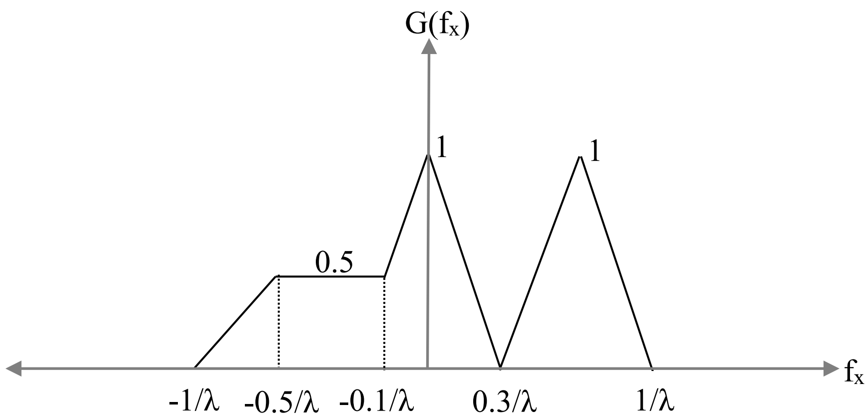
EE176 Midterm **Student Name:** **Student ID#**
(**OPEN BOOKS AND NOTES – NO INTERNET ACCESS**)

Total grade for all questions: 87
Give 13 credits to all students by default

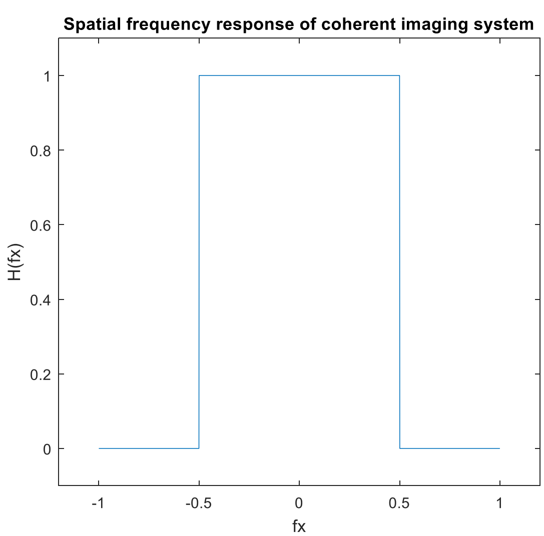
(1) (a) 10pts - For an object that has a spatial frequency spectrum given by $G(f_x)$ as shown below, plot the spatial frequency spectrum of the image, i.e., $U_i(f_x)$, that is formed under a coherent imaging system that has a numerical aperture of 0.5

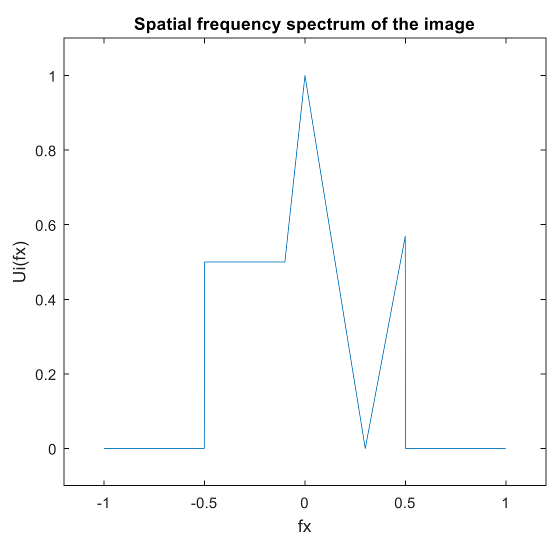
(b) 10 pts - How would your answer to (a) change if the imaging system was spatially incoherent with a numerical aperture of 0.5. **For part (b) only**, you can assume $G(f_x)$ refers to the spatial frequency spectrum of the object's intensity. This will simplify your analysis.

λ denotes the wavelength of light in each case, and f_x is the spatial frequency along x direction.



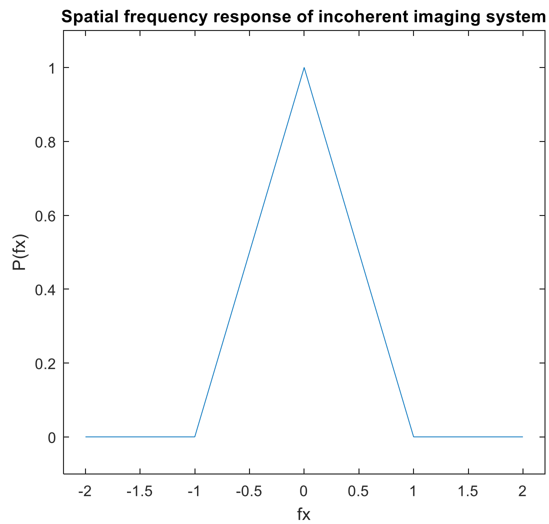
(a)

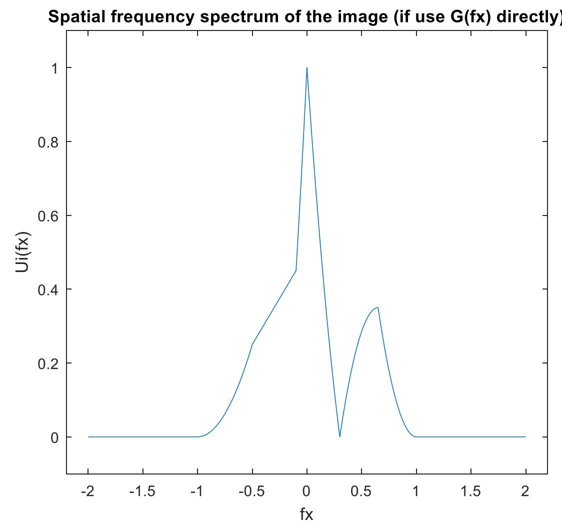




- 5 credits: Show H(fx) correctly, either figure or words that state the cut-off frequency is at 0.5/λ is fine (if final answer has cut-off frequency at 0.5/λ, also give full credits on this part).
- 4 credits: Show the spectrum of the image is roughly in the shape of the above figure, which has sharp stopping frequencies.
- 1 credits: At the right cut-off frequency, Ui(fx) is roughly at 0.5 (0.5686 more precisely).

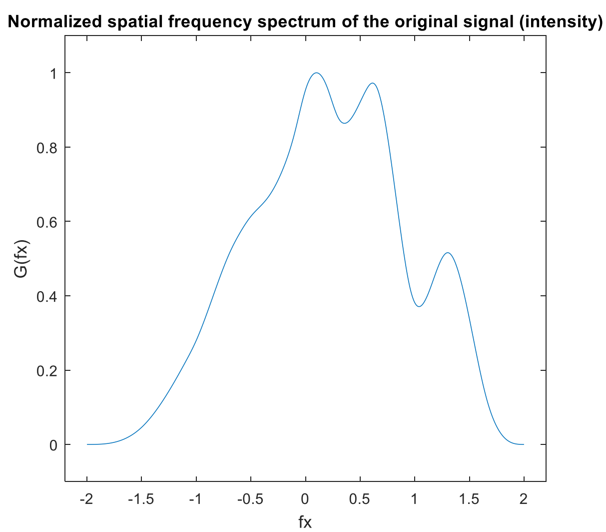
(b)

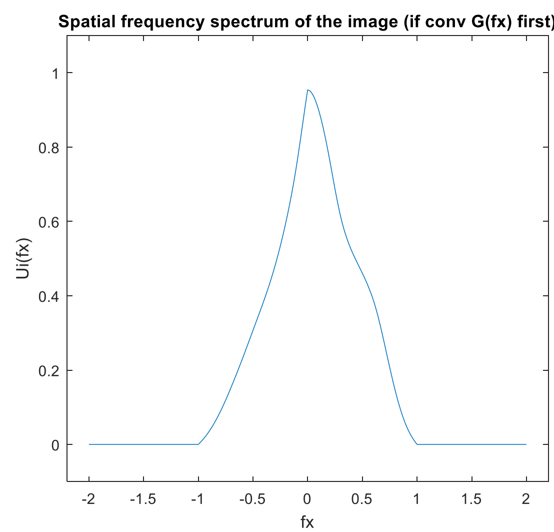




- 5 credits: Show H(fx) correctly, either figure or words that state the cut-off frequency is at $1/\lambda$ is fine (if final answer has cut-off frequency at $1/\lambda$, and the final plot can indicate the student knows that it is inclined line, also give full credits on this part).
- 1 credit: Give 1 credit to each part of the line for Ui(fx), it doesn't need to be accurate, but the tendency must be correct, i.e. if it is a straight line or not. **Note**: There are 6 parts on this curve, if only gets wrong on one part of the curve, give full credits for the question, if all correct, give 1 extra bonus point on the final grade.

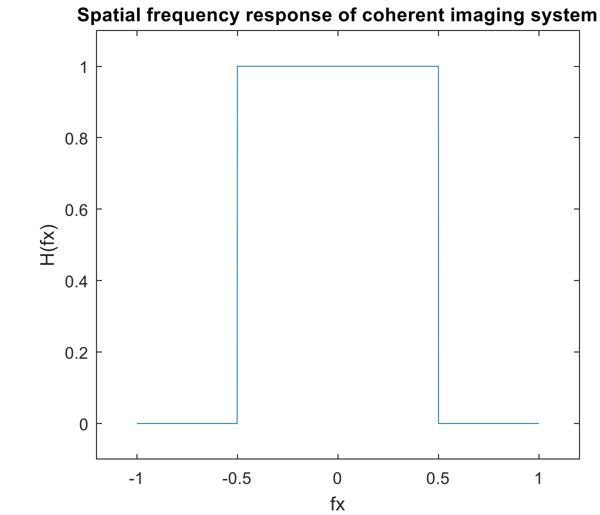
For students who did **not** use the hint to simplify the question:

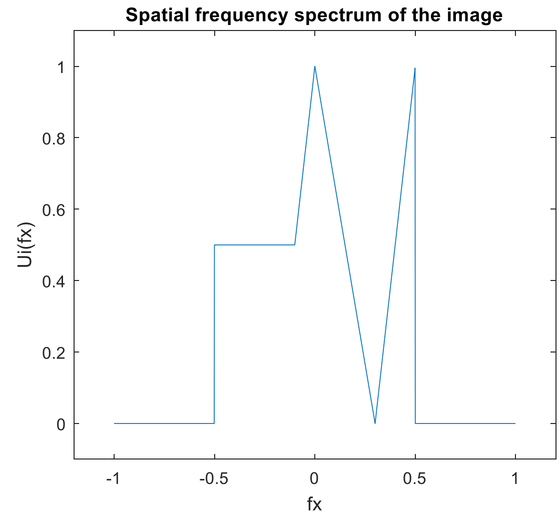


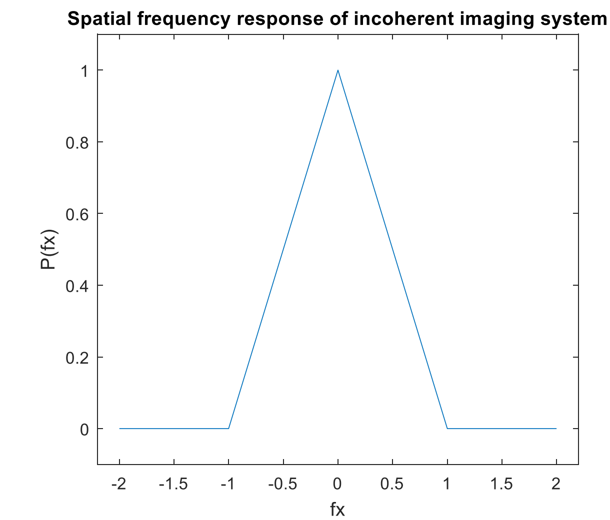


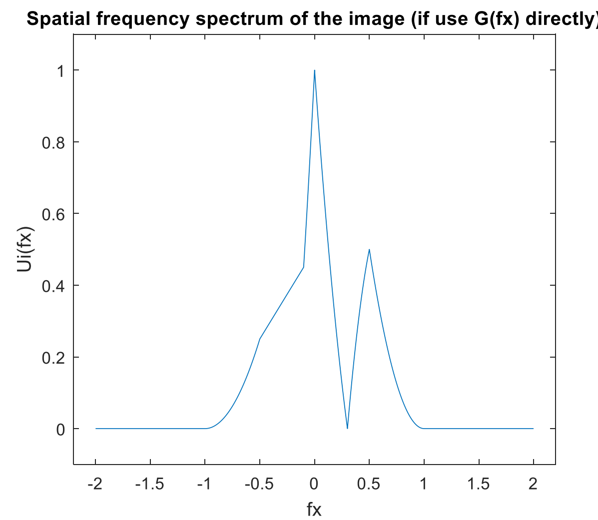
- 5 credits: Show P(fx) correctly (if final answer is correct, this can be skipped), either figure or words that state the cut-off frequency is at $1/\lambda$.
- 1 credit: Show the relationship between P(fx), G(fx), and Ui(fx) correctly.
- 2 credits: Show G(fx) roughly in the shape (this cannot be skipped if the problem is solved this way).
- 2 credits: Show Ui(fx) roughly in the shape.

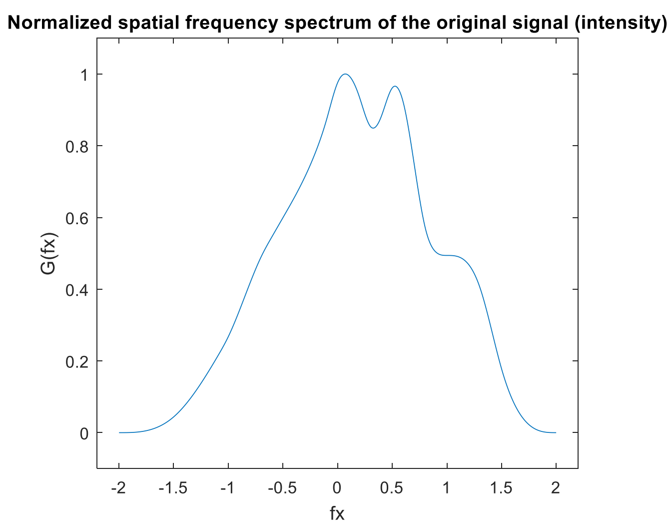
If student think the peak of G(fx) on the right is at $0.5/\lambda$, the answer will look like these, it is also okay:

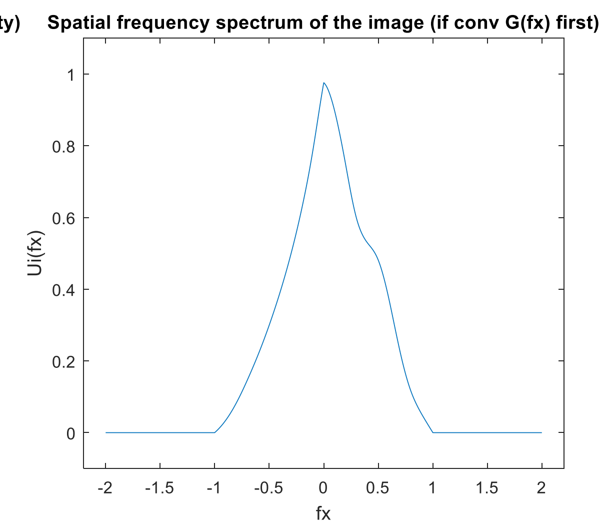












(2) Suppose that you are operating an Excimer laser (with an active material of KrF) which lases at 248nm wavelength. The beam waist of this laser is measured as 5 mm.

(a) 7 pts - Calculate the beam diameter of the Excimer laser after 1 km of propagation in air from the location of the beam waist.

Rayleigh range:

$$z_R = \frac{\pi w_0^2}{\lambda} = \frac{\pi (5\text{mm})^2}{248\text{nm}} = 316.6928\text{m}$$

Beam radius:

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2} = 5\text{mm}\sqrt{1 + \left(\frac{1000\text{m}}{316.6928\text{m}}\right)^2} = 16.561\text{mm}$$

Beam diameter:

$$d = 2w(z) = 33.122\text{mm}$$

- 2 credits: Rayleigh range equation is correct
- 2 credits: Beam radius equation
- 1 credit: Beam diameter (the time 2 relationship)
- 2 credit: Get the final answer or put all numbers in the correct locations (some people didn't have the calculator, it is okay that student don't have the result, but all numbers must be put in the correct locations in their final answer). In addition, unit **must** be correct.

(b) 7 pts - Calculate the radius of curvature of the wavefront of the laser at the same distance (1 km) from the beam waist.

Rayleigh range:

$$z_R = \frac{\pi w_0^2}{\lambda} = \frac{\pi (5\text{mm})^2}{248\text{nm}} = 316.6928\text{m}$$

Radius of curvature:

$$R(z) = z\left[1 + \left(\frac{z_R}{z}\right)^2\right] = 1000\text{m}\left[1 + \left(\frac{316.6928\text{m}}{1000\text{m}}\right)^2\right] = 1100.2943\text{m}$$

- 2 credits: Rayleigh range equation is correct, if answered correctly in (a), give these credits by default.
- 3 credits: Radius of curvature equation
- 2 credit: Get the final answer or put all numbers in the correct locations (some people didn't have the calculator, it is okay that student don't have the result, but all numbers must be put in the correct locations in their final answer). In addition, unit **must** be correct.

(3) Suppose that you are working to find cure to cancer.
For this purpose, suppose also that you frequently label cells with fluorescent markers to make cancer cells glow among many other healthy cells such that you can tell them apart from the rest of the "good" cells. In one experiment, assume that to label these cells you have used a fluorophore that emits light (when pumped appropriately) at 600 nm wavelength (red color).
(a) 7 pts - For this fluorescent cell imaging experiment, using an objective lens that has a numerical aperture (NA) of 0.75, what would be the expected spatial resolution in your images? Just for your information the typical size of a cancer cell would be ~20-50 μm.

The Abbe diffraction limited spatial resolution is:

$$r = \frac{\lambda}{2\text{NA}} = \frac{600\text{nm}}{1.5} = 400\text{nm}$$

- 4 credits: State the equation correctly, Rayleigh is also fine.
- 3 credits: Get to the final answer.

(b) 7 pts - For the same cell imaging experiment, what would be the diffraction limited spatial resolution in air? This question assumes that now you have the best objective lens that operates in air, where the refractive index (n) is 1.0.

The Abbe diffraction limited spatial resolution is:

$$r = \frac{\lambda}{2\text{NA}} = \frac{\lambda}{2n\sin\theta}$$

Since it is the 'best objective', then sinθ should be 1, hence:

$$r = \frac{\lambda}{2n} = \frac{600\text{nm}}{2} = 300\text{nm}$$

- 2 credit: State the Abbe equation correctly, Rayleigh is also fine.
- 2 credits: Have the equation for numerical aperture, it can be the above equality equation (NA = $n$sinθ), or an inequality equation stating that NA $\leqslant n$.
- 1 credits: Have NA = n because it is the 'best objective' (Do **not** take this by default, if the numerical aperture equation is not stated, don't give this credit, i.e., if write $r=\lambda/2n$ directly and get to the final answer, minus 3 credits in total).
- 2 credits: Get to the final answer.

(c) 7 pts - To break the diffraction limit of light, and to achieve a better resolution than your answer to (b), what would you do? In other words, how would you break the diffraction limit of light? Name at least one microscopy technique that can break the diffraction limit of light and briefly discuss how it works.

Near-field Scanning Optical Microscope (SNOM, NSOM): Capture evanescent wave.
Stimulated Emission Depletion Microscopy (STED): Have an extra STED beam to reject fluorescence.
Photo-Activated Localization Microscopy (PALM): Compute each fluorescent molecule center.
Structured Illumination Microscopy (SIM): Each time sample a shifted frequency spectrum, then fuse together.

- 2 credits: Mention a name for the microscope (other techniques, if correct, are also okay).
- 5 credits: Explained the working principle.

(4) 7 pts - Using pulsed lasers you can cut steel and rapidly manufacture prototypes. Similarly, using such lasers you can also remove tissue. In the same context, briefly discuss how the removal of tattoo and LASIK operation are different from each other. In other words, can you use the same pulsed laser that is designed for tattoo removal for LASIK operation? Why / Why not?

No, the same pulsed laser that is designed for tattoo removal can not be used for LASIK operation.
Tattoo removal working principle:
Use pulsed laser, with high peak power (the so-called Q-switching or Q-spoiling mode) to penetrate skin and shatter the ink pigments into particles that can be cleared by body lymphatic system. Different color of tattoo ink is removed with different wavelength, typically around 1000 nm.
LASIK working principle:
Use the excimer laser to vaporize the tissue in a finely controlled manner without damaging the adjacent stroma. No burning with heat or actual cutting is required to ablate the tissue. The layers of tissue removed are tens of micrometers thick. Typical working wavelength is around 200 nm.

- 1 credit: Answer ‘No’.
- 3 credits: Explain the working principle of tattoo removal laser.
- 3 credits: Explain the working principle of LASIK.

(5) 15 pts – For a plane wave, which one of the following spatial frequency pairs defines an evanescent wave in air:

(i) $(f_x, f_y) = (1.5/\lambda, 0)$ ---- Evanescent Wave? Yes or No? Why
(ii) $(f_x, f_y) = (0.2/\lambda, 0.1/\lambda)$ ---- Evanescent Wave? Yes or No? Why
(iii) $(f_x, f_y) = (1/\lambda, 0.01/\lambda)$ ---- Evanescent Wave? Yes or No? Why

f_x, f_y refer to spatial frequencies along x and y, respectively, and λ is the wavelength of the light source. Assume that the refractive index of air is 1.

Evanescent wave is defined by having $f_x^2+f_y^2>(n/\lambda)^2$, with the assumption of n=1, it simplified to $f_x^2+f_y^2>(1/\lambda)^2$, hence:

(i) $(f_x, f_y) = (1.5/\lambda, 0)$
Yes, because: $f_x^2 + f_y^2 = 2.25/\lambda^2 > 1/\lambda^2$.

(ii) $(f_x, f_y) = (0.2/\lambda, 0.1/\lambda)$
No, because: $f_x^2 + f_y^2 = 0.05/\lambda^2 < 1/\lambda^2$.

(iii) $(f_x, f_y) = (1/\lambda, 0.01/\lambda)$
Yes, because: $f_x^2 + f_y^2 = 1.0001/\lambda^2 > 1/\lambda^2$.

- 1 credit: State 'Yes' or 'No' correctly.
- 4 credits: Reason is correct, it can be an equation, or a plot. And for (i), it is also okay to say that fx is already in the evanescent field, so it must be evanescent wave.

(6) (a) 5pts - How would you physically detect an evanescent wave?

Physically, the evanescent wave can be detected by placing the detector in the near-field. The reason is evanescent wave is decaying exponentially, it must be captured in the near-field before it decays too much.

- 4 credits: Mention the key word 'near-field'.
- 1 credit: Mention the key word 'decay', this is important, it is the reason (drawing a figure to demonstrate this is also fine).

(b) 5pts - Why would you bother to detect such evanescent waves? What is useful and desired about them? Briefly explain your answers.

Evanescent wave is defined by having $f_x^2+f_y^2>(n/\lambda)^2$, which represents the higher frequency component of the field. Being able to detect the evanescent wave will enable us to obtain a more complete frequency spectrum of the field with these higher frequency components. In the space domain, this means higher resolution.

- 5 credits: Mention the key word 'high frequency' or 'high resolution'.