SOLUTIONS

UCLA Department of Electrical Engineering
EE170C – Photonic Sensors and Solar Cells
Spring 2015
Midterm, April 29 2015, (1:40 minutes)

Name ______________________________ Student number_________________________

This is a closed book exam – you are allowed 1 page of notes (front+back).

Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in.

Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focusing on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can using symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Topic	Max Points	Your points
Problem 1	Photodetection and APD	30	
Problem 2	Detection limits	40	
Problem 3	Photodiodes	30	
Total		100	

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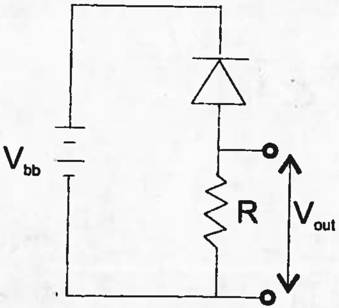
1. Detection (30 points)

Consider a Si photodiode with the following characteristics: area= 0.2 mm^2, quantum efficiency η=0.7 (measured at a wavelength of λ=900 nm), and dark current I_d=1 pA The stray capacitance is 1 pF..



(a) (10 points) Find the minimum detectable power at 900 nm assuming that your detection bandwidth is B=1Hz.

Find NEP assuming R doesn't contribute Johnson noise (quantum limited regime).

$$1 = \frac{\sigma\, NEP}{\sqrt{2eI_dB}} \qquad NEP = \frac{\sqrt{2eI_dB}}{\sigma}$$

For λ=900nm $\frac{h\nu}{e} = 1.38\,V$

$$\sigma = \frac{\eta e}{h\nu} = 0.50\ A/W$$

$$\boxed{NEP = 1.13\times10^{-15}\,W}$$

(b) (10 points) What value should you choose for the load resistance *R* so that you are in the quantum limited detection regime?

Choose R such that

$$\frac{4k_BTB}{R} \ll 2eI_dB$$

$$R \gg \frac{4kT}{2eI_d} = \frac{2 \times 26\,\text{mV}}{10^{-12}\,\text{A}}$$

$$\boxed{R \gg 5.2 \times 10^{10}\ \Omega}$$

Error in wording of question:

Should have asked for dark current limited detection regime. Quantum limited detection implies dark current is negligible. Full credit for alternate answer:

$$i_n^2 = 2eB(I_{ph} + I_d) + \frac{4kTB}{R}$$

(c) (10 points) Now consider that this photodiode is replaced by an APD with the same characteristics, except with gain M=100 and noise factor F=1.3. (You may assume that the dark current is also multiplied by the same factor). How do your answers to (a) and (b) change? Given your answer, what is the primary advantage of using an APD for detection of low –level light.

How does a) change? $I_d \to$ ~~[illegible]~~ MI_d

$$1 = \frac{\sigma M \, NEP}{\sqrt{2e I_d M^2 B F}}$$

$$NEP = \frac{\sqrt{2e I_d F B}}{\sigma}$$

$$= 1.28 \times 10^{-15} W$$

The M factors cancel, so NEP is only slightly increased by a factor of $\sqrt{F}$

b)? $\frac{4kTB}{R} \ll 2e I_d M^2 B F$

$$R \gg \frac{2kT}{M^2 e I_d F}$$

$$\boxed{R \gg 4.0 \times 10^6 \, \Omega}$$

The amplification drastically reduces the value ~~[illegible]~~ of R needed to reach ~~[illegible]~~ ~~limited~~ detection limited by dark current (factor of $1/M^2$).

This will ~~reduce~~ increase detection bandwidth by factor of M^2 from $\ll$ 3Hz to $\ll$ 40 kHz

2. (40 points) Detection limits

(a) (15 points) In class, we usually assumed that the minimum detectable power (i.e. NEP) was either limited by background signal current or dark current. Consider the case if the detector was cooled and such that I_d is negligible, and there is no significant background current. Derive an expression for the noise equivalent power NEP in this case.

$$1 = \cancel{\frac{\sigma NEP}{\sqrt{2eI_p}}} \quad \frac{I_{ph}}{\sqrt{2eI_{ph}B}} = \frac{\sqrt{I_{ph}}}{\sqrt{2eB}} = \frac{\sqrt{\sigma NEP}}{\sqrt{2eB}} = \frac{\eta e}{h\nu}\frac{NEP}{2eB}$$

$$\boxed{NEP = \frac{2B h\nu}{\eta}}$$

(b) (10 points) Consider a photodetector for which $\eta=1$, and for light with $h\nu=1\text{eV}$. What is the value of NEP in this case (assume $B=1$ Hz). How small must I_d be to reach this limit?

$$NEP = 2\times \frac{1.6\times10^{-19}J}{s} = 3.2\times10^{-19}\,W$$

This produces photo current of $I_{ph} = \frac{e\,2h\nu B}{h\nu}$

$$\boxed{I_{ph} = 2eB = 3.2\times10^{-19}A}$$

I_d must be made smaller than this number - very small!

(c) (15 points) Explain what the BLIP Detectivity figure of merit is. Is BLIP detectivity more important for photodetectors at λ=1 μm or λ=10 μm? Why?

Background Limited Infrared Performance : This is the
(Intrinsic)
value of NEP or D^* produced by the background ~~sig~~ photocurrent (& its shot noise) when observing a back scene & its thermal radiation. It is more important at 10 um because the blackbody radiation is much stronger (at 300K) at 10 um than 1 um.

3. (30 points) Photodiodes
   (a) Why are III-V photodiodes typically faster (larger cutoff frequency) than silicon based structures? Give two reasons.

→ Larger absorption coeff α allows thinner absorption regions & shorter transit times.

→ III-V have larger mobility & saturation velocities.

   (b) pin photodiodes often have a dark current value that is less dependent on bias voltage than p-n photodiodes. Explain why this might be the case.

The Photodiode dark current is primarily determined by the thermal generation $I_d = e g_i A (L_p + L_n + W)$ where W is the depletion region width.

For a p-n diode $W \propto \sqrt{|V|}$ in reverse bias, which leads to an increase in dark current $I_d \propto \sqrt{|V|}$ with reverse bias.

For a pin, most of the voltage drop is across the "i" layer, which is independent of bias. This leads to a dependence of W that is slower than $\sqrt{|V|}$.

(c) Explain the tradeoff between high speed operation and quantum efficiency in a conventional photodiode.

The ultimate limit for high speed is the transit time limit: $f_{\tau d} \sim \frac{1}{2\pi \tau_d}$ where $\tau_d = \frac{W}{V_{avg}}$

where W is the depletion thickness. However, in III-V's nearly all of the absorption takes place in the depletion region. If you decrease W, you also decrease the quantum efficiency as the length available for absorption decreases.

$$\eta = 1 - e^{-\alpha W} \approx \alpha W. \quad (\alpha W << 1$$

This leads to an approximate constant bandwidth/eff product for surface incident photodiodes.

$$f_{\tau d}\,\eta \approx \frac{V_{avg}}{2\pi W}\alpha W \approx \frac{V_{avg}\alpha}{2\pi}.$$

Blackbody radiation energy density in cavity: $\rho(\nu)=\dfrac{8\pi h\nu^3}{c^3}\dfrac{1}{e^{h\nu/k_BT}-1}$ (J Hz^{-1} m^{-3})

Spectral Radiance from Blackbody surface:

$R(\nu)=\dfrac{2h\nu^3}{c^2}\dfrac{\cos\theta}{e^{h\nu/k_BT}-1}$ (W ster^{-1} Hz^{-1} m^{-2}) $R(\lambda)=\dfrac{2hc^2}{\lambda^5}\dfrac{\cos\theta}{e^{hc/\lambda k_BT}-1}$ (W ster^{-1} m^{-1} m^{-2})

Radiant Emittance from Blackbody surface: $E=\dfrac{\pi^2(k_BT)^4}{60c^2\hbar^3}=\sigma_BT^4$ (W m^{-2})

Semiconductors

Fermi-Dirac distribution for electrons: $f(E)=\dfrac{1}{e^{(E-E_F)/kT}+1}$, for holes: $f(E)=1-\dfrac{1}{e^{(E-E_F)/kT}+1}$,

Equilibrium Carrier concentrations $n_0=\displaystyle\int_{E_C}^{\infty}f(E)N(E)dE$

Equilibrium Carrier concentrations in non-degenerate limit (E_C-$E_F \gg kT$ and E_F-$E_V \gg kT$).

$$n_0=N_Ce^{-(E_C-E_F)/kT}\quad,\ p_0=N_Ve^{-(E_F-E_V)/kT}$$

$$N_C=2\left(\frac{2\pi m_n^*kT}{h^2}\right)^{3/2}\quad,\ N_V=2\left(\frac{2\pi m_p^*kT}{h^2}\right)^{3/2}$$

$$n_i=\sqrt{n_0p_0}=\sqrt{N_CN_V}e^{-E_g/2kT}$$

Intrinsic Fermi Level $E_i=\dfrac{kT}{2}\ln\left(\dfrac{N_V}{N_C}\right)+\dfrac{E_V+E_C}{2}$

Conductivity of semiconductor: $\sigma=ne\mu_n+pe\mu_p$

Semiconductor electron/hole mobility $\mu_n=\dfrac{e\bar{t}_n}{m_n^*}$ $\mu_p=\dfrac{e\bar{t}_p}{m_p^*}$

Einstein relation for diffusion coeff: $D=\dfrac{kT}{e}\mu$ Diffusion length $L=\sqrt{D\tau}$

Ideal p-n diode (assumes abrupt junction)

Contact potential $V_0=\dfrac{kT}{e}\ln\left(\dfrac{N_AN_D}{n_i^2}\right)$,

Depletion (transition) region width $W=\sqrt{\dfrac{2\epsilon(V_0-V)}{e}\left(\dfrac{1}{N_A}+\dfrac{1}{N_D}\right)}$, $x_{n0}=W\dfrac{N_A}{N_A+N_D}$

Maximum Electric field within depletion region $\mathcal{E}_{\max}=\sqrt{\dfrac{2e(V_0-V)}{\epsilon}\left(\dfrac{N_AN_D}{N_A+N_D}\right)}$

Ideal diode I-V: $I=eA\left(\dfrac{D_p}{L_p}p_n+\dfrac{D_n}{L_n}n_p\right)\left(e^{eV/k_BT}-1\right)=I_0\left(e^{eV/k_BT}-1\right)$

Depletion (junction) Capacitance $C_j=\epsilon A\sqrt{\dfrac{e}{2\epsilon(V_0-V)}\dfrac{N_AN_D}{N_A+N_D}}=\dfrac{\epsilon A}{W}$

Transit time (average): $\tau_d \approx \frac{w}{2}\left(\frac{1}{v_e}+\frac{1}{v_h}\right)$

Noise and photodetection

Fourier Transform:

$$V(\omega)=\int_{-\infty}^{\infty} v(t)e^{-j\omega t}dt$$

$$v(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} V(\omega)e^{j\omega t}d\omega$$

Single sided spectral density over interval *T*: $S_T(\nu)=\frac{2}{T}\left|I_T(\omega)\right|^2$

Johnson noise $\left\langle i_J^2\right\rangle = 4k_BTB/R$ Shot noise $\left\langle i_s^2\right\rangle = 2eIB$

Shot noise with avalanche gain M $\left\langle i_s^2\right\rangle = 2eIBM^2F$

Signal to noise ratio: $SNR_P=\frac{I_{ph}^{\;2}}{\left\langle i_n^2\right\rangle}$ $SNR_I=\frac{I_{ph}}{\sqrt{\left\langle i_n^2\right\rangle}}$

Detectivity $D^*=\frac{\sqrt{AB}}{NEP}$

Constants (SI units):

ε_0=8.85x10^{-12} F/m (or C^2 N^{-1} m^{-2}) μ_0=4π x10^{-7} H/m (or N A^{-2})

e = 1.6x10^{-19} C k_B = 1.38e^{-23} J/K

h = 6.626x10^{-34} J s $\hbar$ = 1.055x10^{-34} J s

1 eV=1.6e-19 J

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