about the electric field inside the wire ?

(a) it is zero inside the conductor

(b) it is constant everywhere

(c) it varies linearly from one end to the other

(d) it points radially outwards

[**6**] The velocity of free charges inside a wire

(a) increases with charge density

(b) increases with length of the wire for a given electric field

(c) increases linearly with electric field

(d) does only depend on the resistivity

[**7**] A certain material is repelled by a permanent magnet. Which of the following would be an appropriate μ_r ?

(a) $\mu_r = 0$

(b) $\mu_r = 0.6$

(c) $\mu_r = -2.3$

(d) $\mu_r = 15$

[**8**] An electric dipole in the field of a second electric dipole experiences

(a) both a net torque and a net force

(b) only a net torque

(c) no force at all since it is at rest

[**9**] Imagine two wires crossing each other at an right angle. If you drive a current through both wires, what will happen ?

(a) the wires will attract each other

(b) both wires will rotate

(c) both wires will align

(d) nothing will happen

[**10**] A homogeneous electric field is created by an infinite sheet of positive charge. If you place a conducting plate in front of the charge sheet what will happen to the electric field behind the plate ?

(a) The plate will charge up and will increase the field since it is closer

(b) The plate will act as a shield and cancel out the electric field behind it

(c) The electric field behind the plate will not change

(d) You create a capacitor and change the field between the two conductors

[11] Which of the following can increase the inductance of a conducting system

(a) charging it

(b) increasing the current flowing through it

(c) increasing its resistance

(d) changing its geometry

[12] A dipole antenna radiates electromagnetic waves preferably in which direction ?

(a) perpendicular to the dipole

(b) along the dipole axis

(c) at an oblique angle

(d) It depends on the frequency

[13] An electromagnetic wave propagates in the z direction. If the magnetic field points in the x direction, in which direction does the electric field point ?

(a) x

(b) y

(c) z

(d) -y

(e) -x

[14] A wave equation is given as $\partial^2 A/\partial z^2 = c \cdot \partial^2 A/\partial t^2$. What is the velocity of the wave ?

(a) c

(b) $\sqrt{c}$

(c) $1/c$

(d) $1/\sqrt{c}$

[15] If you fill the space between the plates of a charge capacitor with a dielectric what will happen to the electric field in that space ?

(a) it will stay the same, changing the charge on the plates

(b) it will increase

(c) it will decrease

(d) can't tell

Problem 2

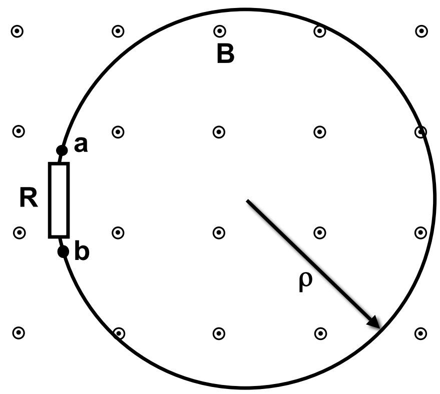
(5/45)

Use $\vec{J} = \sigma \cdot \vec{E}$ to prove that two resistors R_1 and R_2 in parallel behave like one resistor with a total resistance of $R_{tot} = (R_1 R_2)/(R_1 + R_2)$.

Problem 3

(5/45)

A circular conductor in the x-y plane is embedded in an external magnetic field $\vec{B} = 0.2 \cdot cos(120\pi t)\ \vec{a}_z\ T$. Assume that the conductor joining the two ends of the resistor R is perfect (the magnetic field produced by $I(t)$ is negligible).



(a) Find both $V_{ab}(t)$ and $I(t)$ as a function of time t.

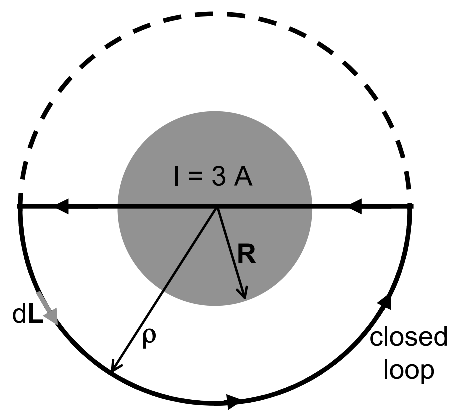
(b) In which direction does the induced current flow if the magnetic field (pointing out of the paper plane) decreases in time ?

(c) How would the result from a) change if $\vec{B}$ would point into the $\vec{a}_x$ direction ?

Problem 4

(7/45)

A long straight wire of circular cross-section carries a uniformly distributed current of 3 A into the paper plane and has a radius of R = 3 cm. The closed loop shown in the sketch is a semi-circle of radius $\rho = 4$ cm that passes through the center of the wire. For this closed loop without using Ampere's law, find (a) the magnetic field along the circular arc, (b) the line integral of $\vec{B}$ along the arc and (c) the line integral of $\vec{B}$ along the straight section. Find the line integral around the entire loop and explain how your result agrees with Ampere's law.



Problem 5

(5/45)

A 1.5 pF plate capacitor is connected to a time varying voltage source V(t) = 12 V $\cdot cos(2\pi f t)$, where $f = 60\ Hz$. Calculate the displacement current inside the capacitor for a plate area of 1 mm^2. Assume that the field is homogeneous and that the charging time of the capacitor is much shorter than 1/f.

Problem 6

A long straight conductor of 0.2 mm radius carries a uniformly-distributed current of 2A (constant).

(a) Find $\vec{J}$ within the conductor

(b) Find the vector magnetic field inside the conductor

(c) Show that $\nabla \times \vec{H} = \vec{J}$ within the conductor.

Useful equations:

Divergence

$$\nabla \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla \cdot \vec{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

$$\nabla \cdot \vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r sin\theta}\frac{\partial}{\partial \theta}(D_\theta sin\theta) + \frac{1}{r sin\theta}\frac{\partial D_\phi}{\partial \phi}$$

Gradient

$$\nabla V = \frac{\partial V}{\partial x}\vec{a}_x + \frac{\partial V}{\partial y}\vec{a}_y + \frac{\partial V}{\partial z}\vec{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\vec{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\vec{a}_\phi + \frac{\partial V}{\partial z}\vec{a}_z$$

$$\nabla V = \frac{\partial V}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{a}_\theta + \frac{1}{r sin\theta}\frac{\partial V}{\partial \phi}\vec{a}_\phi$$

Curl

$$\nabla \times \vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\vec{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\vec{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\vec{a}_z$$

$$\nabla \times \vec{H} = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right)\vec{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\vec{a}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi}\right]\vec{a}_z$$

$$\nabla \times \vec{H} = \frac{1}{r sin\theta}\left[\frac{\partial(H_\phi sin\theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi}\right]\vec{a}_r + \frac{1}{r}\left[\frac{1}{sin\theta}\frac{\partial H_r}{\partial \phi} - \frac{\partial(r H_\phi)}{\partial r}\right]\vec{a}_\theta + \frac{1}{r}\left[\frac{\partial(r H_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}\right]\vec{a}_\phi$$