Solutions to Midterm 2

Fall 2009

Solution to 1.

2 i) $\underline{\nabla} \cdot \underline{J} = -\partial \rho_v / \partial t$

2 ii) $W_E = \frac{1}{2} \int \varepsilon_0 E^2 dv$

2 iii) $I = dQ/dt$

2 iv) $\underline{J} = -\rho_v \mu_e \underline{E} = \sigma \underline{E} = \rho_v \underline{U}_{drift}$.

2 v) $E_t = 0$

2 $D_N = \rho_s$

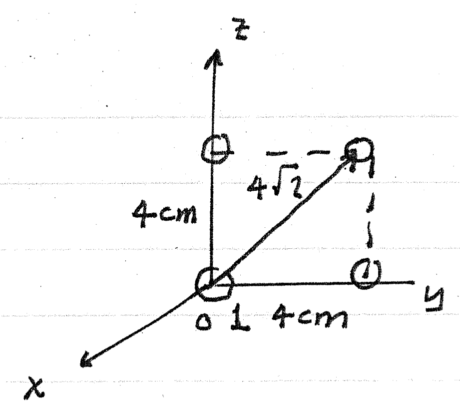
2 vi) $\underline{D} = \varepsilon_0 \underline{E}_{in} + \underline{P} = \varepsilon_0 \varepsilon_r \underline{E}_{in} = \varepsilon_0 (1 + \chi_e) \underline{E}_{in}$

2 vii) $E_{tan1} = E_{tan2}$ or $\dfrac{D_{tan1}}{D_{tan2}} = \dfrac{\varepsilon_1}{\varepsilon_2}$ $\quad C = \dfrac{Q}{V} = \dfrac{\varepsilon S}{d}$

2 $D_{N1} = D_{N2}$ or $\varepsilon_1 E_{N1} = \varepsilon_2 E_{N2}$.

2 viii) $R = V/I = L/\sigma S$.

Solution to 2.



Potential at the origin due to the other 3 charges

4 $V_1 = V_{12} + V_{13} + V_{14}$

4 $= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right]$

4 $= \frac{10^{-9}}{4\pi\epsilon_0}\left[\frac{1}{0.04} + \frac{1}{0.04} + \frac{1}{0.04\sqrt{2}}\right]$

$= 608$ volts.

Thus the energy stored in the potential field is

4 $W_{total} = \frac{1}{2}(4)\,10^{-9}\,V_1$

4 $= \frac{1}{2}(4)\,10^{-9}\,608 = 1.2\times10^{-6}$ J.

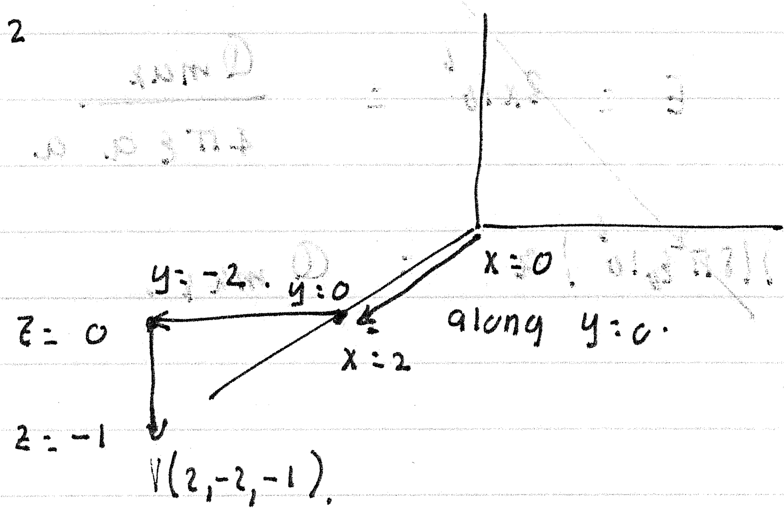
Solution to 3

a) Work done = potential difference = $-\int \underline{E} \cdot d\underline{L}$

8 $V_{PQ} = -\int_0^2 (y+1)\Big|_{y=0} dx - \int_0^{-2} (x-1)\Big|_{x=2} dy - \int_0^{-1} 2\, dz.$

$-2 \quad +2 \quad +2$

2 $= 2$ volts.



8 b) $V_{MN} = -\int_{-2}^{3} (y+1)\Big|_{y=-3} dx - \int_{-3}^{2} (x-1)\Big|_{x=3} dy - \int_4^{-1} 2\, dz.$

$= -\left[-2x\right]_{-2}^{3} - \left[2y\right]_{-3}^{2} - \left[2z\right]_{4}^{-1}$

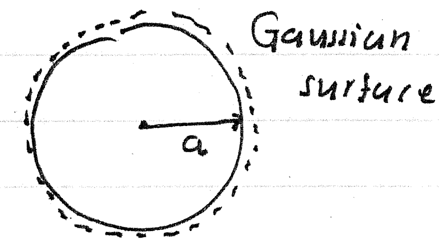
2 $= -\left[-6-4\right] - \left[4+6\right] - \left[-2-8\right]$

$= +10 \quad \cancel{-10} + \cancel{10}$

$= 10$ volts.

Solution to 4.

2 $\oint_S \underline{D} \cdot d\underline{s} = Q_{encl}$



2 $D = D_r \underline{a}_r = \varepsilon_0 E_r \underline{a}_r$

2 $\varepsilon E_r \cdot 4\pi a^2 = Q_{encl}$

$E_r = Q / 4\pi\varepsilon_0 r^2$

$\underline{E} = -\nabla V$

2 or $V = -\int \underline{E} \cdot d\underline{L}$.

$V_a = Q / 4\pi\varepsilon_0 a$

2

But capacitance

2 $C = Q/V = C\big|_{r=a} = Q/V_a = Q \Big/ \left(Q / 4\pi\varepsilon_0 a \right) = 4\pi\varepsilon_0 a.$

$= 7 \times 10^{-4}$ F

2 Let $E_a = E_{breakdown} = 3 \times 10^6$ V/m $= \dfrac{Q_{max}}{4\pi\varepsilon_r a^2}$

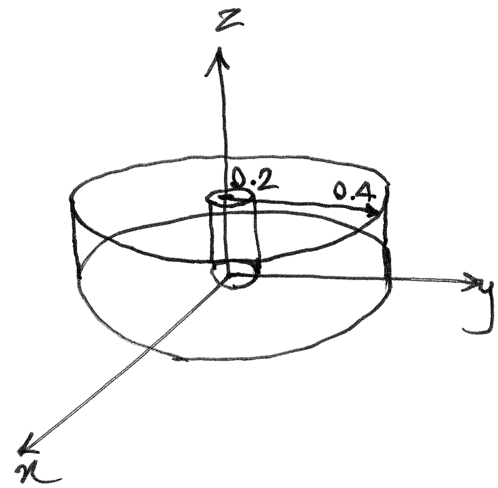
2 $Q_{max} = 4\pi\varepsilon_0 a^2 E_{breakdown}$

$= (C)\, a\, E_{breakdown} = 1.35 \times 10^{10}$ C

2 $V_a = \dfrac{1.35 \times 10^{10}}{4\pi\varepsilon_0 a} = \dfrac{1.35 \times 10^{10}}{7 \times 10^{-4}} \sim 1.9 \times 10^{13}$ Volts

2

Q5.



$$I = \oint_S \underline{J} \cdot \underline{ds} = \int_V \underline{\nabla} \cdot \underline{J}\, dv$$

$$\underline{\nabla} \cdot \underline{J} = \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{25}{\rho}\right) + \frac{1}{\rho} \frac{\partial (0)}{\partial \varphi} + \frac{\partial}{\partial z}\left(\frac{-20}{\rho^2}\right)$$

$$\Rightarrow \underline{\nabla} \cdot \underline{J} = 0$$

$$\Rightarrow I = \oint_S \underline{J} \cdot \underline{ds} = \int_V 0\, dv = 0$$

(20)