EE2 Midterm 2 Fall 2012:

1.

(a) Determine the amount of work needed to transfer two charges of 40 nC and -50 nC from infinity to locations (0,0,1) and (2,0,0) respectively.

(b) If $V = 2x^2 + 6y^2\text{(Volts)}$ in free space find the energy stored in a volume defined by $-1 \le x \le 1,\ -1 \le y \le 1$, and $-1 \le z \le 1$.

2.

(a) The electric field intensity in a polyethelene ($\varepsilon_r = 2.55$) filled, parallel plate capacitor is 10 kV/m. If the distance between the plates is 1.5 mm. Calculate:

(a) $\vec{D}$

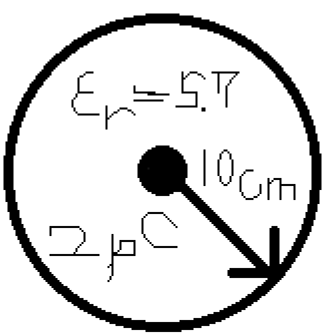
(b) $\vec{P}$

(c) The surface charge density on the plates.

(d) The surface charge density $(\frac{C}{m^2})$ of polarization.

(e) The potential difference between the plates.

(b) A dielectric sphere $\varepsilon_r = 5.7, r = 10cm$ has a $2\ pC$ of charge placed at center.



Calculate:

(a) The surface density of polarization charge $(\frac{C}{m^2})$ on the surface of the sphere.

(b) The force exerted by the charge on a $-4\ pC$ charge on the top of the sphere.

3.

(a) Polarization is defined as

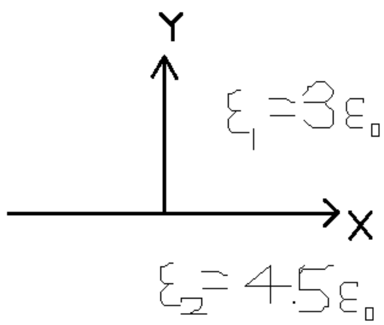
$$\vec{P} = \frac{1}{\Delta V}\sum_{m=1}^{n\Delta V} P_i$$

$$\vec{P_i} = Q\vec{d}$$

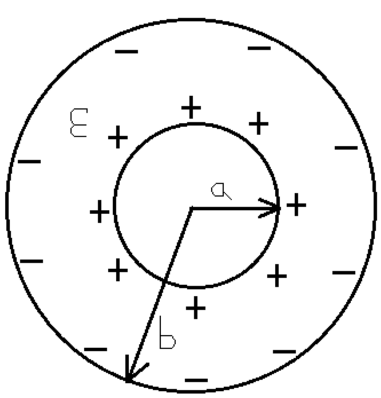
At a particular temperature and pressure, helium gas contains $5 \times 10^{25}\ atoms/m^3$. If a $10kV/m$ field is applied to the gas, the average electron cloud shifts $10^{-18}\ m$. What is the dielectric constant (ε_r) of He?

(b) A $10\ mC$ charge is embedded in wood, $\varepsilon_r = 4$. If the charge is located at the center, find $\vec{P}$ at $r = 1$.

4. Given that $\overrightarrow{E_1} = 10\widehat{a_x} - 6\widehat{a_y} + 12\widehat{a_z}\ {}^{V}/_{m}$ in the accompanying figure, find $\overrightarrow{P_1}$ and $\overrightarrow{E_2}$ and the angle $\overrightarrow{E_2}$ makes with the y-axis.



5. A spherical capacitor is made from two concentric spherical conductor separated by a dielectric ε as shown. Assume that charges $+Q$ and $-Q$ are deposited on the inner and outer conductor respectively. Show that the capacitance is given by $C = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}}$.



6. Write down the expressions for the following:

1) Continuity equation

2) Boundary conditions for conductor – vacuum interface.

3) General expression for resistance of a conductor.

4) Poisson's Equation

5) LaPlace's Equation

6) Gauss's Law

7) Maxwell's Equation

8) Energy stored in a capacitor

9) Divergence theorem

Point

EE 1 Midterm2 Fall 2012 Solutions

1 (a) $W = W_1 + W_2$ — 1

$= 0 + Q_2 V_{21}$ — 1

$= Q_2 \dfrac{Q_1}{4\pi\varepsilon_0 \, |(2,0,0)-(0,0,1)|}$ — 1

$= \dfrac{40\times10^{-9} \times (-50\times10^{-9})}{4\pi \times 8.85\times10^{-12} \, |(2,0,-1)|} = \dfrac{40\times9\times(-50)10^{-9}}{\sqrt{5}}$ — 1

$\simeq -8.05 \ \mu J$ — 1

1 (b). $E = -\nabla V = -\dfrac{\partial V}{\partial x} a_x - \dfrac{\partial V}{\partial y} a_y$ — 1

$= -4x \, a_x - 12y \, a_y \quad V/m$ — 1

$W = \dfrac{1}{2}\varepsilon_0 \iiint E^2 \, dv$ — 1

$= \dfrac{1}{2}\varepsilon_0 \int_{x=-1}^{1} \int_{y=-1}^{1} \int_{-1}^{1} (16x^2 + 144y^2) \, dx\,dy\,dz$ — 1

$= \dfrac{1}{2}\varepsilon_0 \left[16\,(8/3) + 144\,(8/3) \right]$ — 1

$= 1.9 \ nJ$

a) Solution 2.

a) $D = \varepsilon_0 \varepsilon_r E = \frac{10^{-9}}{36\pi} \times (2.55) \times 10^{+4} = 225.4 \ nC/m^2$ 1

b) $P = \chi_e \varepsilon_0 E = (1.55) \times \frac{10^{-9}}{36\pi} \times 10^4 = 137 \ nC/m^2$ 1

c) $\rho_s = \underline{D} \cdot \underline{a}_n = \pm D_n = \pm 225.4 \ nC/m^2$ 1

d) $\rho_{ps} = \underline{P} \cdot \underline{a}_n = \pm P_n = \pm 137 \ nC/m^2$ 1

e) $V = Ed = 10^4 (1.5 \times 10^{-3}) = 15 V$. 1

b) a) $\underline{E} = \frac{Q}{4\pi\varepsilon r^2} \underline{a}_r$ 1

$\underline{P} = \chi_e \varepsilon_0 \underline{E} = \frac{\chi_e Q}{4\pi \varepsilon_r r^2} \underline{a}_r$ 1

$\rho_{ps} = \underline{P} \cdot \underline{a}_r = \frac{(\varepsilon_r - 1)}{4\pi\varepsilon_r r^2} Q = \frac{(4.7)(2) \times 10^{-12}}{4\pi (5.7) 100 \times 10^{-4}}$ 1

$= 13.12 \ pC/m^2$ 1

b) $\underline{F} = \frac{Q_1 Q_2}{4\pi\varepsilon r^2} \underline{a}_r = \frac{(-4)(2) \times 10^{-24}}{4\pi \frac{10^{-9}}{36\pi} (5.7) \times 100 \times 10^{-4}}$ 2

$= -1.26 \ \underline{a}_r \ pN$ 1

k.

Solution to 3.

a) $|\underline{P}| = n Q d = 2ned = \chi_e \varepsilon_0 E$ 1

$$\chi_e = \frac{2ned}{\varepsilon_0 E} = \frac{2 \times 5\times10^{25} \times 1.6\times10^{-19} \times 10^{-18}}{\frac{10^{-9}}{36\pi} \times 10^{+4}}$$ 1 1

$\varepsilon_r = 1 + \chi_e = 1.000182.$ 1

b) $$\underline{E} = \frac{Q}{4\pi \varepsilon_0 \varepsilon_r r^2} \underline{a}_r$$ 1

$$\underline{P} = \chi_e \varepsilon_0 \underline{E} = \frac{\chi_e Q}{4\pi \varepsilon_r r^2} \underline{a}_r = \frac{3(10)10^{-3}}{4\pi(4)(1)^2} \underline{a}_r$$ 2

$= 596.8 \; \underline{a}_r \; \mu C/m^2.$ 1

Solution to 4.

a) $\underline{P}_1 = \varepsilon_0 \chi_{e1} \underline{E}_1 = \frac{2\times10^{-9}}{36\pi}(10,-6,12)$ 1

$= 0.1768\ \underline{a}_x - 0.1061\ \underline{a}_y + 0.2122\ \underline{a}_z\ \text{nC/m}^2$ 1

b) $E_{1n} = -6\underline{a}_y$ 1

$E_{1t} = E_{2t} = 10\ \underline{a}_x + 12\ \underline{a}_z$ V/m 1

$D_{2n} = D_{1n} = \varepsilon_2 E_{2n} = \varepsilon_1 E_{1n}$ 1

$E_{2n} = \frac{\varepsilon_1}{\varepsilon_2} E_{1n} = \frac{3\varepsilon_0}{4.5\varepsilon_0}(-6\ \underline{a}_y) = -4\underline{a}_y$ 1

$\underline{E}_2 = 10\ \underline{a}_x - 4\ \underline{a}_y + 12\ \underline{a}_z$ V/m 1

c) $\tan\theta_2 = \frac{E_{2t}}{E_{2n}} = \frac{\sqrt{10^2+12^2}}{4} = 3.905$ $\theta = \sout{75.4^0}$ 75.64^0. 3

Solution to 5

$C = Q/V$ 1

$Q = \varepsilon \oint \underline{E} \cdot d\underline{s} = \varepsilon E_r \, 4\pi r^2$ 2

or $\underline{E} = \dfrac{Q}{4\pi\varepsilon r^2} a_r$ 1

The potential difference

$$V = -\int_b^a \underline{E} \cdot d\underline{L} = -\int_b^a \frac{Q}{4\pi\varepsilon r^2} a_r \cdot dr \, a_r$$ 2

$$= \frac{Q}{4\pi\varepsilon} \left[\frac{1}{a} - \frac{1}{b} \right]$$ 2

$$C = \frac{Q}{V} = \frac{Q \, 4\pi\varepsilon}{Q \left[\frac{1}{a} - \frac{1}{b} \right]} = \frac{4\pi\varepsilon}{[1/a - 1/b]}$$

2

(10)

Solution to 6

1) $\nabla \cdot J = -\partial \rho_v / \partial t$ 1

2) $E_t = 0$ 2

$D_n = \rho_s$

3) $R = V/I = \dfrac{-\int E \cdot dl}{\int J \cdot ds} = \dfrac{-\int E \cdot dl}{\int \sigma E \cdot ds}$ 1

4) $\nabla^2 V = -\rho_v / \varepsilon$ 1

5) $\nabla^2 V = 0$ subject to boundary conditions 1

6) $\oint D \cdot ds = Q_{enclosed}$ 1

7) $\nabla \cdot D = \rho_v$ 1

8) $\Delta W = \frac{1}{2} C V^2$ 1

9) $\oint_S D \cdot ds = \int_V (\nabla \cdot D)\, dv$. 1