ECE170A Midterm

Nov. 10th 11am - 3pm PST

Due: submit the PDF version of your final mlx file by Nov.10th 15:00 PST on Gradescope

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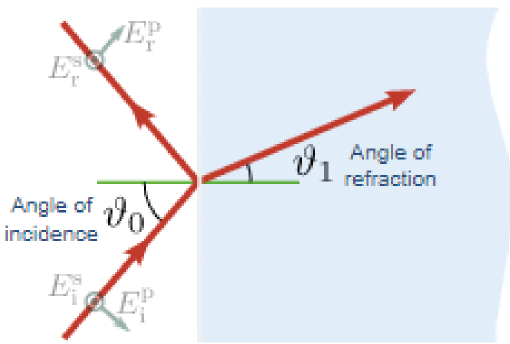
Problem 1 Fresnel Equations and Reflectance



Suppose the light beam of wavelength $2\,\mu m$ incident from air to the material, given the refractive index in the air $n_0 = 1$ and the refractive index in the material $n_1 = 1.74$. Calculate the reflectance of p-polarized component (parallel to the incidence plane) and s-polarized (perpendicular to the incidence plane) component respectively and plot the reflectance of p-polarized component and s-polarized component vs. incidence angle (from 0-90 degrees).

You should have two curves in the same plot. The horizontal axis is the incidence angle (in degree), the vertical axis is the reflectance.

$E_r = E_{\mathrm{ro}}\exp(j(\omega t - k_r \cdot r))$

The reflection coefficients, $r_\perp$ and $r_\parallel$ can be found as follows:

$r_\perp = \frac{E_{\mathrm{ro},\perp}}{E_{\mathrm{io},\perp}} = \frac{\cos\theta_i - \sqrt{[n^2 - \sin^2\theta_i]}}{\cos\theta_i + \sqrt{[n^2 - \sin^2\theta_i]}}$ and

$r_\parallel = \frac{E_{\mathrm{ro},\parallel}}{E_{\mathrm{io},\parallel}} = \frac{\sqrt{[n^2 - \sin^2\theta_i]} - n^2\cos\theta_i}{\sqrt{[n^2 - \sin^2\theta_i]} + n^2\cos\theta_i}$, where $n = \frac{n_1}{n_0}$ in this case. In general, $n = \frac{n_2}{n_1}$ where n_2 is the refractive material and n_1 is the incident material.

The reflectance of the S-polarized component, R_s , and the P-polarized component, R_p can be found using:

$R_s = |r_\perp|^2$ and $R_p = |r_\parallel|^2$.

Rearranging, we obtain:

$$R_s = \left| \frac{n_0\cos\theta_i - n_1\sqrt{1 - \left(\frac{n_0}{n_1}\sin\theta_i\right)^2}}{n_0\cos\theta_i + n_1\sqrt{1 - \left(\frac{n_0}{n_1}\sin\theta_i\right)^2}} \right|^2$$

$$R_p = \left| \frac{n_0\sqrt{1 - \left(\frac{n_0}{n_1}\sin\theta_i\right)^2} - n_1\cos\theta_i}{n_0\sqrt{1 - \left(\frac{n_0}{n_1}\sin\theta_i\right)^2} + n_1\cos\theta_i} \right|^2$$

where n has been separated into n_0 and n_1.

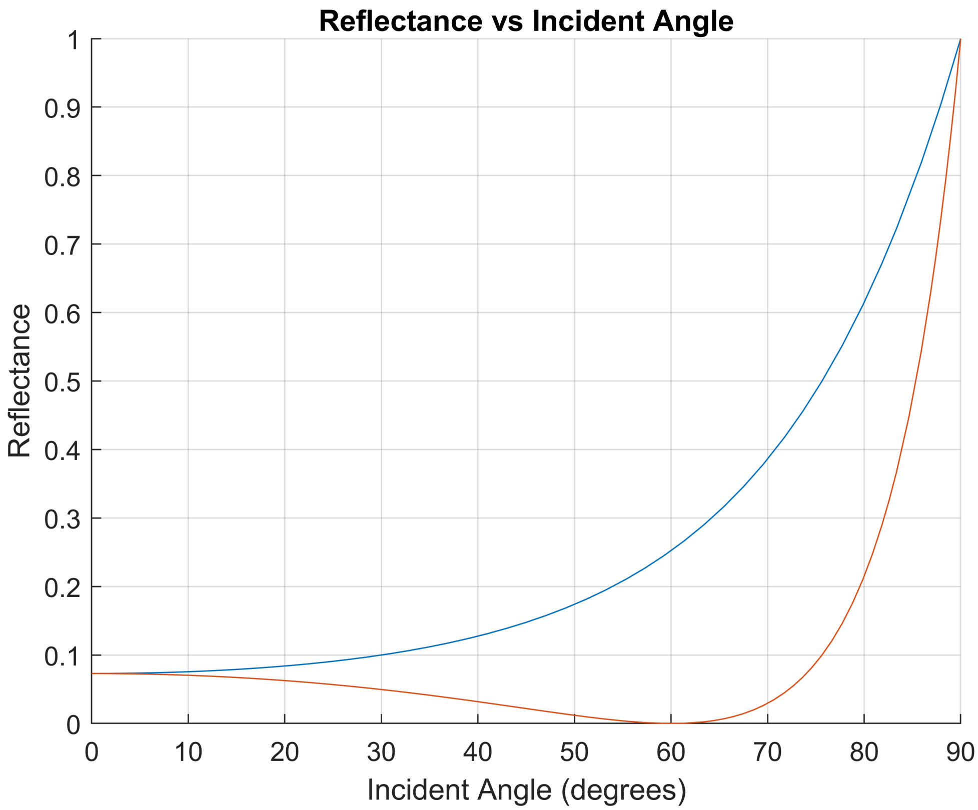
```
n0 = 1;
n1 = 1.74;
figure;
hold on;
grid;
fplot(@(theta) abs((-n1*sqrt(1-((n0/n1)*sin(deg2rad(theta)))^2)+n0*cos(deg2rad(theta)))/ ...
    (n1*sqrt(1-((n0/n1)*sin(deg2rad(theta)))^2)+n0*cos(deg2rad(theta))))^2, [0,90], "DisplayNam
```

```
Warning: Function behaves unexpectedly on array inputs. To improve performance, properly vectorize your
 function to return an output with the same size and shape as the input arguments.
```

```
fplot(@(theta) abs((n0*sqrt(1-((n0/n1)*sin(deg2rad(theta)))^2)-n1*cos(deg2rad(theta)))/ ...
    (n0*sqrt(1-((n0/n1)*sin(deg2rad(theta)))^2)+n1*cos(deg2rad(theta))))^2, [0,90], "DisplayNam
```

```
Warning: Function behaves unexpectedly on array inputs. To improve performance, properly vectorize your
 function to return an output with the same size and shape as the input arguments.
```

```
title("Reflectance vs Incident Angle");
xlabel("Incident Angle (degrees)");
ylabel("Reflectance");
```




```
legend off;
%funS = abs((-n1*sqrt(1-((n0/n1)*sin(deg2rad(theta)))^2)+n0*cos(deg2rad(theta)))/ ...
%    (n1*sqrt(1-((n0/n1)*sin(deg2rad(theta)))^2)+n0*cos(deg2rad(theta))))^2
%funP = abs((n0*sqrt(1-((n0/n1)*sin(deg2rad(theta)))^2)-n1*cos(deg2rad(theta)))/ ...
%    (n0*sqrt(1-((n0/n1)*sin(deg2rad(theta)))^2)+n1*cos(deg2rad(theta))))^2
```

The S-polarized reflectance is the blue line, while the P-polarized reflectance is the orange line.

Problem 2 Diffraction Angles of Reflection Gratings

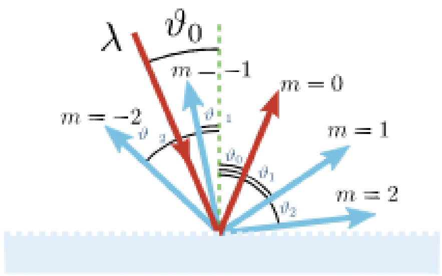
Consider a reflection grating with a period $d = 1.25\ \mu m$. A collimated light wave is incident on the grating at an angle of 25°. Find the diffraction angle of order $m = -3,\ -2,\ -1,\ 1$ and plot the diffraction angles vs. wavelengths λ (from $400\,\mathrm{nm}$ to $1600\mathrm{nm}$).

You should have 4 curves in the same plot. The horizontal axis is the wavelength (in nm), the vertical axis is the diffraction angle (in degree)

note:

1. not every wavelength in the range is valid for all orders of diffraction angles, you may need to first find a cut-off wavelength for each order. For wavelength greater than the cut-off wavelength, there will be no valid solution because the value of sin() function cannot be larger than 1.

2. the diffraction angles could have negative values. Given the light is incident from the left of the normal, we define the reflected light on the right of the normal to have positive diffraction angle, the reflected light on the left of the normal to have negative diffraction angle. (In the figure below for example, the cases where m=0,1,2 have positive angles and the cases where m=-1, -2 have negative values.)



Oblique Incidence:

$d(\sin\theta_m - \sin\theta_i) = m\lambda$

$(\sin\theta_m) = \frac{m\lambda}{d} + \sin\theta_i$

$\sin\theta_i = \sin(25^o)$

Plugging in numbers for m and setting $\sin\theta_m$ as 1 or -1, as appropriate (for wavelengths > 0), the following cutoff wavelengths can be obtained:

m = 1 : $\lambda = 7.217 * 10^{-7}$m

m = -1: $\lambda = 1.778 * 10^{-6}$m

m = -2: $\lambda = 8.891 * 10^{-7}$m

m = -3: $\lambda = 5.928 * 10^{-7}$m

```
d = 1250
```

```
d = 1250
```

```
thetai = deg2rad(25);
cut1 = 721.7;
cutn1 = 1778;
cutn2 = 889.1;
cutn3 = 592.8;
lambda1 = linspace(400, min(cut1, 1600), 1001);
lambdan1 = linspace(400, min(cutn1, 1600), 1001);
lambdan2 = linspace(400, min(cutn2, 1600), 1001);
lambdan3 = linspace(400, min(cutn3, 1600), 1001);
fun1 = rad2deg(asin(1*lambda1/d + sin(thetai)));
funn1 = rad2deg(asin(-1*lambdan1/d + sin(thetai)));
funn2 = rad2deg(asin(-2*lambdan2/d + sin(thetai)));
```

```
funn3 = rad2deg(asin(-3*lambdan3/d + sin(thetai)));
figure;
hold on;
plot(lambda1, fun1, "DisplayName", "m=1");
plot(lambdan1, funn1, "DisplayName", "m=-1");
plot(lambdan2, funn2, "DisplayName", "m=-2");
plot(lambdan3, funn3, "DisplayName", "m=-3");
```

```
Warning: Imaginary parts of complex X and/or Y arguments ignored.
```

```
xlabel("Lambda (nm)");
ylabel("Diffraction Angle (degrees)");
title("Diffraction Angle vs Wavelength");
grid;
legend;
```