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ECE170A Midterm Exam

Date: **11/6/18 4-5:50 pm**

1.

Snell's law and lateral beam displacement What is the displacement of a laser beam passing through a glass plate of thickness 2 mm and refractive index 1.570 if the angle of incidence is 40°? (See Figure 1.14)

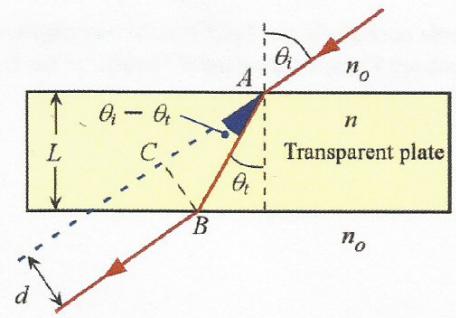

Figure 1.14 Lateral displacement of light passing obliquely through a transparent plate

2. Note: No worry about the spectroscopic ellipsometry measurements, the given photon energy is 1.5 eV in this problem

Complex refractive index Spectroscopic ellipsometry measurements on a germanium crystal at a photon energy of 1.5 eV show that the real and imaginary parts of the complex relative permittivity are 21.56 and 2.772 respectively. Find the complex refractive index. What is the reflectance and absorption coefficient at this wavelength? How do your calculations match with the experimental values of $n = 4.653$ and $K = 0.298$, $R = 0.419$ and $\alpha = 4.53 \times 10^6$ m^{-1}?

3.

Fabry-Perot optical cavity from a ruby crystal Consider a ruby crystal of diameter 1 cm and length 10 cm. The refractive index is 1.78. The ends have been silvered and the reflectances are 0.99 and 0.95 each. What is the nearest mode number that corresponds to a radiation of wavelength 694.3 nm? What is the actual wavelength of the mode closest to 694.3 nm? What is the mode separation in frequency and wavelength? What are the finesse F and Q-factors for the cavity?

4.

A multimode fiber Consider a multimode fiber with a core diameter of 100 μm, core refractive index of 1.4750, and a cladding refractive index of 1.4550 both at 850 nm. Consider operating this fiber at $\lambda = 850$ nm. (a) Calculate the V-number for the fiber and estimate the number of modes. (b) Calculate the wavelength beyond which the fiber becomes single mode. (c) Calculate the numerical aperture. (d) Calculate the maximum acceptance angle. (e) Calculate the modal dispersion $\Delta\tau$ and hence the bit rate × distance product.

5.

Dispersion at zero dispersion coefficient Since the spread in the group delay $\Delta\tau$ along a fiber depends on the $\Delta\lambda$, the linewidth of the source, we can expand $\Delta\tau$ as a Taylor series in $\Delta\lambda$. Consider the expansion at $\lambda = \lambda_0$ where $D_{ch} = 0$. The first term with $\Delta\lambda$ would have $d\Delta\tau/d\lambda$ as a coefficient, that is D_{ch}, and at λ_0 this will be zero; but not the second term with $(\Delta\lambda)^2$ that has a differential, $d^2\Delta\tau/d\lambda^2$ or $dD_{ch}/d\lambda$. Thus, the dispersion at λ_0 would be controlled by the slope S_0 of D_{ch} vs. λ curve at λ_0. Show that the chromatic dispersion at λ_0 is

$$\Delta\tau = \frac{L}{2}S_0(\Delta\lambda)^2$$

A single mode fiber has a zero-dispersion at $\lambda_0 = 1310$ nm, dispersion slope $S_0 = 0.090$ ps nm^2 km. What is the dispersion for a laser with $\Delta\lambda = 1.5$ nm? What would control the dispersion?

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1. $L = 2mm,\ n = 1.570,\ \theta_i = 40°\quad n_0 = 1$

$$d = L\sin\theta_i\left(1 - \frac{\cos\theta_i}{\sqrt{(n/n_0)^2 - \sin^2\theta_i}}\right)$$
$$= 2mm\,\sin 40°\left(1 - \cos 40° / \sqrt{1.570^2 - \sin^2 40}\right)$$
$$d = 0.598\ mm$$

2. $E = 1.5eV,\ \varepsilon_r = 21.56 - i2.772,\ \varepsilon_r' = 21.56,\ \varepsilon_r'' = 2.772$

$$n = \sqrt{\tfrac{1}{2}\left(\varepsilon_r' + \sqrt{\varepsilon_r'^2 + \varepsilon_r''^2}\right)}$$
$$= \sqrt{\tfrac{1}{2}\left(21.56 + \sqrt{(21.56)^2 + (2.772)^2}\right)}$$
$$n = 4.65282$$

$$K = \sqrt{\frac{(\varepsilon_r'^2 + \varepsilon_r''^2)^{1/2} - \varepsilon_r'}{2}} = \sqrt{\frac{(21.56^2 + 2.772^2)^{1/2} - 21.56}{2}}$$
$$K = 0.29788$$

$N = n - iK$

$N = 4.65282 - j0.29788$

$$R = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2}$$
$$= \frac{(4.65282-1)^2 + (0.29788)^2}{(4.65282+1)^2 + (0.29788)^2}$$
$$R = 0.41918$$

$\alpha = 2k_0K \qquad E = h\nu \Rightarrow \nu = 3.622\cdot10^{14}$

$\lambda = c/\nu$

$\frac{2\pi}{k} = \frac{c}{\nu} \Rightarrow k = \frac{2\pi\nu}{c}$

$= 7586098\ 1/m$

$\alpha = 2k_0K = 2\cdot 7586098\ 1/m \cdot 0.29788$

$$\alpha = 4.52\cdot10^{6}\ m^{-1}$$

These are very close to the experimental values.

3. $d = 1cm$, $L = 10cm$, $n = 1.78$, $R_1 = 0.99$, $R_2 = 0.95$, closest mode to $\lambda = 694.3nm$?

$$m = \frac{2nL}{\lambda} = \frac{2 \cdot 1.78 \cdot 10cm}{694.3nm}$$

512746 ~1

$$m = 512746.65$$

$$\Rightarrow m = 512747 \text{ is closest to } 694.3nm$$

$$\lambda_m = \frac{2nL}{m} = \frac{2 \cdot 1.78 \cdot 10cm}{512747}$$

$$\Rightarrow \lambda_m = 694.2995\ nm \text{ at } m = 512747$$

Average reflectance is $\frac{0.99 + 0.95}{2} = 0.97$

Finesse $F = \frac{\pi\sqrt{R}}{1-R} = \frac{\pi\sqrt{0.97}}{1-0.97} \Rightarrow \cancel{F = 103.137}$

Mode separation $\Delta\nu_m = \frac{c}{2nL} = \frac{3 \cdot 10^8 m/s}{2 \cdot 1.78 \cdot 10cm}$

$$\cancel{\Delta\nu_m = 8.427 \times 10^8\ Hz}$$

$$\Delta\lambda = \frac{\lambda_m^2}{2nL} = \frac{(694.2995nm)^2}{2 \cdot 1.78 \cdot 10cm}$$

$$\cancel{\Delta\lambda = 1.354\ pm}$$ ~~(picometers)~~

$Q = m_o F$

$m_o = 512747$

$Q = 512747 \cdot 103.137 \Rightarrow \cancel{Q = 52883187.34}$

4. $a = 50\mu m$, $n_{core} = 1.4750$, $n_{clad} = 1.4550$ at $\lambda = 850nm$

a) $V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi \cdot 50\mu m}{850nm} \cdot \sqrt{1.475^2 - 1.455^2}$

$V = 89.47$

# of modes $M = V^2/2 = \frac{89.47^2}{2}$

$M = 4002.483$

$\Rightarrow$ 4002 modes

b) Cutoff wavelength λ_c when $V = 2.405 = \frac{2\pi a}{\lambda_c}\sqrt{n_1^2 - n_2^2}$

$\lambda_c = \frac{2\pi a}{2.405}\sqrt{n_1^2 - n_2^2} = \frac{2\pi \cdot 50\mu m}{2.405}\sqrt{1.475^2 - 1.455^2}$

$\lambda_c = 31.62\mu m$

c) $NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.475^2 - 1.455^2}$

$NA = 0.24207$

d) Maximum acceptance angle α_{max}

$\sin\alpha_{max} = \left(\frac{NA}{n_0}\right)$ $\quad n_0 = 1$

$\alpha_{max} = \arcsin(NA) = \arcsin(0.24207)$

$\alpha_{max} = 14.01°$ Total acceptance angle $= 2\alpha_{max} = 28.02°$

e) Modal dispersion $\sigma = 0.29\,\Delta\tau$

$\Delta\tau = L \cdot \frac{n_1 - n_2}{c} = L \cdot \frac{1.475 - 1.455}{3 \cdot 10^8 m/s}$

$\Delta\tau = 6.667 \cdot 10^{-11} s/m \cdot L$

$BL = \frac{0.25L}{\sigma} = \frac{0.25L}{0.29 \cdot 6.667 \cdot 10^{-11} \frac{s}{m} \cdot L} = \frac{0.25}{0.29 \cdot 6.667 \cdot 10^{-11} s/m}$

$BL = 1.293 \cdot 10^{10} \frac{bits}{sec} \cdot m$

5. $\frac{\Delta\tau}{L} = |D_{ch} + D_m| \Delta\lambda$

$$\approx \frac{1}{1!}\frac{d(\Delta\tau)}{d\lambda}\Big|_{\lambda=\lambda_0}\cdot\Delta\lambda + \frac{1}{2!}\frac{d^2(\Delta\tau)}{d\lambda^2}\Big|_{\lambda=\lambda_0}\cdot(\Delta\lambda)^2 \quad \text{Taylor expansion}$$

$$= D_{ch}\cdot\Delta\lambda + \frac{1}{2}\frac{dD_{ch}}{d\lambda}\Big|_{\lambda=\lambda_0}(\Delta\lambda)^2 \qquad D_{ch} = 0, \text{ so}$$

$$\frac{\Delta\tau}{L} = \frac{1}{2}\frac{dD_{ch}}{d\lambda}\Big|_{\lambda=\lambda_0}\cdot(\Delta\lambda)^2 \qquad S_0 = \frac{dD_{ch}}{d\lambda}\Big|_{\lambda=\lambda_0}$$

$$\Delta\tau = \frac{L}{2}S_0\cdot(\Delta\lambda)^2$$

$\lambda_0 = 1310\,nm$, $S_0 = 0.09\frac{ps}{nm^2\cdot km}$, $\Delta\lambda = 1.5\,nm$

$$\Delta\tau = \frac{L}{2}S_0\cdot(\Delta\lambda)^2 = \frac{L}{2}\cdot 0.09\frac{ps}{nm^2\cdot km}\cdot(1.5\,nm)^2$$

$$\Delta\tau = 0.10125\,\frac{ps}{km}\cdot L \qquad L = 1\,km \Rightarrow \Delta\tau = 0.10125\,ps$$

Dispersion would be controlled by $\Delta\lambda$ and the length of the fiber and the slope of D_0 at $\lambda = 1310\,nm$. Since $\Delta\lambda = 1.5\,nm$, there is some wavelength variation in the light along the fiber, so some wavelengths travel faster as they penetrate further into the cladding (where n is lower) and so they travel faster, while others travel slower, causing dispersion, due to $\Delta\lambda$ and S_0.