EE 172 Spring 2006 Midterm

Answer all questions.

Problem #	
1	
2	
3	
4	
5	
Total	

UID: ______________________________

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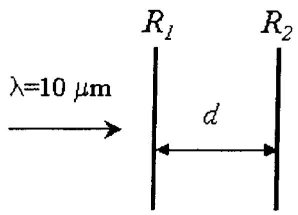
1. (24 pts, 3pts for each)The intensity transmission of a Fabry-Perot cavity is found to be

$$T(kd) = \frac{I_{trans}}{I_{inc}} = \left\{ \frac{(1-R_1)(1-R_2)}{(1-\sqrt{R_1R_2})^2 + 4\sqrt{R_1R_2}\sin^2(kd)} \right\}$$, where R_1, R_2 are the power reflectivities of the mirrors. Assume it is in free space so n=1.



(1) What is the condition of kd in order to have the maximum transmission?
(2) What is the definition of FWHM ($\Delta\nu_{1/2}$)?
(3) If the FSR of the cavity is 150MHz, what is the cavity length d?
(4) If R_1=0.85 and R_2=0.9, what is the photon lifetime τ_p?
(5) What is Q of the cavity?
(6) What is Finesse of the cavity?
(7) If the gain medium inside the cavity has a bandwidth of 1.2GHz. How many longitudinal modes can exist inside the cavity?
(8) If there are multiple longitudinal modes oscillating inside the cavity, propose a method to do longitudinal mode control such that only one longitudinal mode can oscillate inside the cavity.

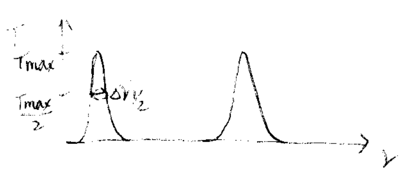
(1)

Round trip phase shift = $2\pi q$

$\Rightarrow kd = \pi q$ $\quad$ q is an integer #

(2)

$\Delta\nu_{1/2}$ is the full width at half maximum in the transmission spectrum #



3.

$FSR = \frac{c}{2d} \Rightarrow 150\times10^6 = \frac{3\times10^8}{2d} \Rightarrow d = 1\ (m)$ #

(4)

$\tau_p = \frac{\tau_{RT}}{1-S} = \frac{2d/c}{1-R_1R_2} = \frac{(150\times10^6)^{-1}}{1-0.85\times0.9} = 2.837\times10^{-8}\ (sec)$ #

(5)

$$Q = \tau_p \omega_0 = 2.837\times10^{-8} \times \frac{3\times10^{8}\times 2\pi}{10\times10^{-6}} = 5.348\times10^{6}$$

OR

$$Q = \frac{2\pi d}{\lambda_0} \times \frac{(R_1R_2)^{1/4}}{1-(R_1R_2)^{1/2}} = \frac{2\pi}{10^{-5}} \times \frac{(0.85\times0.9)^{1/4}}{1-(0.85\times0.9)^{1/2}} = 4.7\times10^{6}$$

(6)

$$\Delta\nu_{1/2} = \frac{\omega_0}{2\pi Q} = \frac{1}{2\pi\tau_p} = 5.61 \ (MHz)$$

$$F = \frac{150}{5.61} = 26.74 \ \#$$

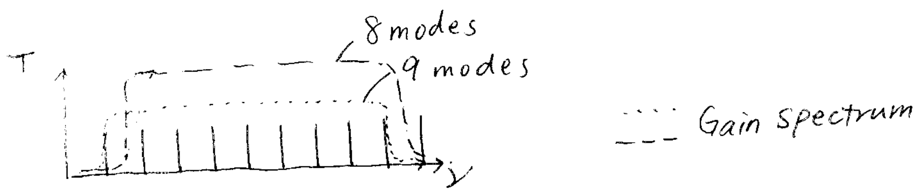
OR

$$\Delta\nu_{1/2} = \frac{c}{2d\pi}\,\frac{1-\sqrt{R_1R_2}}{(R_1R_2)^{1/4}} = \frac{3\times10^{8}}{2\pi} \times \frac{1-\sqrt{0.85\times0.9}}{(0.85\times0.9)^{1/4}} = 6.4 \ (MHz)$$

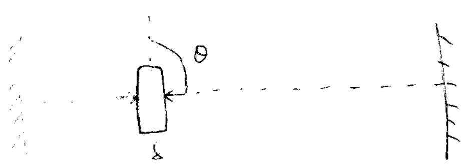
$$F = \frac{150}{6.4} = 23.47 \ \#$$

(7)

$$\text{Number of modes} = \frac{1.2\times10^{9}}{FSR} = \frac{1.2\times10^{9}}{150\times10^{6}} = 8 \text{ or } 9$$



(8)



Insert an intracavity etalon, then FSR now becomes

$$FSR = \frac{c}{2ns}$$

By varying the tilting angle of the etalon, we can control the FSR such that only one longitudinal mode is within the bandwidth of the gain medium. #

2. (18 pts)The spot size and the radius of curvature of a Gaussian beam are given by

$$w^2(z) = w_0^2(1+(\frac{z}{z_0})^2),\ R(z) = z(1+(\frac{z_0}{z})^2),\ \text{where } z_0 = \frac{\pi}{\lambda}w_0^2$$

(1) (4 pts)What are the beam characteristics when $z = 0$?
(2) (4 pts)What are the physical significances of Rayleigh length z_0?(List at least two)
(3) (3 pts)What is the spot size expression when $z >> z_0$?
(4) (4 pts)Derive the expansion angle from (3). Draw a diagram to show the definition of the angle.
(5) (3 pts)A certain Nd:YAG laser with wavelength 1μm has a smallest spot size $w_0 = 0.3$mm, what is the divergence angle of the laser beam?

(1) when $z=0$

a. Spot size is minimum and has the value $w(z=0) = w_0$

b. wave front is planar at $z=0$ $(R \to \infty)$ #

(2) At $z = z_0$,

a. $w(z) = \sqrt{2}\, w_0 \Rightarrow w^2(z) = 2w_0^2$ Cross section area of the beam is twice as that at $z=0$

b. Peak intensity $I(z_0) = \frac{1}{2} I(0)$

c. Radius of curvature of the wavefront $R(z_0) = 2z = 2z_0$ and is at its minimum $(R \to \infty$, maximum at $z=0$ and $z \to \infty)$

d. Longitudinal phase shift is $\frac{\pi}{4}$ w.r.t. $z=0$

(3)

$$w^2(z) = w_0^2\left[1+\left(\frac{z}{z_0}\right)^2\right]$$

Since $z >> z_0$

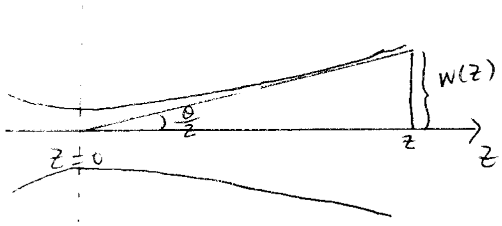
$$\Rightarrow \frac{z}{z_0} >> 1$$

so

$$w^2(z) \simeq w_0^2\left(\frac{z}{z_0}\right)^2 \Rightarrow w(z) \simeq \frac{w_0}{z_0} z = \frac{\lambda}{\pi w_0} z$$

#

4.



$$\tan\left(\frac{\theta}{2}\right) = \frac{W(z)}{z} = \frac{\lambda}{\pi W_0}$$

Use paraxial approximation $\Rightarrow \tan\left(\frac{\theta}{2}\right) \simeq \frac{\theta}{2}$

$$\theta \simeq \frac{2\lambda}{\pi W_0}$$ #

5.

$\lambda = 1\ \mu m$

$W_0 = 0.3\ mm$

$$\theta = \frac{2 \times 10^{-6}}{\pi \times 0.3 \times 10^{-3}} = 2.122 \times 10^{-3}\ (rad)$$

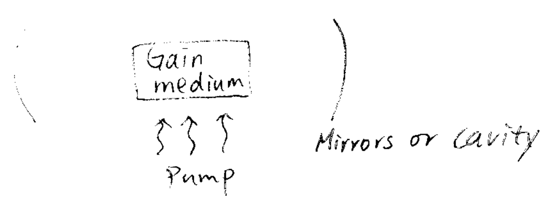
$$= 0.122^{\circ}\ (degree)$$

#

3. (22 pts) Explain briefly the following concepts:
   (1) (2 pts)What does "LASER" stand for?
   (2) (6 pts)What are the 3 basic components to build up a laser? Explain the purpose for each component briefly.
   (3) (3 pts)Describe spontaneous emission in an atomic system.
   (4) (4 pts)A photon can be generated by both spontaneous emission and stimulated emission. But what are the differences in the characteristics of these two processes(List at least two)?
   (5) (3 pts)Describe absorption process in an atomic system.
   (6) (2 pts)Line shape function(List at least one definition)
   (7) (2 pts)Explain why a laser beam diverges less than a normal flashlight.

(1) Light Amplification by Stimulated Emission of Radiation

(2)

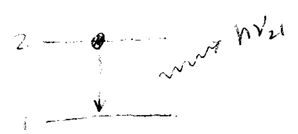


a. Gain medium: produce and amplify coherent radiation by stimulated emission

b. Pump: provide the necessary energy for gain medium to reach population inversion

c. Cavity: provide positive optical feedback

(3)



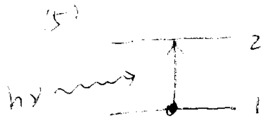
The atom at energy state 2 decays spontaneously to state 1 and emits a photon with frequency ν_{21} but random phase, polarization or direction.

(4)

Spontaneous emission: The process is spontaneous without external excitation. The photon emitted is random in phase, polarization and direction.

Stimulated emission: The process is induced by external radiation (or photons) and the photon generated has the same phase, polarization and direction as the
frequency,
original photon.

(5)



An atom originally at energy state 1 absorbs a photon and converts to state 2.

(6) Refer to text. (Sec 7.4)

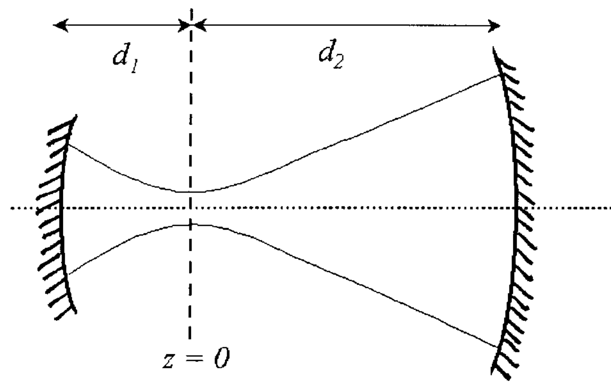
(7)

A laser beam diverges much less than a flashlight because the laser beam is a <u>Gaussian</u> beam and thus it exhibits <u>high spatial coherence</u> than flashlight.

4. (18 pts) A laser generates a field given by

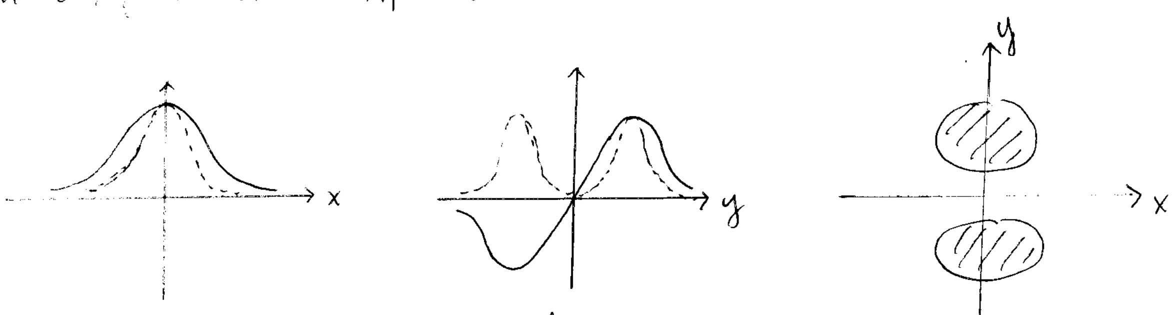
$$E_{m,p} = E\frac{yw_0}{w^2(z)}\exp[\frac{-(x^2+y^2)}{w^2(z)}]\exp[-j\frac{k(x^2+y^2)}{2R(z)}]\exp\{-j[kz-(1+m+p)\tan^{-1}(\frac{z}{z_0})]\}$$

(1) (4pts)Identify the mode (i.e., $TEM_{m,p}$; m=? p=?)
(2) (8pts)Plot the electric field and intensity distribution as a function of x and y. Then, plot the intensity pattern in the xy plane.
(3) (6pts)Find the explicit expression for the resonant frequency $\nu_{m,p,q}$ (in terms of d_1, d_2, and z_0) in the cavity shown below. Express you answer with the m and p obtained in (1).



(1) $m = 0$, $p = 1$ since $H_1(u) \propto u$

(2)



solid line : Electric field
dotted line : Intensity

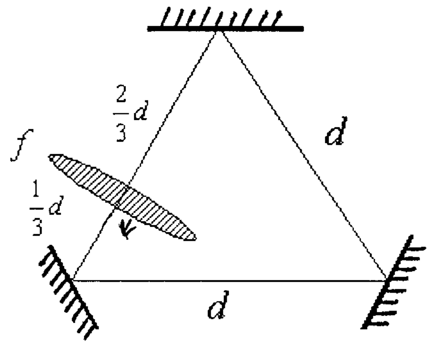
(3)

$$\left[kd_1 - (1+m+p)\tan^{-1}\left(\frac{d_1}{z_0}\right)\right] + \left[kd_2 - (1+m+p)\tan^{-1}\left(\frac{d_2}{z_0}\right)\right] = \pi q$$

$$\Rightarrow k(d_1+d_2) - (1+m+p)\left[\tan^{-1}\left(\frac{d_1}{z_0}\right) + \tan^{-1}\left(\frac{d_2}{z_0}\right)\right] = \pi q$$

$$\Rightarrow \nu_{m,p,q} = \frac{c}{2(d_1+d_2)}\left\{q + \frac{1}{\pi}\left[\tan^{-1}\left(\frac{d_1}{z_0}\right) + \tan^{-1}\left(\frac{d_2}{z_0}\right)\right]\right\}$$ #

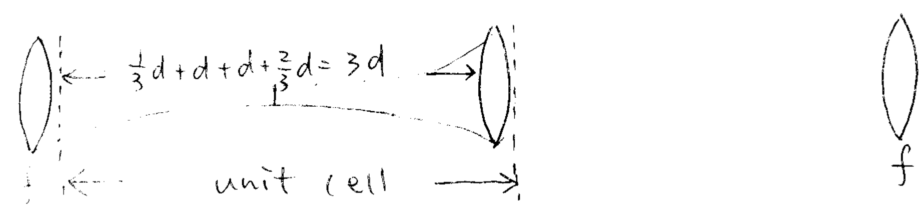
5. (18 pts)Consider the cavity shown below. Assume it is a stable cavity.



The ABCD matrix for distance z in free space is $\begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix}$. The ABCD matrix for a thin lens with focal length f is $\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$. q-parameter for a Gaussian beam is $\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi n w^2(z)}$. ABCD law for Gaussian beams is $\frac{1}{q_2} = \frac{C + D(\frac{1}{q_1})}{A + B(\frac{1}{q_1})}$

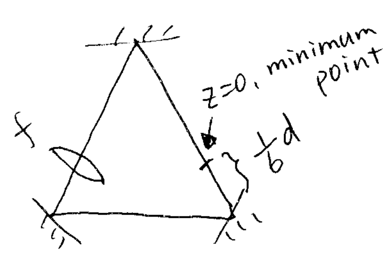
(1) (6 pts)Sketch an equivalent-lens waveguide of this cavity and label a unit cell.
(2) (4 pts)Identity the plane z = 0 inside the cavity where the spot size of the Gaussian beam is minimum.
(3) (8 pts)Show relevant proofs to support your answer in (2). (You may find the following info useful: the solution of the quadratic function $ax^2 + bx + c = 0$ is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

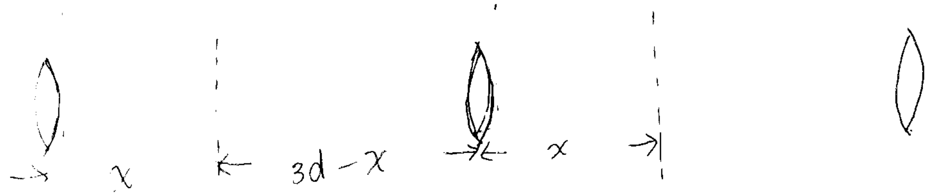
(1) Start right after lens in c.c.w. direction



(2) z = 0 plane is at the center of the unit cell, so in the cavity

$\frac{3d}{2} - \frac{1}{3}d - d = \frac{1}{6}d$





Suppose the minimum spot size plane is x away from lens (in c.c.w direction)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & 3d-x \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\frac{x}{f} & (1-\frac{x}{f})(3d-x)+x \\ \frac{-1}{f} & \frac{-1}{f}(3d-x)+1 \end{bmatrix}$$

since $\frac{1}{q_1} = \frac{1}{q_2} = s$

$$s = \frac{C+Ds}{A+Bs} \Rightarrow As + Bs^2 = C + Ds$$

$$\Rightarrow Bs^2 + (A-D)s - C = 0$$

$$\Rightarrow s = \frac{(D-A) \pm \sqrt{(D-A)^2 + 4BC}}{2B}$$

$$Re[s] = \frac{1}{R(z)} = \frac{1}{\infty} = 0 \Rightarrow \frac{D-A}{2B} = 0 \Rightarrow D = A$$

$$Im[s] = \frac{-\lambda}{\pi n w_0^2}$$

Therefore

$$1 - \frac{x}{f} = \frac{-1}{f}(3d-x) + 1$$

$$\Rightarrow x = 3d - x$$

$$\Rightarrow x = \frac{3d}{2} \quad \#$$