SOLUTIONS

UCLA Department of Electrical Engineering
EE170C – Photonic Sensors and Solar Cells
Spring 2016
Midterm, May 4 2016, (1:40 minutes)

Name ______________________________ Student number__________________________

This is a closed book exam – you are allowed 1 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Photodetection	50	
Problem 2	Photodiode concepts	50	
Total		100	

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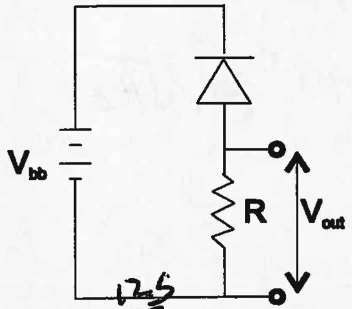
1. Photodetection (50 points)

Consider detection using a Silicon pin photodiode with area 1.06x1.06 mm^2 whose data sheet is shown in the back of the exam.



12.5

(a) (~~10~~ points) Given the specs on the data sheet, what is the photodetector quantum efficiency η at λ = 850 nm?

$\sigma = 0.6\ A/W = \frac{\eta e}{h\nu}$ $\qquad h\nu = 1.46 eV$

$$\boxed{\eta = 0.6 \times \frac{h\nu}{e} = 0.88}$$

12.5

(b) (~~10~~ points) Assume that this detector is viewing a 300K scene with a full 2π steradian field of view (i.e. half-space). What is the background photocurrent produced I_{bg}? Assume that the detector has a filter in front with a passband bandwidth of Δλ = 100 nm with a central wavelength of λ=850 nm.

$R(\nu) = \frac{2h\nu^3}{c^2} \frac{\cos\theta}{e^{h\nu/kT}-1}$ $\qquad R(\lambda) = \frac{2hc^2}{\lambda^5} \frac{\cos\theta}{e^{h\nu/kT}-1} \approx \frac{2hc^2}{\lambda^5} \cos\theta\, e^{-h\nu/kT}$

Total received power $A\Omega = A'\Omega'$ $\qquad h\nu/kT >> 1$ $\quad e^{-h\nu/kT} \approx 4\times10^{-25}$

$\iint_{2\pi\, ster.} R(\lambda)\, \Delta\lambda A = \frac{2hc^2}{\lambda^5} \pi e^{-h\nu/kT} \Delta\lambda A = 3.3\times10^{-10} \Delta\lambda A = 3.3\times10^{-23} W$

$$\boxed{I_{bg} \approx 2\times10^{-23} A}$$

Very small! Negligible

(c) (10 points) Based upon the dark current specs at 2V reverse voltage, and ignoring any contribution of Johnson noise from a load resistor, what is the NEP (in units W Hz$^{-1/2}$), and what is the detectivity D^* (units cm Hz$^{1/2}$ W^{-1})?

$I_d = 30\,pA$

$$1 = \frac{\sigma\, NEP}{\sqrt{2eI_dB}} \qquad \frac{NEP}{\sqrt{B}} = \frac{\sqrt{2eI_d\cancel{B}}}{\sigma}$$

$NEP = 5.2\times10^{-15}\, W/\sqrt{Hz}$

$$\boxed{D^* = \frac{\sqrt{AB}}{NEP} = 2\times10^{13}\, cm\, Hz^{1/2}/W}$$

(d) (~~10~~ 12.5 points) For R=50 Ω and specs provided, what is the RC-time constant limited bandwidth of the detector? How does that compare with the listed cutoff frequency? Given that, what is your best guess about the physical mechanism that is limiting high frequency performance? Explain qualitatively.

$f_{RC} = \frac{1}{2\pi RC} = 636\, MHz$ $\qquad C = 5pF$ @ $-2V$

This is much higher than $f_{cutoff} \sim 10\, MHz$ listed in specs.
RC is not the limit - it is more likely an ~~i~~ intrinsic limit such as diffusion limited response.

(e) (~~10~~ 13.5 points) Assume I am trying to measure the power of a 100mW laser operating at exactly λ=850 nm, using R=50Ω . What is the Signal-to-noise ratio for current assuming B=1 Hz? Is this the quantum-limited or thermal-limited detection regime?

$I_{ph} = 0.060\, A \gg I_d$ (neglect dark current)

$$SNR = \frac{I_{ph}}{\sqrt{2eI_{ph}B + \frac{4kTB}{R}}}$$

$$= \frac{I_{ph}}{\sqrt{2eB}\sqrt{I_{ph} + \frac{2kT}{eR}}}$$

$$\boxed{SNR = 4.33\times10^{8}}$$

4

$\frac{2kT}{eR} = 0.001\, A$

$\underline{0.06 > 0.001 A}$

Quantum limited

2. **(50 points) Photodiode concepts**

(a) (12.5 points) Describe two advantages of a *pin* photodiode compared to a *pn* photodiode.

A pin Photodiode has a depletion region with approximately constant E-field. This allows the E-field to be set at the optimum value to minimize transit time + increase speed.

A pin photodiode can choose the width of the depletion region by choosing the "i" layer thickness, independent of doping of "p" + "n" regions. This allows heavily doped p, n regions for low parasitic series resistance.

(b) (12.5 points) Why are III-V photodiodes typically faster (larger cutoff frequency) than silicon based structures? Give two reasons.

III-Vs generally have larger mobility + larger V_{sat} compared to silicon. This leads to shorter transit times.

III-V have larger absorption coeff ($\alpha \gtrsim 10^4 \text{ cm}^{-1}$) compared to silicon ($\alpha \sim 100 - 10^3 \text{ cm}^{-1}$). This allows one to use thin depletion region absorbers with high quantum eff and short transit times.

(c) (12.5 points) Explain qualitatively/physically: Why does the high frequency response of an APD depend upon the ratio of ionization coefficients for electrons and holes: α/β ?

When $\alpha \gg \beta$ or $\beta \gg \alpha$, impact ionization will only occur for a single carrier type (electron or hole). This leads to single pass amplification, and the transit time is approximatly the same as for a conventional p-n.

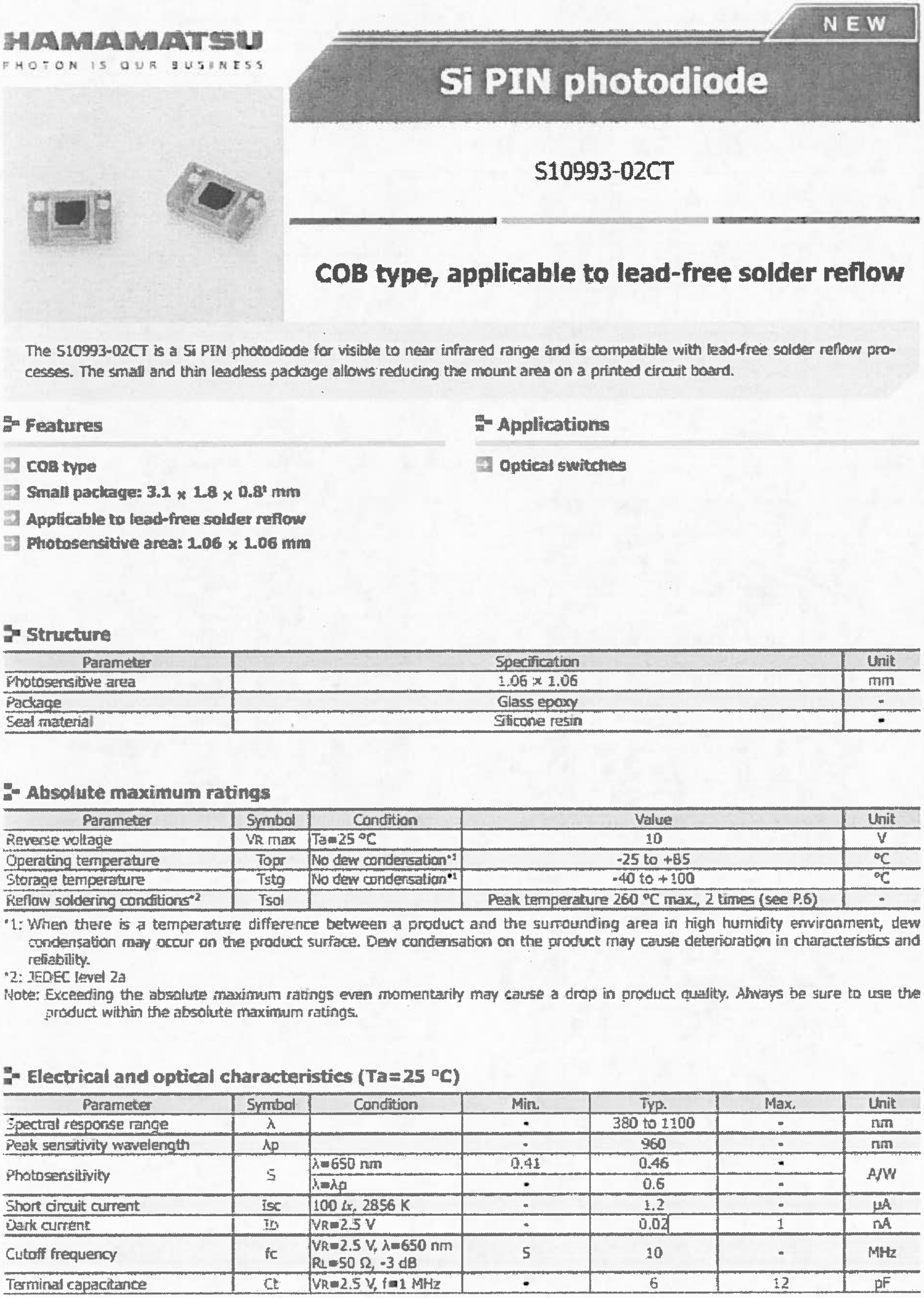
When $\alpha \approx \beta$, then impact ionization is likely for both electrons & holes, so you get a positive feedback, & multipass amplification. The effective transit time becomes much larger. ~~as carriers~~

(d) (12.5 points) Why do photoconductive detectors have a different fundamental BLIP detectivity limit than photodiode detectors?

Photo conductive detectors experience noise associated with both generation + recombination events, which are random & uncorrelated.

Photodiode detectors generally don't exhibit recombination noise since the depletion region is shorter + the velocity is higher.

Photoconductors have 2x the shot noise ~~a~~ power density as photodiodes, hence their D^*_{BLIP} limit is $\sqrt{2}$ factor smaller.



HAMAMATSU

PHOTON IS OUR BUSINESS

NEW

Si PIN photodiode

S10993-02CT

COB type, applicable to lead-free solder reflow

The S10993-02CT is a Si PIN photodiode for visible to near infrared range and is compatible with lead-free solder reflow processes. The small and thin leadless package allows reducing the mount area on a printed circuit board.

Features

- COB type
- Small package: 3.1 × 1.8 × 0.8^t mm
- Applicable to lead-free solder reflow
- Photosensitive area: 1.06 × 1.06 mm

Applications

- Optical switches

Structure

Parameter	Specification	Unit
Photosensitive area	1.06 × 1.06	mm
Package	Glass epoxy	-
Seal material	Silicone resin	-

Absolute maximum ratings

Parameter	Symbol	Condition	Value	Unit
Reverse voltage	V_R max	Ta=25 °C	10	V
Operating temperature	Topr	No dew condensation[1]	-25 to +85	°C
Storage temperature	Tstg	No dew condensation[1]	-40 to +100	°C
Reflow soldering conditions[2]	Tsol		Peak temperature 260 °C max., 2 times (see P.6)	-

[1] When there is a temperature difference between a product and the surrounding area in high humidity environment, dew condensation may occur on the product surface. Dew condensation on the product may cause deterioration in characteristics and reliability.

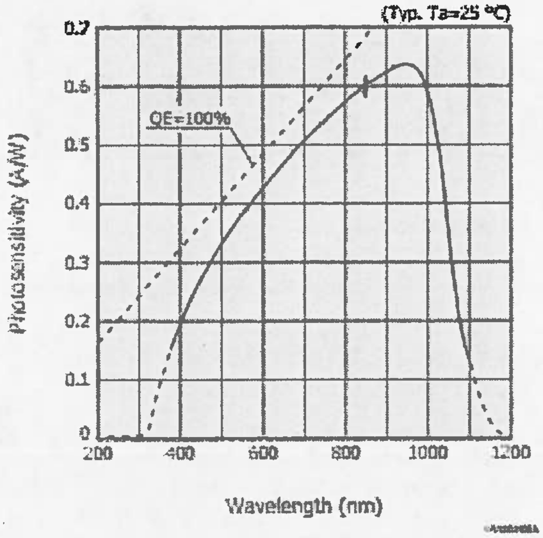
[2] JEDEC level 2a

Note: Exceeding the absolute maximum ratings even momentarily may cause a drop in product quality. Always be sure to use the product within the absolute maximum ratings.

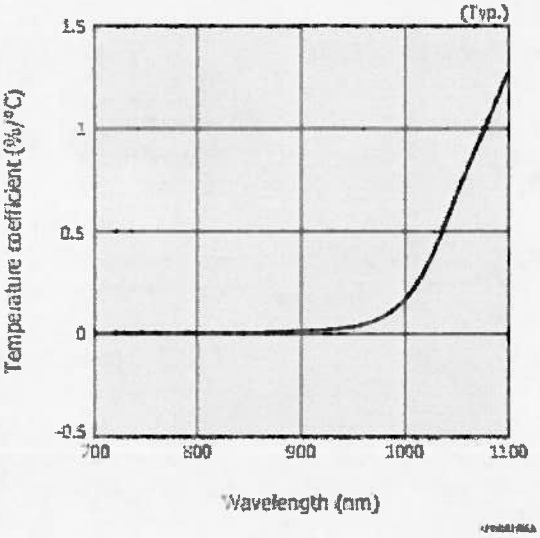
Electrical and optical characteristics (Ta=25 °C)

Parameter	Symbol	Condition	Min.	Typ.	Max.	Unit
Spectral response range	λ		-	380 to 1100	-	nm
Peak sensitivity wavelength	λp		-	960	-	nm
Photosensitivity	S	λ=650 nm	0.41	0.46	-	A/W
		$\lambda=\lambda p$	-	0.6	-	
Short circuit current	Isc	100 lx, 2856 K	-	1.2	-	μA
Dark current	I_D	V_R=2.5 V	-	0.02	1	nA
Cutoff frequency	fc	V_R=2.5 V, λ=650 nm R_L=50 Ω, -3 dB	5	10	-	MHz
Terminal capacitance	Ct	V_R=2.5 V, f=1 MHz	-	6	12	pF

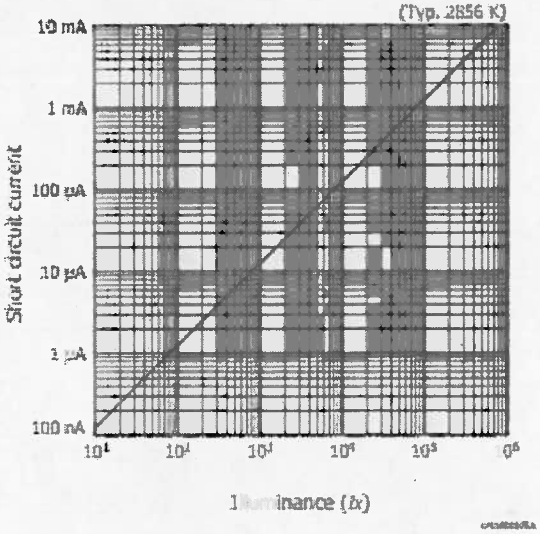
Spectral response



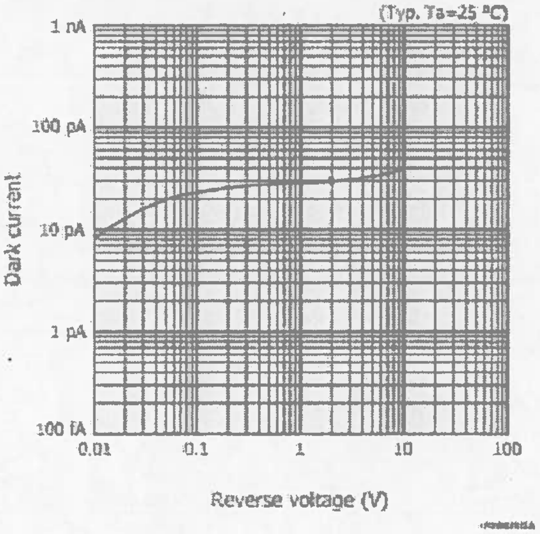
Photosensitivity temperature characteristics



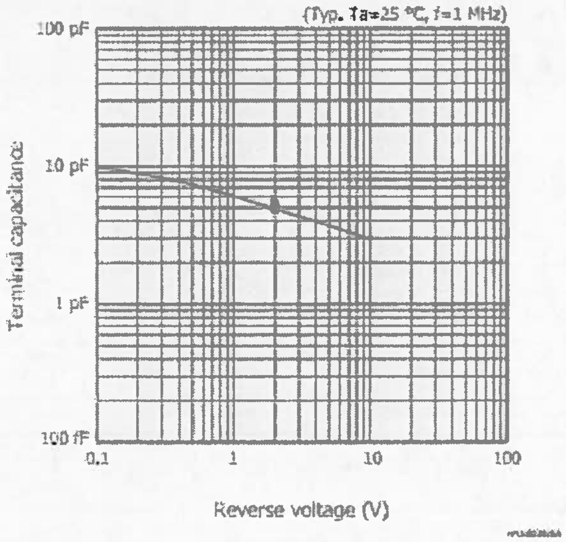
Linearity



Dark current vs. reverse voltage



Terminal capacitance vs. reverse voltage



Blackbody radiation energy density in cavity: $\rho(\nu)=\frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/k_BT}-1}$ (J Hz^{-1} m^{-3})

Spectral Radiance from Blackbody surface:

$R(\nu)=\frac{2h\nu^3}{c^2}\frac{\cos\theta}{e^{h\nu/k_BT}-1}$ (W ster^{-1} Hz^{-1} m^{-2}) $R(\lambda)=\frac{2hc^2}{\lambda^5}\frac{\cos\theta}{e^{hc/\lambda k_BT}-1}$ (W ster^{-1} m^{-1} m^{-2})

Radiant Emittance from Blackbody surface: $E=\frac{\pi^2(k_BT)^4}{60c^2\hbar^3}=\sigma_BT^4$ (W m^{-2})

Semiconductors

Fermi-Dirac distribution for electrons: $f(E)=\frac{1}{e^{(E-E_F)/kT}+1}$, for holes: $f(E)=1-\frac{1}{e^{(E-E_F)/kT}+1}$,

Equilibrium Carrier concentrations $n_0=\int_{E_C}^{\infty}f(E)N(E)dE$

Equilibrium Carrier concentrations in non-degenerate limit (E_C-$E_F \gg kT$ and E_F-$E_V \gg kT$).

$$n_0=N_Ce^{-(E_C-E_F)/kT}\quad,\ p_0=N_Ve^{-(E_F-E_V)/kT}$$

$$N_C=2\left(\frac{2\pi m_n^*kT}{h^2}\right)^{3/2}\quad,\ N_V=2\left(\frac{2\pi m_p^*kT}{h^2}\right)^{3/2}$$

$$n_i=\sqrt{n_0p_0}=\sqrt{N_CN_V}e^{-E_g/2kT}$$

Intrinsic Fermi Level $E_i=\frac{kT}{2}\ln\left(\frac{N_V}{N_C}\right)+\frac{E_V+E_C}{2}$

Conductivity of semiconductor: $\sigma=ne\mu_n+pe\mu_p$

Semiconductor electron/hole mobility $\mu_n=\frac{e\overline{t_n}}{m_n^*}$ $\mu_p=\frac{e\overline{t_p}}{m_p^*}$

Einstein relation for diffusion coeff: $D=\frac{kT}{e}\mu$ Diffusion length $L=\sqrt{D\tau}$

Ideal p-n diode (assumes abrupt junction)

Contact potential $V_0=\frac{kT}{e}\ln\left(\frac{N_AN_D}{n_i^2}\right)$,

Depletion (transition) region width $W=\sqrt{\frac{2\epsilon(V_0-V)}{e}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)}$, $x_{n0}=W\frac{N_A}{N_A+N_D}$

Maximum Electric field within depletion region $\mathcal{E}_{\max}=\sqrt{\frac{2e(V_0-V)}{\epsilon}\left(\frac{N_AN_D}{N_A+N_D}\right)}$

Ideal diode I-V: $I=eA\left(\frac{D_p}{L_p}p_n+\frac{D_n}{L_n}n_p\right)\left(e^{eV/k_BT}-1\right)=I_0\left(e^{eV/k_BT}-1\right)$

Depletion (junction) Capacitance $C_j=\epsilon A\sqrt{\frac{e}{2\epsilon(V_0-V)}\frac{N_AN_D}{N_A+N_D}}=\frac{\epsilon A}{W}$

Transit time (average): $$\tau_d \approx \frac{w}{2}\left(\frac{1}{v_e}+\frac{1}{v_h}\right)$$

Noise and photodetection

Fourier Transform:

$$V(\omega)=\int_{-\infty}^{\infty} v(t)e^{-j\omega t}dt$$

$$v(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} V(\omega)e^{j\omega t}d\omega$$

Single sided spectral density over interval T: $$S_T(\nu)=\frac{2}{T}\left|I_T(\omega)\right|^2$$

Johnson noise $\langle i_J^2\rangle = 4k_BTB/R$ Shot noise $\langle i_s^2\rangle = 2eIB$

Shot noise with avalanche gain M $\langle i_s^2\rangle = 2eIBM^2F$

Signal to noise ratio: $$SNR_P=\frac{I_{ph}{}^2}{\langle i_n^2\rangle} \qquad SNR_I=\frac{I_{ph}}{\sqrt{\langle i_n^2\rangle}}$$

Detectivity $$D^*=\frac{\sqrt{AB}}{NEP}$$

Constants (SI units):

$\varepsilon_0=8.85\times10^{-12}$ F/m (or $C^2 N^{-1} m^{-2}$)

$\mu_0=4\pi\times10^{-7}$ H/m (or $N A^{-2}$)

$e = 1.6\times10^{-19}$ C

$k_B = 1.38e^{-23}$ J/K

$h = 6.626\times10^{-34}$ J s

$\hbar = 1.055\times10^{-34}$ J s

1 eV=1.6e-19 J

Material properties of Silicon and GaAs

All parameters at room temp	Silicon	GaAs
Crystal Structure	Diamond	Zincblende
a *(lattice constant)*	5.43 Å	5.65 Å
Mass density	2.33 g/cm^3	5.31 g/cm^3
Relative permittivity ϵ_r	11.8	13.2
Refractive index *n*		
E_g	1.11 eV	1.43 eV
μ_e (for intrinsic/low doping)	1350 cm^2/V s	8500 cm^2/V s
μ_h (for intrinsic/low doping)	480 cm^2/V s	400 cm^2/V s
m^*_e	$0.26m_0$	$0.067m_0$
m^*_h	$0.49m_0$	$0.5m_0$
Effective DOS N_c	2.8×10^{19} cm^{-3}	4.7×10^{17} cm^{-3}
Effective DOS N_v	1.0×10^{19} cm^{-3}	7.0×10^{18} cm^{-3}
n_i	1.5×10^{10} cm^{-3}	2×10^{6} cm^{-3}
D_n (for intrinsic/ low doping)	35 cm^2 s^{-1}	220 cm^2 s^{-1}
D_p (for intrinsic/ low doping)	12.5 cm^2 s^{-1}	10 cm^2 s^{-1}

Useful equations

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