[redacted]

EE1 Midterm1 Name [redacted] 70

* $\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m Student ID # [redacted]

1 Point charges Q_1 and Q_2 are respectively located at $(4,0,-3)$ and $(2,0,1)$. If Q_2 is 4 nC find Q_1 such that

a) The electric field intensity has no z component at point $(5,0,6)$

b) The force on a unit test charge has no x-component at point $(5,0,6)$

8

12

TB 11/7/12

a)
$$\begin{array}{r} 5\ 0\ 6 \\ -4\ 0\ 3 \\ \hline 1\ 0\ 3 \end{array} \qquad \begin{array}{r} 5\ 0\ 6 \\ -2\ 0\ 1 \\ \hline 3\ 0\ 5 \end{array} \qquad (2)$$

$$\frac{Q_1}{4\pi\varepsilon_0} \cdot \frac{(1\bar{a}_x + 3\bar{a}_z)}{(\sqrt{10})^3} + \frac{4nC}{4\pi\varepsilon_0} \frac{(3\bar{a}_x + 5\bar{a}_z)}{(\sqrt{34})^3}$$

a) $$\frac{1\,\bar{a}_z}{4\pi\varepsilon_0}\left(\frac{3Q_1}{(\sqrt{10})^3} + \frac{4nC\,(5)}{(\sqrt{34})^3}\right) \neq 0$$

$$Q_1 = -\frac{4nC\,(5)}{(\sqrt{34})^3} \times \frac{(\sqrt{10})^3}{3} = \boxed{-1.0\cancel{63}\ nC}$$

b) $$\frac{(1C)\bar{a}_x}{4\pi\varepsilon_0}\left(\frac{Q_1}{(\sqrt{10})^3} + \frac{4nC\,(3)}{(\sqrt{34})^3}\right) = 0$$

$$Q_1 = -\frac{4nC(3)}{(\sqrt{34})^3} \times (\sqrt{10})^3 = \boxed{-1.\cancel{9}14\ nC}$$

2 Determine the total charge

8

(a) On line $0 < x < 5$ m if $\rho_L = 12x^2$ mC/m

$\int_0^5 12x^2\, dx$

$\left[\frac{12x^3}{3}\right]_0^5$

$\left[4x^3\right]_0^5$

$4(5)^3 - 0 = \boxed{500 \text{ mC}}$

(b) On a cylinder $\rho = 3$, $0 < z < 4$ m if $\rho_s = \rho z^2$ nC/m^2

$\rho = 3$

$\int \rho_s\, dS$

$\int_0^4 \int_0^{2\pi} \rho z^2 \, \rho\, d\phi\, dz$

$2\pi\rho^2 \int_0^4 z^2\, dz$

$18\pi \left[\frac{z^3}{3}\right]_0^4$

$\boxed{1206.372 \text{ nC}}$

(c) Within the sphere $r = 4$ m if $\rho_v = \frac{10}{r\sin\theta}$ C/m^3.

$\int \rho_v\, dV$

$\int_0^{\pi}\int_0^{2\pi}\int_0^4 \frac{10}{r\sin\theta} r^2 \sin\theta\, dr\, d\phi\, d\theta$

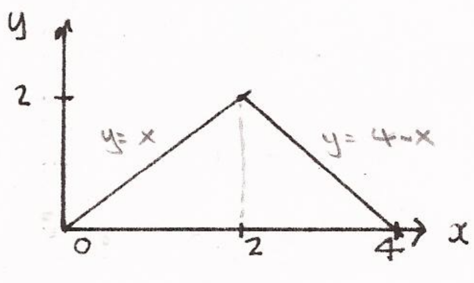
$\int_0^{\pi}\int_0^{2\pi}\int_0^4 10r\, dr\, d\phi\, d\theta$

$2\pi^2 \left[\frac{r^2}{5}\right]_0^4$

$2\pi^2 \left[\frac{4^2}{5} - 0\right]$

$\boxed{63.165 \text{ C}}$

(d) How much charge is enclosed in the triangular region if $\rho_s = 6xy$ C/m^2



$\int_0^2 6xy\, dx + \int_2^4 6xy\, dx$

$\int_0^2 6x(x)\,dx + \int_2^4 6x(4-x)\, dx$

$\int_0^2 6x^2\, dx + \int_2^4 24x - 6x^2\, dx$

$\left[2x^3\right]_0^2 + \left[12x^2 - 2x^3\right]_2^4$

$\left[2(2)^3 - 0\right] + \left[(12(4)^2 - 2(4)^3) - (12(2)^2 - 2(2)^3)\right] = \boxed{48 \text{ C}}$

3 Let $\underline{A} = -3\,\underline{a}_x + \underline{a}_y - 2\,\underline{a}_z$

$\underline{B} = 2\,\underline{a}_x - 5\,\underline{a}_y + \underline{a}_z$

$\underline{C} = \underline{a}_y + 4\,\underline{a}_z$.

7

determine

4 a) Smaller of the two angles between $\underline{A}$ and $\underline{B}$

$A \cdot B = |A||B| \cos\phi$

$\cos\phi = \frac{A \cdot B}{|A||B|}$

$\therefore \phi = \cos^{-1}\left(\frac{A \cdot B}{|A||B|}\right)$

$\phi = 129.371^\circ$ $180^\circ - \phi$

$(-3a_x + 1a_y - 2a_z) \cdot (2a_x - 5a_y + 1a_z)$

$-6 - 5 - 2 = -13 = A \cdot B$ ✓

$|A| = \sqrt{(-3)^2 + (1)^2 + (-2)^2} = \sqrt{14}$ ✓

$|B| = \sqrt{(2)^2 + (-5)^2 + (1)^2} = \sqrt{30}$ ✓

3 b) The ~~component~~ vector component of $\underline{A}$ along $\underline{C}$

$= \left(A \cdot \frac{C}{|C|}\right) \cdot \frac{C}{|C|}$

$= \left((-3a_x + a_y - 2a_z) \cdot \frac{(a_y + 4a_z)}{\sqrt{(1)^2 + (4)^2}}\right) \cdot \left(\frac{a_y + 4a_z}{\sqrt{(1)^2 + (4)^2}}\right)$

$= \left(\frac{1-8}{\sqrt{17}}\right) \cdot \left(\frac{a_y + 4a_z}{\sqrt{17}}\right) = -0.412\,\bar{a}_y - 1.647\,\bar{a}_z$ ✓

0 c) $(\underline{A} \times \underline{B} \cdot C)$

$$\begin{vmatrix} i & j & k \\ -3 & 1 & -2 \\ 2 & -5 & 1 \end{vmatrix} \begin{matrix} i & j \\ -3 & 1 \\ 2 & -5 \end{matrix}$$

$[1-(-3)]i + [(-4)-10]j + [15-2]k$

$(4\bar{a}_x - 14\bar{a}_y + 13\bar{a}_z)(a_y + 4a_z)$

$-14 + 52 = \boxed{38}$ ✗

4 a A volume charge density $\rho_v = 4\rho^2 z \cos\phi$ nC exists inside a wedge defined by

$0 < \rho < 2$

$0 < \phi < \pi/4$

$0 < z < 1$

14

7 How much charge is contained inside the wedge?

$\int \rho_v \, dv$

$$\int_0^1 \int_0^{\frac{\pi}{4}} \int_0^2 4\rho^2 z \cos\phi \, \rho \, d\rho \, d\phi \, dz$$

$$4 \int_0^1 \int_0^{\frac{\pi}{4}} \int_0^2 \rho^3 z \cos\phi \, d\rho \, d\phi \, dz$$

$$4 \int_0^2 \rho^3 \, d\rho \int_0^{\frac{\pi}{4}} \cos\phi \, d\phi \int_0^1 z \, dz$$

$$4 \left[\frac{\rho^4}{4}\right]_0^2 \left[\sin\phi\right]_0^{\frac{\pi}{4}} \left[\frac{z^2}{2}\right]_0^1$$

$$4\left[\frac{2^4}{4} - 0\right]\left[\sin\frac{\pi}{4} - \sin 0\right]\left[\frac{(1)^2}{2} - 0\right] = \boxed{11.314 \text{ nC}}$$

7 4 b A spherical shell extending from $r = 2$ cm to $r = 4$ cm has a uniform charge density $\rho_v = 5 \text{ mC/m}^3$. How much charge is contained?

$Q = \int \rho_v \, dv$ ρ_v is const so pull it out of integral

$Q = \rho_v \int dv$

$Q = \rho_v V$ $\int dV$ = volume.

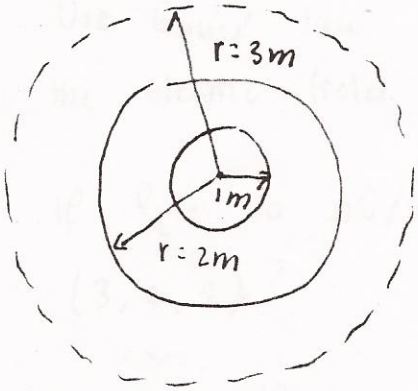
$= \rho_v \frac{4}{3}\pi (r_2^3 - r_1^3)$

$= 5 \text{mC/m}^3 \; \frac{4}{3}\pi (0.04\text{m}^3 - 0.02\text{m}^3)$

$= \boxed{0.00117 \text{ mC}}$

5 If spherical surfaces $r = 1m$ and $r = 2m$ respectively carry uniform surface charge densities of $8 \ nC/m^2$ and $-6 \ nC/m^2$, find $\underline{D}$ at $r = 3 \ m$.

11



$Q_{enc} = \int \rho_{s_1} \partial S_1 + \int \rho_{s_2} \partial S_2$

$Q_{enc.} = \rho_{s_1} \int \partial S_1 + \rho_{s_2} \int \partial S_2$

$\oint \bar{D} \cdot \partial \bar{S} = Q_{enc.}$

$\oint \bar{D} \cdot \partial \bar{S} = Q_{enc.}$

$\bar{D} \oint \partial \bar{S} = Q_{enc.}$

$\bar{D} \, 4\pi r_3^2 = (8nC/m^2) 4\pi r_1^2 + (-6nC/m^2) 4\pi r_2^2$

$\bar{D} \cancel{4\pi} r_3^2 = \cancel{4\pi} (8nC/m^2 \, r_1^2 - 6nC/m^2 \, r_2^2)$

because of symmetry, D is constant over the gaussian surface

$\bar{D} = \dfrac{8nC/m^2 \, r_1^2 - 6nC/m^2 \, r_2^2}{r_3^2}$

9

$\boxed{\bar{D} = -1.778 \ \bar{a}_\rho}$

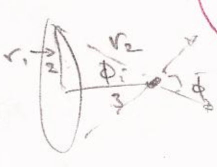
b) A ring placed along $y^2 + z^2 = 4$, $x = 0$ carries a uniform charge of $5 \ \mu C/m$.
Find $\underline{D}$ at $P(3, 0, 0)_m$

$\phi = \tan^{-1} \frac{2}{3}$

2

$r = \sqrt{4+9}$

$r = \sqrt{13}$



$\partial D = \dfrac{\partial Q}{4\pi r^2} a_\rho$

$D = \dfrac{2\cancel{\pi} r_1 \rho_L}{4\cancel{\pi} r_2^2} \cos\phi \, a_x$

Symmetry, only $\bar{a}_x$ component remains

$\boxed{\bar{D} = 1.600 \times 10^{-7} \ \frac{C}{m^2} \bar{a}_x}$

$\partial Q = \rho$

$2\pi r$

$2\pi \cancel{r} \times 5 \times 10^{-6} \ C/m$

4π

6 An infinitely long but thin wire carries a line charge density of ρ_L C/m and lies along the z axis

Use Gauss' law to calculate an expression for the electric field $\underline{E}(\rho, \phi, z)$.

14

If $\rho_L = 10$ nC/m what is the electric field at (3, 4, 5)?

cart.

$$\oint \bar{D}\cdot d\bar{S} = Q_{enc}$$

$$\bar{D}\oint d\bar{S} = \rho_L L$$

how? $\bar{D}\,2\pi\rho K = \rho_L K$

$$\bar{D} = \frac{\rho_L}{2\pi\rho}\,\bar{a}_\rho \qquad \bar{D} = \epsilon_0 \bar{E}$$

7

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\,\bar{a}_\rho}$$

line along z-axis

$$\rho = \sqrt{x^2+y^2}$$

$$\rho = \sqrt{3^2+4^2} = 5$$

$$\bar{E}(3,4,5) = \frac{10\times10^{-9}\ C/m}{2\pi\epsilon_0\,(5m)}\,\bar{a}_\rho$$

7

$$\bar{E}(3,4,5) = \boxed{36\ \frac{V}{m}\ \bar{a}_\rho}$$

Question 7 (a) Two point charges $Q_1 = 2\,nC$ and $Q_2 = -4\,nC$ are located at $(1,0,3)$ and $(-2,1,5)$ respectively. Determine the potential at $P(1,-2,3)$.

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

$\overline{1P}$: $(1,-2,3) - (1\ 0\ 3) = 0\ -2\ 0$

$\overline{2P}$: $1\ -2\ 3 - (-2\ 1\ 5) = 3\ -3\ -2$

by super position

$$V = \sum_{n}^{n} V_{mp}$$

$|\overline{1P}| = 2$

$|\overline{2P}| = \sqrt{22}$

$$V = \frac{2\times10^{-9}\,C}{4\pi\epsilon_0(2)} + \frac{-4\times10^{-9}\,C}{4\pi\epsilon_0(\sqrt{22})}$$

4

$$V = \boxed{1.325\,V}$$

8

(b) Given that a spherically symmetric charge distribution is given by

$$\rho_v = \rho_0\left(1-(r/a)\right)^2 \quad \text{for} \quad r \le a$$
$$= 0 \quad r \geq a$$

Find $\underline{E}$ and V for $r \geq a$.

$$\int_0^{2\pi}\int_0^{\pi}\int_a^{r} \rho_0\left(1-\left(\tfrac{r}{a}\right)\right)^2 r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$\oint E\cdot d\bar{S} = \frac{Q_{enc}}{\epsilon_0}$$

$$2\pi\left[\cos\theta\right]_0^{\pi} \int_a^{r} \rho_0\left(r-\frac{r^2}{a}\right)^2 dr$$

$$\bar{E}\oint d\bar{S} = \frac{\int \rho_v\, dV}{\epsilon_0}$$

$4\pi\rho_0$

$$\bar{E}\,4\pi r^2 = \frac{\int \rho_0\left(1-\left(\frac{r}{a}\right)\right)^2 dV}{\epsilon_0}$$

$$\left(\frac{r-\frac{r^2}{a}}{3}\right)^3 \frac{1}{1-\frac{2r}{a}}$$

4

$$\bar{E} = \frac{\int \rho_0\left(1-\left(\frac{r}{a}\right)\right)^2 dV}{4\pi r^2 \epsilon_0}$$

$$V = -\int E\cdot dr$$