EE170A Principles of Photonics
Electrical Engineering, UCLA

Fall 2014 Midterm Exam

In-class 1 hr 50 min, Closed Book   Nov. 13, 2014   Prof. J.M. Liu

1. (10%) At a given optical frequency, the optical susceptibility tensors of several materials are measured with respect to an arbitrary set of orthogonal coordinates in space, as listed below. Identify each material whether it is (1) a dielectric or a magnetic material and (2) an optically lossless or lossy material.

$$A:\boldsymbol{\chi}=\epsilon_0\begin{bmatrix}2.3 & 0.1+i0.2 & 0\\ 0.1+i0.2 & 2.7 & i0.2\\ 0 & i0.2 & 2.4\end{bmatrix},\ B:\boldsymbol{\chi}=\begin{bmatrix}2.0+i0.1 & -i0.3 & 0\\ i0.3 & 1+i0.2 & 0\\ 0 & 0 & 1.5\end{bmatrix},$$

$$C:\boldsymbol{\epsilon}=\epsilon_0\begin{bmatrix}1.59 & 0.13 & -0.16\\ 0.13 & 1.59 & 0.11\\ -0.16 & 0.11 & 1.41\end{bmatrix},\ D:\boldsymbol{\chi}=\epsilon_0\begin{bmatrix}1.9 & 0.2 & 0.3\\ 0.2 & 2.8 & 0.1\\ 0.3 & 0.1 & 2.5+i0.2\end{bmatrix},\ E:\boldsymbol{\chi}=\epsilon_0\begin{bmatrix}1.30 & -i0.35 & 0\\ i0.35 & 1.25 & 0.15\\ 0 & 0.15 & 1.40\end{bmatrix}.$$

2. (10%) A crystal has the following permittivity tensor at $\lambda = 0.50$ μm,

$$\boldsymbol{\epsilon}=\epsilon_0\begin{bmatrix}5.481 & 0 & 0\\ 0 & 5.267 & 0.214\\ 0 & 0.214 & 5.267\end{bmatrix}.$$

(a) Find the principal *indices* of the crystal at this wavelength.
(b) Is the crystal birefringent or nonbirefringent? If it is birefringent, is it uniaxial or biaxial? If it is used to make a quarter-wave plate, what is the minimum thickness of the plate?

3. (12%) A symmetric step-index planar InGaAsP/InP waveguide has a core index of $n_1 = 3.438$ and a cladding index of $n_2 = 3.205$ at the optical wavelength $\lambda = 1.30$ μm.
(a) If the core thickness is chosen to be $d = 1$ μm, how many guided modes are supported by the waveguide? What are they?
(b) If a single-mode waveguide is desired, what is the required core thickness? Is the waveguide truly single-mode if this requirement is met? Name the mode or modes.

4. (6%) Sketch the real and imaginary parts of $\chi(\omega)$ for an atomic system in the normal state of thermal equilibrium as a function of ω near a resonance frequency ω_0 with a relaxation constant γ. Where is anomalous dispersion found in this situation?

5. (4%) The refractive index of diamond at $\lambda = 1.0$ μm is $n = 2.39$. At a specific incident angle, an optical wave at $\lambda = 1.0$ μm reflected from a diamond surface is always linearly polarized no matter what the polarization of the incident wave is. For a diamond surface exposed to the air, what is this specific incident angle that ensures the reflected wave to be linearly polarized? What is the polarization of this reflected wave?

6. (8%) In designing a waveguide coupler of any geometry, what are the three major parameters that have to be considered in order to have a good efficiency? In what order of priority do they have to be considered?

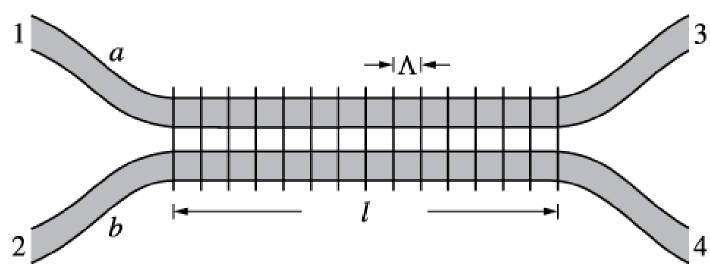
7. (4%) Consider a 3-dB codirectional coupler and a 3-dB contradirectional coupler. Both have perfect phase matching and have the same coupling coefficient κ. What is the length $l_{3\text{dB}}$ of each coupler in terms of $|\kappa|$?

8. (6%) Give an example for the type of waveguide that has each of the following characteristics:
(a) Supports only TEM modes.
(b) Supports only TE and TM modes and always supports both.
(c) Supports only a TE mode, but not any other type of mode.

9. (4%) At room temperature, the bandgap energy of Ge is 0.66 eV. It absorbs photons of energies above its bandgap and transmit diamond transmits those below its bandgap. What is the cutoff wavelength for light to be transmitted through a thick piece of pure Ge?

10. (10%) A fundamental Gaussian laser beam of power $P = 10$ W at a wavelength of $\lambda = 600$ nm is focused to a small spot size for a peak intensity of $I_0 = 2.5$ MW cm^{-2} at its beam waist. What is the beam-waist radius w_0 of the beam. What is the divergence angle of the beam? What are its spot size and peak intensity at a distance of 5 m from the beam waist? If the the spot size is increased to $w_0 = 50$ μm at the beam waist, what are the changes in the peak intensities at the beam waist and at 5 m from the waist, respectively?

11. (20%) A fiber-optic frequency filter is made of two single-mode fibers of different mode propagation constants. They are placed in close contact for a length *l* as shown below. At 1.55 μm optical wavelength, the effective indices for the two fiber modes are $\beta_a = 5.959$ μm^{-1} and $\beta_b = 5.849$ μm^{-1}, respectively, and the coupling coefficient between the two fiber modes is $\kappa = \kappa_{ab} \approx \kappa_{ba} = 2\times10^{-3}$ μm^{-1}. A grating of period Λ is built into the fibers in the coupling section. The input port of the device is port 1. The device is to function as an optical filter for separating the 1.55 μm wavelength from other wavelengths.



(a) If the device is to direct all of the optical power at 1.55 μm wavelength to port 4 and to dump all other wavelengths to port 3, what is the maximum possible coupling efficiency without the grating?
(b) With a first-order grating, what are the values of Λ and l that should be selected to obtain the best efficiency? What is the maximum efficiency if the parameters of the grating are properly chosen?
(c) If the device is to direct the power at 1.55 μm wavelength to port 2, what is the maximum possible coupling efficiency without the grating?
(d) With a first-order grating, what should the grating period Λ be chosen for the best efficiency? In this case, if the length l of the coupler remains the same as that found in (b), what is the efficiency of directing the 1.55 μm light from port 1 to port 2?

12. (6%) Name three important types of interferometers and sketch their basic structures.

Formulas:

$V = \frac{2\pi}{\lambda} d \sqrt{n_1^2 - n_2^2}$

$l_{\lambda/4} = \frac{\lambda}{4\left|n_y - n_z\right|}$

$l_{\lambda/2} = \frac{\lambda}{2\left|n_y - n_z\right|}$

$V < \pi$

$M = \left[\frac{V}{\pi}\right]_{\text{int}}$

$\tan^{-1}\frac{n_2}{n_1}$

$\sin^{-1}\frac{n_2}{n_1}$

$\frac{1239.8}{\lambda}$

$\frac{1}{n_{\text{e}}^2(\theta)} = \frac{\cos^2\theta}{n_{\text{o}}^2} + \frac{\sin^2\theta}{n_{\text{e}}^2}$

$\left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2$

$P = \frac{\pi w^2(z)}{2} I_0(z) = \frac{\pi w_0^2}{2} I_0$

$\Delta\theta = \frac{2\lambda}{\pi n w_0}$

$l_{\text{c}}^{\text{PM}} = \frac{\pi}{2|\kappa|}$

$\eta = \frac{1}{1 + \delta^2 / |\kappa|^2} \sin^2\left(|\kappa| l \sqrt{1 + \delta^2 / |\kappa|^2}\right)$

$\eta = \frac{\sinh^2\left(|\kappa| l \sqrt{1 - \delta^2 / |\kappa|^2}\right)}{\cosh^2\left(|\kappa| l \sqrt{1 - \delta^2 / |\kappa|^2}\right) - \delta^2 / |\kappa|^2}$

$\kappa = \frac{h^2}{\beta d_{\text{E}}} \frac{d_{\text{g}}}{4} \left(\delta_{q,1} + \delta_{q,-1}\right)$

$\kappa = \frac{h^2}{\beta d_{\text{E}}} d_{\text{g}} \frac{\sin \xi q \pi}{q\pi} \mathrm{e}^{-\mathrm{i}\xi q \pi}$

$\eta = \sin^2 |\kappa| l$

$2\delta = \Delta\beta + qK$

$w(z) = w_0 \left[1 + \left(\frac{2z}{k w_0^2}\right)^2\right]^{1/2}$

$\eta = \tanh^2 |\kappa| l$

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1. A: dielectric and lossy; B: magnetic and lossy; C: dielectric and lossless; D: dielectric and lossy; E: magnetic and lossless.

2. **(a)** By diagonalizing the permittivity tensor, the principal dielectric constants are found: $\epsilon_x/\epsilon_0 = 5.481$, $\epsilon_y/\epsilon_0 = 5.481$, and $\epsilon_z/\epsilon_0 = 5.053$. Thus the principal indices are $n_x = \sqrt{\epsilon_x/\epsilon_0} = 2.341$, $n_y = \sqrt{\epsilon_y/\epsilon_0} = 2.341$, and $n_z = \sqrt{\epsilon_z/\epsilon_0} = 2.248$.

(b) Because $n_x = n_y > n_z$, the crystal is birfringent and is negative uniaxial with $n_\text{o} = 2.341$ and $n_\text{e} = 2.248$. The required minimum thickness of the half-wave plate is

$$l_{\lambda/4} = \frac{\lambda}{4|n_\text{e} - n_\text{o}|} = \frac{0.50}{4|2.341 - 2.248|}\ \mu\text{m} = 1.344\ \mu\text{m}.$$

3. With the given parameters,

$$V = \frac{2\pi}{\lambda} d\sqrt{n_1^2 - n_2^2} = \frac{2\pi}{1.30} d\sqrt{3.438^2 - 3.205^2} = 1.914 d\pi, \text{ where } d \text{ is in } \mu\text{m}.$$

(a) With $d = 1\ \mu$m, we find that $V = 1.914 d\pi = 1.914\pi$. Therefore, $M_\text{TE} = M_\text{TM} = 2$. The waveguide supports 2 TE modes and 2 TM modes:
TE_0, TM_0, TE_1, TM_1.

(b) For a symmetric planar waveguide to be single-mode, $V < \pi$. Thus,

$$d < \frac{1}{1.914}\ \mu\text{m} = 0.522\ \mu\text{m}.$$

The waveguide is not truly single-mode because it supports both TE_0 and TM_0 modes.

4. See Fig 1.21(a). Anomalous dispersion is found in a spectral region of spectral width 2γ centered at ω_0 in the frequency range $\omega_0 - \gamma < \omega < \omega_0 + \gamma$.

5. The reflected light is TE-polarized at an incident angle of the Brewster angle

$$\theta_\text{B} = \tan^{-1}\frac{2.39}{1} = 67.295^\text{o}.$$

6. The three most important parameters to be considered are (i) the coupling coefficient, κ, which has to be nonzero and preferably large, (ii) phase matching, which has to be close enough so that $|\delta/\kappa| \ll 1$, ideally perfect with $\delta = 0$, and (iii) the coupler length, which has to be proper chosen to maximize the efficiency. The relative importance of the three factors is in the order given.

7. For a phase-matched 3-dB codirectional coupler,

$$\eta = \sin^2 |\kappa| l_{3\text{dB}} = \frac{1}{2} \quad \Rightarrow \quad l_{3\text{dB}} = \left(n + \frac{1}{4}\right)\frac{\pi}{|\kappa|} \quad \text{for} \quad n = 1, 2, \ldots .$$

For a phase-matched 3-dB contradirectional coupler,

$$= \tanh^2 |\kappa| l_{3\text{dB}} = \frac{1}{2} \quad \Rightarrow \quad l_{3\text{dB}} = \frac{0.88}{|\kappa|}.$$

8. (a) A metallic coaxial cable. (b) A symmetric planar dielectric waveguide. (c) A single-mode asymmetric planar dielectric waveguide.

9. $\lambda_\text{g} = \dfrac{1.2398 \text{ eV } \mu\text{m}}{E_\text{g}} = \dfrac{1.2398}{0.66} \ \mu\text{m} = 1.8785 \ \mu\text{m}.$ Light of a wavelength longer than λ_g is transmitted.

10. We have $P = 10$ W, $\lambda = 600$ nm, and the peak intensity $I_0 = 2.5 \text{ MW cm}^{-2} = 2.5\times10^{10} \text{ W m}^{-2}$ at the beam waist. Therefore,

$$I_0 = \frac{2P}{\pi w_0^2} \ \Rightarrow \ w_0 = \left(\frac{2P}{\pi I_0}\right)^{1/2} = \left(\frac{2\times 10}{\pi\times 2.5\times 10^{10}}\right)^{1/2} \text{ m} \approx 16 \ \mu\text{m}.$$

$$\Delta\theta = \frac{2\lambda}{\pi w_0} = \frac{2\times 600\times 10^{-9}}{\pi\times 16\times 10^{-6}} \text{ rad} = 23.9 \text{ mrad}.$$

At z = 5 m from the beam waist,

$$w(z) = w_0\left[1+\left(\frac{2z}{kw_0^2}\right)^2\right]^{1/2} = w_0\left[1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{1/2}$$

$$= 16\times 10^{-6}\times\left[1+\left(\frac{600\times 10^{-9}\times 5}{\pi\times\left(16\times 10^{-6}\right)^2}\right)^2\right]^{1/2} \text{ m}$$

$$= 59.7 \text{ mm},$$

$$I(z) = \frac{2P}{\pi w^2(z)} = \frac{2\times 10}{\pi\times\left(59.7\times 10^{-3}\right)^2} \text{ W m}^{-2} = 1.79 \text{ kW m}^{-2} = 0.179 \text{ W cm}^{-2}.$$

If w_0 is increased to $w_0 = 50 \ \mu\text{m}$, the peak intensity I_0 at the beam waist is multiplied by a factor of $(16/50)^2$ to 256 kW cm^{-2}. However, because $w(z) \approx \lambda z / \pi w_0$ at $z = 5$ m, $w(z)$ will be reduced by a factor of $16/50$; thus, $I_0(z)$ at $z = 5$ m is increased by a factor of $(50/16)^2$ to 1.75 W cm^{-2}.

11. (a) For this purpose, it is necessary for the coupler to have phase-matched codirectional coupling with a coupling efficiency of 100% at 1.55 μm but zero coupling efficiency at other wavelengths. With $\beta_a = 5.959\ \mu\text{m}^{-1}$ and $\beta_b = 5.849\ \mu\text{m}^{-1}$, the phase mismatch for codiectional coupling is

$$2\delta = \beta_b - \beta_a = 5.849\ \mu\text{m}^{-1} - 5.959\ \mu\text{m}^{-1} = -0.11\ \mu\text{m}^{-1} \quad \Rightarrow \quad \delta = -0.055\ \mu\text{m}^{-1}.$$

For codirectional coupling with $\delta \neq 0$,

$$\eta = \frac{1}{1+\delta^2/|\kappa|^2}\sin^2\left(|\kappa| l\sqrt{1+\delta^2/|\kappa|^2}\right) \le \frac{1}{1+\delta^2/|\kappa|^2}.$$

With $\kappa = \kappa_{ab} \approx \kappa_{ba} = 2\times10^{-3}\ \mu\text{m}^{-1} = 0.002\ \mu\text{m}^{-1}$, the maximum possible efficiency is

$$\eta_{\max} = \frac{1}{1+\delta^2/|\kappa|^2} = \frac{1}{1+0.055^2/0.002^2} = 1.32\times10^{-3}$$

which is really small.

(b) To obtain the best efficiency, the first-order grating period has to be chosen for perfect phase matching:

$$2\delta = \beta_b - \beta_a + K = 0$$

$$\Rightarrow \quad K = \beta_a - \beta_b = 5.959\ \mu\text{m}^{-1} - 5.849\ \mu\text{m}^{-1} = 0.11\ \mu\text{m}^{-1}$$

$$\Rightarrow \quad \Lambda = \frac{2\pi}{K} = \frac{2\pi}{0.11}\ \mu\text{m} = 57.1\ \mu\text{m}.$$

For phase-matched codirectional coupling, the maximum efficiency is $\eta = 100\%$ when the length of the grating is chosen to be

$$l = l_c^{\text{PM}} = \frac{\pi}{2|\kappa|} = \frac{\pi}{2\times0.002}\ \mu\text{m} = 785\ \mu\text{m}.$$

(c) For this purpose, it is necessary for the coupler to have phase-matched contradirectional coupling at 1.55 μm but zero coupling efficiency at other wavelengths. With $\beta_a = 5.959\ \mu\text{m}^{-1}$ and $\beta_b = 5.849\ \mu\text{m}^{-1}$, the phase mismatch for codiectional coupling is

$$2\delta = -\beta_b - \beta_a = -5.849\ \mu\text{m}^{-1} - 5.959\ \mu\text{m}^{-1} = -11.808\ \mu\text{m}^{-1} \quad \Rightarrow \quad \delta = -5.904\ \mu\text{m}^{-1}.$$

For contradirectional coupling with $\delta \neq 0$,

$$\eta = \frac{\sinh^2\left(|\kappa| l\sqrt{1-\delta^2/|\kappa|^2}\right)}{\cosh^2\left(|\kappa| l\sqrt{1-\delta^2/|\kappa|^2}\right) - \delta^2/|\kappa|^2}.$$

With $\kappa = \kappa_{ab} \approx \kappa_{ba} = 2\times10^{-3}\ \mu\text{m}^{-1} = 0.002\ \mu\text{m}^{-1}$ and $\delta = -5.904\ \mu\text{m}^{-1}$, we find that $\delta^2/|\kappa|^2 \approx 8.7\times10^6$ so that the maximum possible efficiency is

$$\eta = \frac{\sin^2(|\delta| l)}{\cos^2(|\delta| l) + \delta^2/|\kappa|^2} \quad \Rightarrow \quad \eta_{\max} = \frac{1}{\delta^2/|\kappa|^2} = \frac{1}{8.7\times10^6} = 1.1\times10^{-7}$$

which is negligibly small.

(d) To obtain the best efficiency, the first-order grating period has to be chosen for perfect phase matching:

$$2\delta = -\beta_b - \beta_a + K = 0$$

$$\Rightarrow \quad K = \beta_a + \beta_b = 5.959\ \mu\text{m}^{-1} + 5.849\ \mu\text{m}^{-1} = 11.808\ \mu\text{m}^{-1}$$

$$\Rightarrow \quad \Lambda = \frac{2\pi}{K} = \frac{2\pi}{11.808}\ \mu\text{m} = 0.532\ \mu\text{m} = 532\ \text{nm}.$$

With perfect phase matching and a coupling length of $l = 785\ \mu\text{m}$ from (a), the coupling efficiency of the contradirectional coupler is

$$\eta = \tanh^2 |\kappa| l = \tanh^2 (0.002 \times 785) = \tanh^2 1.57 = 84\%.$$

12. The three types of interferometers are Fabry−Perot interferometer, Mach−Zehnder interferometer, and Michelson interferometer. Their basic structures and principles are shown: