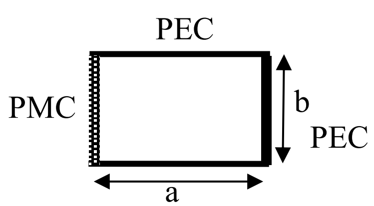
Midterm Exam, EE163A, Fall 2020, Due: Monday 11/16/20 9AM

Policy: Open Book, PDF version of the book and slides are ok to use. You should not use internet, EM software, etc. Calculator is ok to use. Collaboration/discussion is not allowed.

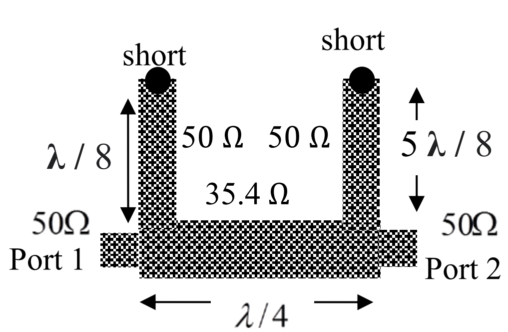
Problem # 1 (30 points): An air-filled rectangular waveguide has inner dimensions of 0.9 inches by 0.4 inches. The excitation frequency is 8 GHz. (a) Calculate $k, f_{c,} k_c, \beta, \lambda_g, V_p, V_g$ for TE_{10} mode. (b) Redo the above calculation for the same waveguide filled with dielectrics $\varepsilon_r = 9$. (c) Find out the frequency range in which the waveguide is single mode (allowing propagation of only one mode) for both air filled and dielectric filled waveguides.

Problem # 2 (35 points): Perfect Magnetic Conductor (PMC) is a counterpart of Perfect Electrical Conductor under the dual relationship of electric field and magnetic field. A PMC satisfies the boundary condition that the tangential magnetic field and normal electric field must be zero. In an air-filled waveguide shown below, the top, bottom, and right plates of the waveguide are made of PEC, while left side is made of PMC. (a) What is the dominant mode propagating in this waveguide and its cut-off frequency? (b) What transverse field components exist for this mode? (c) Draw the electric field template (variation) for this mode in the cross-section.



(a/2>b)

Problem # 3 (35 Points): In the following microstrip line circuit, the two parallel stubs are short-circuited through ground vias. (a) Calculate the ABCD matrix from Port 1 to Port 2. (b) Assuming both source and output ports are matched 50 Ohm, if the voltage at Port 1 is 1 volt, find the amplitude and phase of the output voltage at port 2.



Problem 1

Air Filled Rectangular Waveguide

$a = 0.9\,in \quad , \quad b = 0.4\,in$

$= 2.286\,cm \quad b = 1.016\,cm$

$f = 8\,GHz$

a) $k,\ f_c,\ k_c,\ \beta,\ \lambda_g,\ V_p,\ V_g$ for TE_{10}

$$k = \omega\sqrt{\mu_0\varepsilon_0} = \frac{2\pi \cdot 8 \cdot 10^9}{2.99 \cdot 10^8} = 168.1\ m^{-1}$$

$$k_c = \sqrt{\frac{\pi^2}{a^2} + \frac{(0\pi)^2}{b^2}} = \frac{\pi}{a} = \frac{\pi}{2.286 \cdot 10^{-2}} = 137.4\ m^{-1}$$

$$f_c = \frac{c}{2\pi}\frac{\pi}{a} = \frac{c}{2a} = \frac{2.99 \cdot 10^8}{2 \cdot 2.286 \cdot 10^{-2}} = 6.54\ GHz$$

$$\beta = \sqrt{k^2 - k_c^2} = 96.8\ m^{-1}$$

$$\lambda_g = \frac{2\pi}{\beta} = 6.49\,cm$$

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \cdot 8 \cdot 10^9}{96.8} = 5.19 \cdot 10^8\ m/s$$

$$v_g = \left(\frac{d\beta}{d\omega}\right)^{-1} = \left(\frac{d}{d\omega}\left(\omega^2\mu_0\varepsilon_0 - \frac{\pi^2}{a^2}\right)^{+1/2}\right)^{-1}$$

$$= \left(\frac{1}{\cancel{2}}\left(\omega^2\mu_0\varepsilon_0 - \frac{\pi^2}{a^2}\right)^{+1/2} \cdot \cancel{2}\omega^{-1}\mu_0^{-1}\varepsilon_0^{-1}\right)^{+1}$$

$$= \frac{\beta}{\omega\mu_0\varepsilon_0} = \frac{\beta}{\omega}c^2 = \frac{96.8}{2\pi \cdot 8 \cdot 10^9} \cdot (2.99 \cdot 10^8)^2$$

$$= 1.72 \cdot 10^8\ m/s$$

b. $\varepsilon_r = 9$

value in air

$$\boxed{\begin{aligned}
k_d &= k_a \cdot \sqrt{\varepsilon_r} = 3 \cdot k_a = 504.3\ m^{-1} \\
k_c &= \text{same} = 137.4\ m^{-1} \\
f_{c_d} &= \frac{f_{c_a}}{\sqrt{\varepsilon_r}} = 2.18\ GHz \\
\beta_d &= \sqrt{k_d^2 - k_c^2} = 485.2\ m^{-1} \\
\lambda_{g_d} &= \frac{2\pi}{\beta_d} = 1.29\ cm \\
v_{p_d} &= \frac{\omega}{\beta_d} = \frac{2\pi \cdot 8 \cdot 10^9}{485.2} = 1.04 \cdot 10^8\ m/s \\
v_{g_d} &= \frac{\beta_d}{\omega} \cdot \frac{c^2}{(\sqrt{\varepsilon_r})^2} = \frac{485.2}{2\pi\, 8 \cdot 10^9} \cdot \frac{(2.97 \cdot 10^8)^2}{9} \\
&= 9.59 \times 10^7\ m/s
\end{aligned}}$$

c.) <u>Air-filled</u>

$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

TE_{10}, $f_c = 6.54\ GHz$

~~TE_{01}, $f_c = 14.7\ GHz$~~

TE_{20}, $f_c = 13.1\ GHz$

$$\boxed{\text{Single mode for } 6.54\ GHz < f < 13.1\ GHz}$$

Dielectric filled:

$$f_{c_d} = \frac{f_{c_a}}{\sqrt{\varepsilon_r}} = \frac{f_{c_a}}{3}, \quad \frac{6.54}{3}\text{GHz} = 2.18\text{ GHz}$$

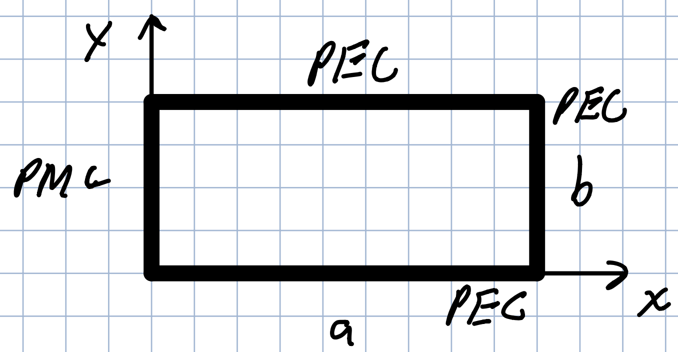
$$\frac{13.1\text{ GHz}}{3} = 4.37\text{ GHz}$$

$$\boxed{\text{Single mode for } 2.18\text{ GHz} < f < 4.37\text{ GHz}}$$

Problem 2

PMC B.C. $H_{\parallel} = 0, \quad E_{\perp} = 0$

a)



$\frac{a}{2} > b$

TE

$H_z \neq 0, \quad E_z = 0$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) h_z = 0, \quad k_c^2 = k_x^2 + k_y^2$$

$$\Rightarrow h_z = (A\cos k_x x + B\sin k_x x)(C\cos k_y y + D\sin k_y y)$$

x – B.C.

$$x = 0, \quad \underset{\leftarrow \text{PMC}}{H_z = 0} \Rightarrow A = 0$$

$$x = a, \quad \underset{\leftarrow \text{PEC}}{E_y = 0} \Rightarrow 0 = \frac{\partial h_z}{\partial x} = (B k_x \cos k_x x)(C\cos k_y y + D\sin k_y y)$$

$$\Rightarrow k_x a = \frac{(2m-1)\pi}{2} \Rightarrow k_x = \frac{(2m-1)\pi}{2a}, \quad m = 1, 2, \ldots$$

y − B.C. ← PEC

$y=0,\ E_x=0 \Rightarrow 0 = \frac{\partial H_z}{\partial y} = B \sin k_x x\,(-C \sin k_y y \cdot k_y + D \cos k_y y \cdot k_y)$

$\Rightarrow D=0$

$y=b,\ E_x=0 \Rightarrow 0 = B \sin k_x x\,(-C \sin k_y y \cdot k_y)$

$k_y b = n\pi \Rightarrow k_y = \frac{n\pi}{b} \quad n=0,1,2,\ldots$

$$\Rightarrow H_z = A' \sin k_x x \cos k_y y\, e^{-j\beta z}$$

$$k_c^2 = \left(\frac{(2m-1)\pi}{2a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

Checking TM $\quad E_z \neq 0,\ H_z = 0$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) e_z = 0$$

$$e_z = (E \cos k_x x + F \sin k_x x) \cdot (G \cos k_y y + H \sin k_y y)$$

B.C − x ← PMC

$x=0,\ H_y=0 \Rightarrow 0 = \frac{\partial E_z}{\partial x} = (-E k_x \sin k_x x + F k_x \cos k_x x) \cdot (G \ldots + \ldots)$

$\Rightarrow F=0$

$x = a, \quad E_z = 0 \Rightarrow E \cos k_x a = 0$

$$k_x a = \frac{(2m-1)\pi}{2}$$

$$k_x = \frac{(2m-1)\pi}{2a} \quad m = 1, 2, \ldots$$

B, C, - y

$y = 0, \quad E_z = 0 \Rightarrow G = 0$

$y = b, \quad E_z = 0 \Rightarrow H \sin k_y b = 0$

$$k_y b = n\pi$$

$$k_y = \frac{n\pi}{b} \quad n = 1, 2, 3, \ldots$$

$$k_c^2 = \left(\frac{(2m-1)\pi}{2a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$E_z = A' \cos k_x x \sin k_y y \, e^{-j\beta z}$$

k_c is lowest when $m = 1, n = 0$

$\rightarrow TM_{10} = 0$ so TE_{10} is the lowest prop. mode.

$$\boxed{TE_{10} \text{ is dominant}}$$

$$f_c = \frac{c}{2\pi} k_c = \frac{c}{2\pi}\sqrt{\left(\frac{2(1)-1)\pi}{2a}\right)^2}$$

$$= \frac{c}{2\pi} \frac{1 \cdot \pi}{2a}$$

$$\boxed{f_c = \frac{c}{4a}}$$

b) <u>TE_{10} Components (Transverse)</u>

$$H_z = A' \sin k_x x \cos k_y y \, e^{-j\beta z} \qquad m=1, n=0$$

$$k_x = \frac{\pi}{2a}, \; k_y = 0 \qquad k_c^2 = \left(\frac{(2m-1)\pi}{2a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$H_z = A' \sin \frac{\pi}{2a} x \, e^{-j\beta z}, \quad k_c = \frac{\pi}{2a} = k_x, \quad \boxed{\beta = \sqrt{k^2 - \left(\frac{\pi}{2a}\right)^2}} \quad \boxed{k = \omega\sqrt{\mu_0 \epsilon_0}}$$

$$H_x = \frac{-j}{k_c^2} \beta \frac{\partial H_z}{\partial x}$$

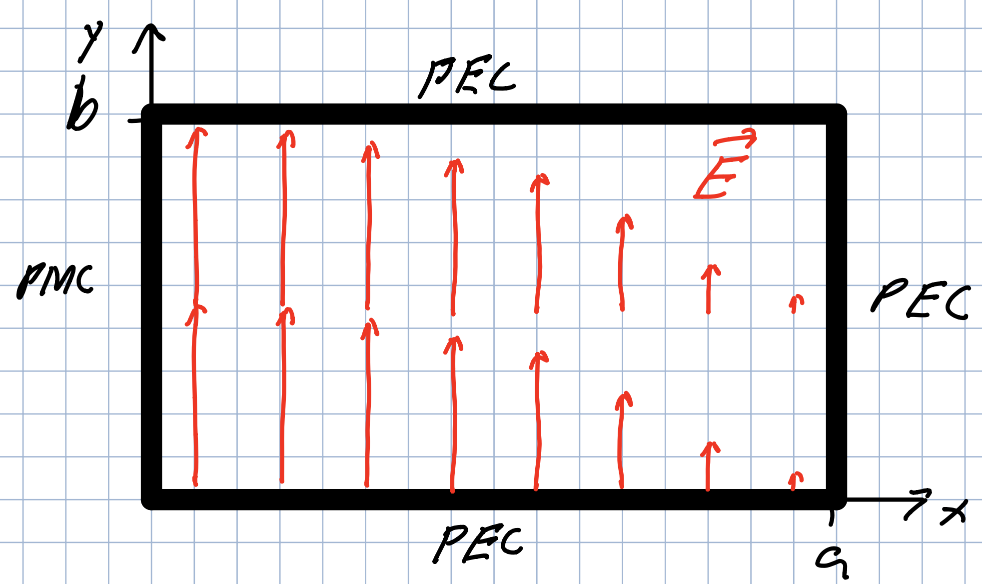
$$= \frac{-j\beta}{k_x^2} A' \cos k_x x \cdot k_x \, e^{-j\beta z}$$

$$= -j\beta A' \frac{2a}{\pi} \cos\left(\frac{\pi}{2a} x\right) e^{-j\beta z}$$

$$E_y = \frac{j}{k_c^2} \omega\mu \frac{\partial H_z}{\partial x}$$

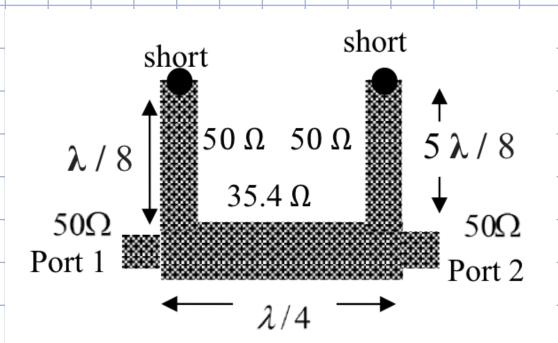
$$= j\omega\mu A' \frac{2a}{\pi} \cos\left(\frac{\pi}{2a} x\right) e^{-j\beta z}$$

c)

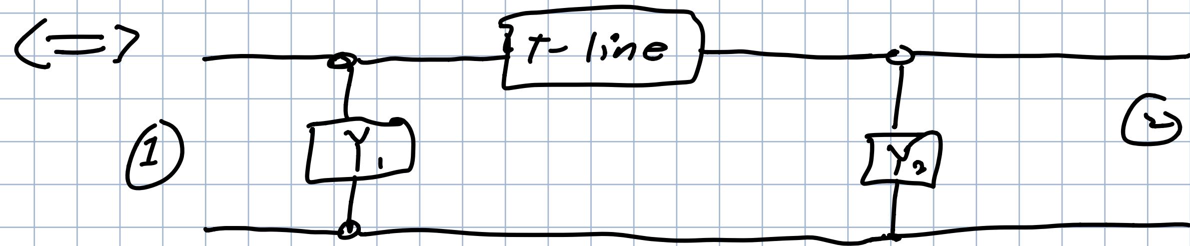


Problem 3

a) ABCD from Port 1 to Port 2



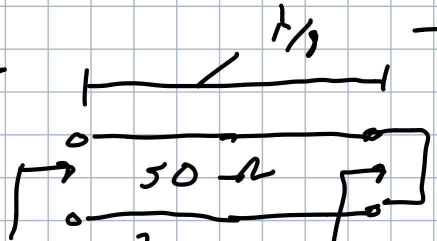
$\Longleftrightarrow$



$Z_0 = 50\Omega$, $l = \lambda/8$

$Z_0 = 50\Omega$, $l = 5\lambda/8$

$\underline{Y_1}$



$Z_1 = \frac{1}{Y_1}$ $\qquad Z_L = 0$

$\rightarrow \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \pi/4$

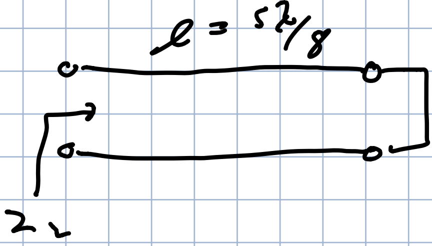
$$Z_1 = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$Z_L = 0 \Rightarrow Z_1 = Z_0 \frac{jZ_0 \tan\beta l}{Z_0}$$

$$= jZ_0 \tan\beta l = j\,50 \tan \pi/4 = j50$$

$$\Rightarrow Y_1 = \frac{-j}{50}$$

$\underrightarrow{Y_2}$



$$\beta\ell = \frac{2\pi}{\lambda}\cdot\frac{5\lambda}{8} = \frac{10\pi}{8} = \frac{5\pi}{4}$$

$$Z_2 = Z_0 \, \frac{0 + jZ_0 \tan\frac{5\pi}{4}}{Z_0 + j0}$$

$$= jZ_0 = j50$$

$$\Rightarrow Y_2 = \frac{1}{Z_2} = \frac{-j}{50} = Y_1$$

ABCD Matrices

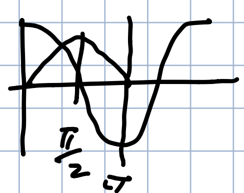
for Y_1, Y_2: $\begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix}$

for T-line: $\begin{bmatrix} \cos\beta\ell & jZ_0\sin\beta\ell \\ j\frac{1}{Z_0}\sin\beta\ell & \cos\beta\ell \end{bmatrix}$, $Z_0 = 35.4\,\Omega$

$\ell = \frac{\lambda}{4}$

$\beta\ell = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{\pi}{2}$

$$\begin{bmatrix} 0 & jZ_0 \\ j\frac{1}{Z_0} & 0 \end{bmatrix}$$



Port 1 to Port 2 ABCD:

$$[Y_1]\,[\text{T-line}]\,[Y_1]$$

$$= \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} \begin{bmatrix} 0 & jZ_0 \\ j\frac{1}{Z_0} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & jZ_0 \\ j/Z_0 & jZ_0Y_1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} jZ_0Y_1 & jZ_0 \\ j\frac{1}{Z_0} + jZ_0Y_1^2 & jZ_0Y_1 \end{bmatrix}$$

$$= \begin{bmatrix} j\frac{35.4}{50}(-j) & j35.4 \\ j\frac{1}{35.4} + j35.4\cdot\frac{(-j)^2}{50^2} & j\frac{35.4}{50}(-j) \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 0.708 & j35.4 \\ j0.01409 & 0.708 \end{bmatrix}}$$

$$j\left(\frac{1}{35.4} - \frac{35.4}{50^2}\right)$$

b) Y mat $\rightarrow$ $Y_{11} = \frac{D}{B}$ $\quad Y_{12} = \frac{BC - AD}{B}$

$Y_{21} = \frac{-1}{B}$ $\quad Y_{22} = \frac{A}{B}$

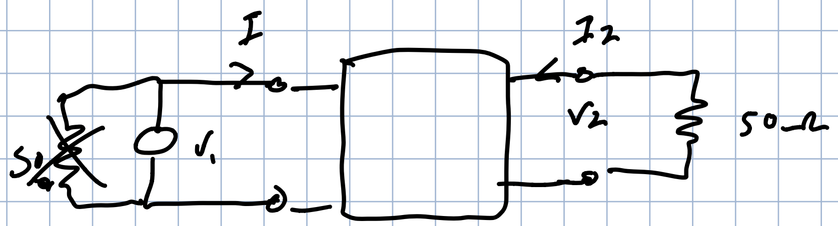
$$\Rightarrow Y_{11} = \frac{jZ_0Y_1}{jZ_0} = Y_1 \qquad Y_{12} = \frac{-1 + -Z_0^2Y_1^2 - (-Z_0^2Y_1^2)}{jZ_0}$$

$$Y_{21} = \frac{-1}{jZ_0} = \frac{j}{Z_0} \qquad = \frac{j}{Z_0}$$

$$Y_{22} = Y_1$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_1 & j/Z_0 \\ j/Z_0 & Y_1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow I_2 = \frac{j}{Z_0} V_1 + Y_1 V_2$$



$$\Rightarrow \frac{V_2}{-I_2} = 50\,\Omega \Rightarrow \frac{-V_2}{50} = I_2$$

$$\Rightarrow \frac{-V_2}{50} = \frac{j}{Z_0} V_1 + Y_1 V_2$$

$$-V_2\left(\frac{1}{50} + Y_1\right) = j\frac{V_1}{Z_0}$$

$$V_2 = \frac{-j\frac{V_1}{Z_0}}{\left(\frac{1}{50} + Y_1\right)}, \quad V_1 = 1$$

$$= -j\frac{1}{35.4\left(\frac{1}{50} + \frac{-j}{50}\right)}$$

$$= -j\frac{50}{35.4(1-j)} = \frac{50\angle -90^\circ}{35.4\sqrt{2}\angle -45^\circ}$$

$$= \frac{50}{35.4\sqrt{2}}\angle -45^\circ$$

$$= 0.999\angle -45^\circ$$

$$= \boxed{1\angle -45^\circ}$$