EE170A Principles of Photonics
Electrical Engineering, UCLA

Fall 2016 Midterm Exam

In-class 2 hrs, Closed Book | Nov. 1, 2016 | Prof. J.M. Liu

1. (10%) At a given optical frequency, the optical susceptibility tensors of several materials are measured with respect to an arbitrary set of rectilinear coordinates in space, as listed below. Identify each material whether it is (1) reciprocal or nonreciprocal and (2) lossless or lossy. Here "lossy" means having an optical loss or gain.

$$A:\chi=\begin{pmatrix}1.45 & \text{i}0.2 & 0\\ -\text{i}0.2 & 1.45 & -\text{i}0.1\\ 0 & \text{i}0.1 & 1.46\end{pmatrix},\ B:\chi=\begin{pmatrix}3.5 & 0.2 & 0.3\\ 0.1 & 3.8 & 0.1\\ 0.3 & 0.4 & 3.9\end{pmatrix},\ C:\chi=\begin{pmatrix}2.5 & 0.3 & 0.2-\text{i}0.5\\ 0.3 & 2.9 & 0.1\\ 0.2-\text{i}0.5 & 0.1 & 2.65\end{pmatrix},$$

$$D:\chi=\begin{pmatrix}2.07 & -\text{i}0.3 & 0\\ \text{i}0.3 & 2.28 & -\text{i}0.1\\ 0 & \text{i}0.1 & 2.40+\text{i}0.2\end{pmatrix},\ E:\chi=\begin{pmatrix}3.30 & 0.25 & 0\\ 0.25 & 3.25 & 0.35\\ 0 & 0.35 & 3.40\end{pmatrix}.$$

2. (10%) A crystal has the following permittivity tensor at $\lambda=1.0$ μm,

$$\epsilon=\epsilon_0\begin{pmatrix}3.1954 & 0 & 0.1608\\ 0 & 3.0625 & 0\\ 0.1608 & 0 & 3.1954\end{pmatrix}.$$

(a) Find the principal *indices* of the crystal at this wavelength, accurate to the third decimal point.
(b) Is the crystal birefringent or nonbirefringent? If it is birefringent, is it uniaxial or biaxial? If it is used to make a quarter-wave plate, what is the minimum thickness of the plate by proper arrangement?

3. (12%) Gold is one of the best conductors such that the free-electron Drude model describes its optical properties reasonably well. In this model, the free electron density of Au is found to be $N=5.90\times10^{28}\ \text{m}^{-3}$. The DC conductivity of Au at $T=273$ K is $\sigma(0)=4.50\times10^{7}\ \text{S m}^{-1}$.

(a) Find the plasma frequency ω_p and the relaxation time τ for Au at $T=273$ K. Also find the cutoff optical frequency ν_p and cutoff wavelength λ_p. For what optical wavelengths is Au expected to be highly reflective? For what wavelengths is it expected to become transmissive?
(b) A surface plasmon mode can exist at the interface between a gold plate and free space. What is the surface plasmon frequency of this interface? For what optical wavelengths can a surface plasmon mode exist at this interface? What is the polarization of this mode?

4. (5%) Sketch the real and imaginary parts of $\chi(\omega)$ for an atomic system in the normal state of a material in thermal equilibrium with its surrounding as a function of ω near a resonance frequency ω_0 with a relaxation constant γ. Where is anomalous dispersion found in this situation?

5. (6%) The principal indices of refraction of $LiNbO_3$, which is a negative uniaxial crystal, at the $\lambda = 1.3$ μm wavelength are $n_x = n_y = n_o = 2.222$ and $n_z = n_e = 2.145$. Find the propagation constant for an optical wave at $\lambda = 1.3$ μm that propagates through a $LiNbO_3$ crystal under each of the following conditions. In each case, does the polarization state change as the wave propagates through the crystal?

(a) Linearly polarized along $\hat{z}$, propagating along $\hat{x}$.

(b) Circularly polarized in the xy plane, propagating along $\hat{z}$.

(c) Circularly polarized in the yz plane, propagating along $\hat{x}$.

6. (6%) In designing a waveguide coupler of any geometry, what are the three major parameters that have to be considered in order to have a good efficiency? List them in the correct order of priority.

7. (8%) A symmetric step-index planar InGaAsP/InP waveguide has a core index of $n_1 = 3.286$ and a cladding index of $n_2 = 3.166$ at the $\lambda = 1.55$ μm wavelength. The core thickness is $d = 1.0$ μm.

(a) At this wavelength, how many guided modes are supported by the waveguide? What are they?

(b) Among the guided modes found in (a), which mode has the largest propagation constant? Which one has the smallest propagation constant?

8. (5%) An optical fiber has a length of $l = 100$ km and an attenuation coefficient of 0.3 dB km^{-1} at the $\lambda = 1.55$ μm wavelength. If the minimum output power required at the end of the fiber is 10 μW, what is the minimum power that is required to be launched into the fiber at the input end?

9. (4%) A waveguide Bragg reflector incorporates a grating of a period Λ for phase-matched contradirectional coupling of a guided mode that has a propagation constant of $\beta = 1.047 \times 10^7$ m^{-1}. The grating does not cause any zeroth-order self-coupling effect. Find the required period of a first-order grating for perfect phase matching. What is the required period for a second-order grating?

10. (4%) An air wedge that has a length of $l = 2$ cm is formed between two flat glass plates by making them in contact at one end but separated at the other end by a wire that has a thickness of $h = 0.5$ mm. It is vertically illuminated with coherent light at the $\lambda = 550$ nm wavelength. What is the period of the interference fringes? How many dark and bright interference fringes appear on the surface of the wedge?

11. (9%) A ring cavity has three mirrors with reflectivities of $R_1 = R_2 = 0.98$ and $R_3 = 0.81$, which are separated at $l_{12} = l_{23} = 0.5$ m and $l_{31} = 0.8$ m.

(a) Find the round-trip length l_{RT}, the round-trip time T, and the round-trip gain factor G_c of the cold cavity.

(b) Find the finesse F, the longitudinal mode frequency spacing $\Delta\nu_L$, and the longitudinal mode width $\Delta\nu_c$ of the cavity.

(c) Find the photon lifetime τ_c, the cavity decay rate γ_c, and the quality factor Q of the cavity at the $\lambda = 532$ nm wavelength.

12. (9%) A linear Fabry–Pérot cavity in free space has a flat left mirror that has a radius of curvature of $\mathcal{R}_1 = \infty$ and a concave right mirror that has a radius of curvature of $\mathcal{R}_2 = 1$ m. The cavity length is $l = 0.75$ m. Is the cavity stable? If it is stable, where is the Gaussian beam waist located? What is the beam waist spot size for an optical wavelength of $\lambda = 851$ nm?

13. (4%) A linear Fabry–Pérot optical cavity that contains an isotropic medium supports three lowest-order Gaussian modes. Name these modes. Which one has the largest propagation constant?

14. (4%) A collimated ray of unpolarized monochromatic optical wave is obliquely incident from the air on the flat surface of a glass plate. At an incident angle of $\theta_i = 56°$, it is found that the reflected light is linearly polarized. What is the polarization of this linearly polarized reflected light? What is the refractive index of the glass plate?

15. (4%) Sketch the basic structures of the Michelson interferometer and the Mach-Zehnder interferometer. Properly identify the input and output ports.