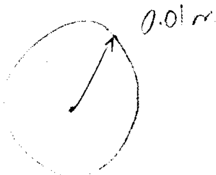
1. A copper sphere of radius 1 cm carries a surface charge density of 1 $\mu C/cm^2$ on its surface in free space. (a)Calculate total energy stored in the electrostatic field. (b) Calculate the capacitance for this sphere.

2. A current filament of $-2a_y$ A lies along the y axis. Find the magnitude and direction of the magnetic field at (a) $P_1(1,0,0)$, (b) $P_2(0,0,2)$.

3. A uniform current sheet is located in free space at z=0, 10 a_x A/m. A current filament of 2 a_x A is parallel to the x axis located at z = 1 m. Find the magnetic flux density, B, at (a) $P_1(0,0,2)$, (b) $P_2(0,2,1)$.

4. Three planar current sheets are located in the free space as follows: 10 a_x A/m at z = 0, 5 a_x A/m at z = 2, and 1 a_x A/m at z = 4. Find the magnetic energy density for all z.

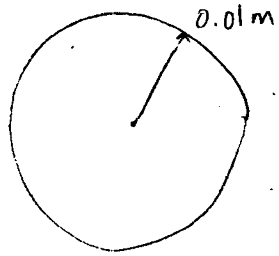
1)



$\rho_s = 1 \frac{\mu C}{cm^2}$

1)

$1\frac{\mu C}{cm^2} \times \left(\frac{100\,cm}{1\,m}\right)^2 = 1\times10^{-2}\,\frac{C}{m^2}$



$1\frac{\mu C}{cm^2}$

inside sphere $E=0$

free space $\Rightarrow \varepsilon_0$

25

$V = -\int_{\infty}^{0.01} \vec{E}\cdot d\vec{l}$

$\oint E\cdot dA = \frac{Q_{en}}{\varepsilon_0}$ $\qquad \rho_s = \frac{Q}{A}$

$\vec{E}(4\pi r^2) = \frac{(1\times10^{-2}C)(4\pi(0.01)^2)}{\varepsilon_0}$

$\vec{E} = \frac{1.26\times10^{-5}}{\varepsilon_0 4\pi r^2}\,\hat{a}_r \qquad r > 0.01\,m \qquad \vec{E}\cdot\vec{E} = E^2$

a) $W_E = \frac{1}{2}\int_{vol} \varepsilon_0 E^2\,dv = \frac{1}{2}\varepsilon_0 \int_0^{2\pi}\int_0^{\pi}\int_{0.01}^{\infty} \frac{1.2756\times10^{10}}{r^4}\, r^2 \sin\theta\, dr\, d\theta\, d\phi$

$\vec{E} = \frac{112943}{r^2}\,\hat{a}_r$

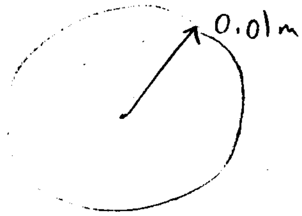
$E^2 = \vec{E}\cdot\vec{E} = \left(\frac{11294}{r^2}\hat{a}_r\right)\cdot\left(\frac{11294}{r^2}\hat{a}_r\right) = 1.\frac{2756\times10^{10}}{r^4}$

$= (5.65\times10^{-2})2\pi \int_0^{\pi}\int_{0.01}^{\infty} \frac{1}{r^2}\sin\theta\, dr\, d\theta$

$= 3.55\times10^{-1}\int_{0.01}^{\infty} \frac{1}{r^2}dr \cdot \int_0^{\pi} \sin\theta\, d\theta$

$W_E = \boxed{70.964\ J}$ ✓

b)



$$\frac{1\,\mu C}{cm^2} = 1\times10^{-2}\,\frac{C}{m^2}$$

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 a \quad \text{for a sphere}$$

$$C = 4\pi\left(8.854\times10^{-12}\right)(0.01)$$

$$\boxed{C = 1.113\times10^{-12}\ \text{F}}$$

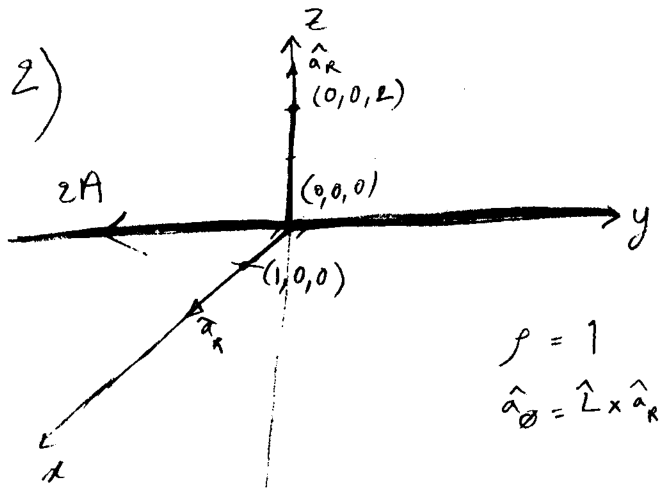
or to prove for a sphere:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{a}_r \qquad Q = (0.01)(4\pi(0.01)^2) \qquad V = -\int_b^a \vec{E}\cdot d\vec{l}$$

$$V = -\int_{\infty}^{0.01} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = 1.129\times10^{7}\ \text{V}$$

$$C = \frac{Q}{V} = \frac{1.256\times10^{-5}}{1.129\times10^{7}} = 1.113\times10^{-12}\ \text{F}$$

2)



$I = -2\,\hat{a}_y$ A

$\rho = 1 \qquad \hat{a}_R = 1\,\hat{a}_x$

$\hat{a}_\phi = \hat{L} \times \hat{a}_R = (-\hat{a}_y) \times (\hat{a}_x) = \hat{a}_z$

at (1,0,0):

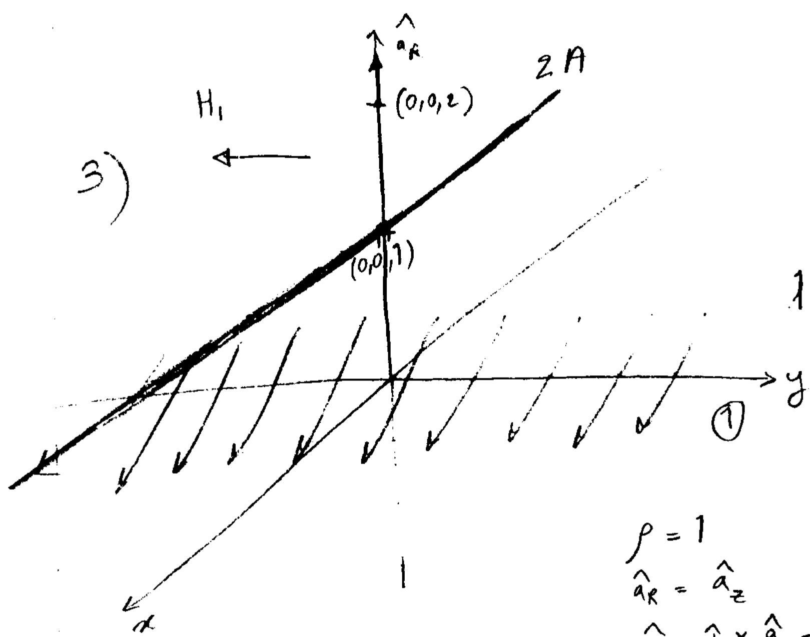
a) $\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi = \frac{2}{2\pi(1)}\hat{a}_z \qquad \boxed{\vec{H} = 0.318\,\hat{a}_z}$ ✓

$\rho = 2 \qquad \hat{a}_R = 2\,\hat{a}_z \qquad \hat{a}_\phi = \hat{L} \times \hat{a}_R = \frac{(-\hat{a}_y) \times (2\,\hat{a}_z)}{2} = -\hat{a}_x$

at (0,0,2):

b) $\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi = \frac{2}{2\pi(2)}(-\hat{a}_x) \qquad \boxed{\vec{H} = -0.159\,\hat{a}_x}$ ✓

3)



$\vec{K} = 10\,\hat{a}_x\ \frac{A}{m}$

$\rho = 1$

$\hat{a}_R = \hat{a}_z$

$\hat{a}_\phi = \hat{l} \times \hat{a}_R = (\hat{a}_x) \times (\hat{a}_z) = -\hat{a}_y$

for the sheet.

a) $\vec{H}_1 = \frac{1}{2}\vec{K} \times \hat{a}_n = \frac{1}{2}(10)\,(\hat{a}_x \times \hat{a}_z) = \boxed{-5\,\hat{a}_y}\ \frac{A}{m}$

for the line.

$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi = \frac{2}{2\pi(1)}(-\hat{a}_y) = \boxed{-0.318\,\hat{a}_y}\ \frac{A}{m}$

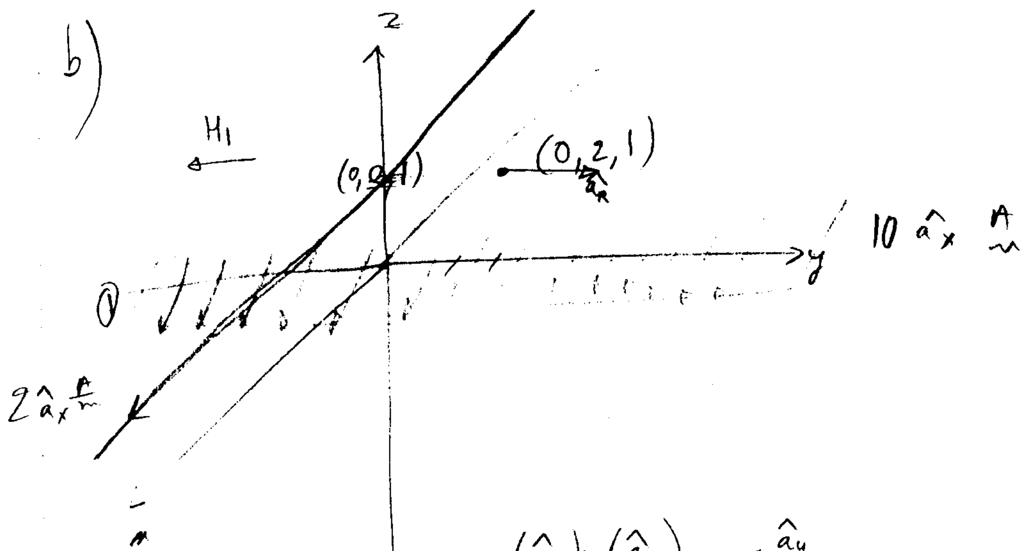
$H_{total} = \vec{H}_1 + \vec{H} = -5\,\hat{a}_y + (-0.318\,\hat{a}_y)$

$\vec{H} = -5.318\,\hat{a}_y$

$\vec{B} = \mu_0\vec{H} = (4\pi \times 10^{-7})(-5.318\,\hat{a}_y)$

✓ $\boxed{\vec{B} = -6.683 \times 10^{-6}\,\hat{a}_y\ T}$

b)



$$(\hat{a}_x)\times(\hat{a}_z) = -\hat{a}_y$$

from sheet:

$$\vec{H}_1 = \frac{1}{2}\vec{K}\times\hat{a}_n = \frac{10}{2}(-\hat{a}_y) = \boxed{-5\,\hat{a}_y}\ \frac{A}{m}$$

from line:

$$\rho = 2$$

$$\hat{a}_R = 2\,\hat{a}_y \qquad \hat{a}_\phi = \hat{l}\times\hat{a}_R = \frac{(\hat{a}_x)\times(2\,\hat{a}_y)}{2} = \hat{a}_z$$

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi = \frac{2}{2\pi(2)}\hat{a}_z = \boxed{0.159\ \hat{a}_z}\ \frac{A}{m}$$

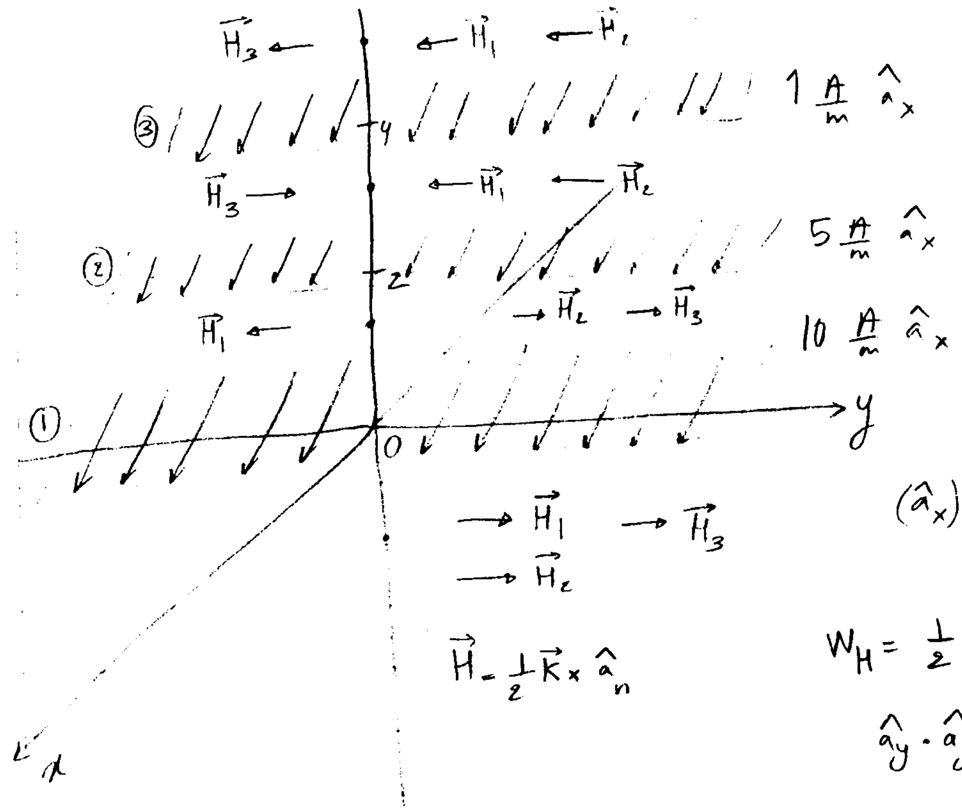
$$\vec{H}_{total} = \vec{H}_1 + \vec{H} = -5\,\hat{a}_y + 0.159\,\hat{a}_z$$

$$\vec{B} = \mu_0\vec{H} = (4\pi\times10^{-7})(-5\,\hat{a}_y + 0.159\,\hat{a}_z)$$

$$\boxed{\vec{B} = (-6.28\times10^{-6}\,\hat{a}_y + 2\times10^{-7}\,\hat{a}_z)\ T}$$

4)

25



μ_0

$(\hat{a}_x) \times -\hat{a}_z = \hat{a}_y$

$\vec{H} = \frac{1}{2}\vec{K} \times \hat{a}_n$

$W_H = \frac{1}{2}\vec{B}\cdot\vec{H} \quad \frac{J}{m^3}$

$\hat{a}_y \cdot \hat{a}_y = 1$

- for $z < 0$:

$\vec{H} = \vec{H}_1 + \vec{H}_2 + H_3 = \frac{1}{2}(10)\,\hat{a}_y + \frac{1}{2}(5)\,\hat{a}_y + \frac{1}{2}(1)\,\hat{a}_y = 8\,\hat{a}_y \quad \frac{A}{m}$

$\vec{B} = \mu_0 \vec{H} = (4\pi \times 10^{-7})(8)\,\hat{a}_y = 1.01 \times 10^{-5}\,\hat{a}_y \quad T$

$W_H = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\left(1.01\times10^{-5}\hat{a}_y\right)\cdot\left(8\,\hat{a}_y\right) = \boxed{4.02 \times 10^{-5} \ \frac{J}{m^3}}$ ✓

- for $0 < z < 2$:

$\vec{H} = H_2 + H_3 - H_1 = \frac{5}{2}\hat{a}_y + \frac{1}{2}\hat{a}_y - \frac{10}{2}\hat{a}_y = -2\,\hat{a}_y \quad \frac{A}{m}$

$\vec{B} = \mu_0 \vec{H} = (4\pi\times10^{-7})(-2)\,\hat{a}_y = -2.51 \times 10^{-6}\,\hat{a}_y \quad T$

$W_H = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\left(-2.51\times10^{-6}\hat{a}_y\right)\cdot\left(-2\,\hat{a}_y\right) = \boxed{2.513 \times 10^{-6} \ \frac{J}{m^3}}$ ✓

- for $2 < z < 4$:

$\vec{H} = -H_1 - H_2 + H_3 = \frac{10}{2}(-\hat{a}_y) + \frac{5}{2}(-\hat{a}_y) + \left(\frac{1}{2}\,\hat{a}_y\right) = -7\,\hat{a}_y \quad \frac{A}{m}$

$\vec{B} = \mu_0\vec{H} = (4\pi\times10^{-7})(-7\,\hat{a}_y) = -8.796 \times 10^{-6}\,\hat{a}_y \quad T$

$W_H = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\left(-8.796\times10^{-6}\hat{a}_y\right)\cdot\left(-7\,\hat{a}_y\right) = \boxed{3.08\times10^{-5} \ \frac{J}{m^3}}$ ✓

- for $z > 4$:

$\vec{H} = -H_1 + -H_2 + -H_3 = \frac{10}{2}(-\hat{a}_y) + \frac{5}{2}(-\hat{a}_y) - \frac{1}{2}\hat{a}_y = -8\,\hat{a}_y \quad \frac{A}{m}$

$\vec{B} = \mu_0\vec{H} = (4\pi\times10^{-7})(-8\,\hat{a}_y) = -1.01\times10^{-5}\,\hat{a}_y \quad T$

$W_H = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\left(-1.01\times10^{-5}\hat{a}_y\right)\left(-8\,\hat{a}_y\right) = \boxed{4.02\times10^{-5} \ \frac{J}{m^3}}$