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Problem #1 (34 points). A dipole antenna and a circular loop antenna are both designed to operate at 1MHz. Each antenna has a maximum dimension of 1 meter and is made of 4mm diameter copper wire.

- (a) What are the directivities of both antennas? +6
- (b) Compute the impedance of both antennas. +6
- (c) Find the radiation efficiency of both antennas. +4
- (d) If both antennas are radiating the same amount of power of 100Watt, find the strength of the maximum electric field at 10 meter away for both antennas and compare them. +6
- (e) Redo the loop antenna case for (b)-(d) when the loop is now made of 10 turns. +6

a)  $D_{dipole} = 1.5 = 1.76 \text{ dB}$   
 $D_{loop} = 1.5 = 1.76 \text{ dB}$   
 ↑ dual of dipole

b)  $Z = R + jX$   
 $\sigma_{copper} = 5.8 \times 10^7$   
 $a = 2 \text{ mm}$   
 Dipole:  $R_s = \sqrt{\frac{\omega \mu}{2\sigma}}$   
 $= \sqrt{\frac{2\pi(1 \text{ MHz})(4\pi \times 10^{-7})}{2(5.8 \times 10^7)}}$   
 $= 2.61 \times 10^{-4}$   
 $R_{loss} = \frac{R_s}{2\pi a} \cdot l$   
 $= \frac{2.61 \times 10^{-4}}{2\pi(2 \times 10^{-3})}$   
 $= 0.021$   
 $R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2$   $\lambda = 300 \text{ m}$   
 $= 8.77 \times 10^{-3}$   
 $X = -\frac{120}{\pi} \ln\left(\ln\left(\frac{l}{2a}\right) - 1\right)$   
 $= -51812$

$Z_{dipole} = 0.03 - j51812 \Omega$

$Z = R + j\omega L$   
 Loop:  $R_s = 2.61 \times 10^{-4}$   
 $R_{loss} = \frac{R_s}{2\pi a} (2\pi b)$   
 $= \frac{2.61 \times 10^{-4}}{2\pi(2 \times 10^{-3})} (2\pi(0.5))$   
 $= 0.065$   
 $R_r = 31200 \left(\frac{S}{\lambda^2}\right)^2$   $S = \pi b^2$   
 $= 31200 \left(\frac{\pi(0.5)^2}{300^2}\right)^2$   
 $= 2.38 \times 10^{-6}$   
 $L = \mu b \left(\ln\left(\frac{8l}{a}\right) - 2\right)$   
 $= 4\pi \times 10^{-7} (0.5) \left(\ln\left(\frac{8(0.5)}{2 \times 10^{-3}}\right) - 2\right)$   
 $= 3.52 \times 10^{-6}$

$Z_{loop} = 0.065 + j(2\pi(1 \times 10^6))(3.52 \times 10^{-6})$   
 $= 0.065 + j2.12 \Omega$

Continued on back

$$c) e_r = \frac{R_r}{R_r + R_{Ls}}$$

$$\text{Dipole: } e_r = \frac{8.77 \times 10^{-3}}{8.77 \times 10^{-3} + 0.021}$$

$$= \boxed{29.5\%} \quad 24\%$$

$$\text{Loop: } e_r = \frac{2.35 \times 10^{-6}}{2.35 \times 10^{-6} + 0.065}$$

$$= \boxed{3.66 \times 10^{-5}}$$

$$d) P = 100 \text{ W} \quad r = 10 \text{ m}$$

$$r_{fl} = \frac{2D^2}{\lambda}$$

$$S = \frac{|E|^2}{2\eta_0} \quad \eta_0 = 120\pi$$

$$\approx 0.015 \ll r$$

$\Rightarrow$  far field (same  $|E|$  for dipole & loop)

$$S = \frac{P}{4\pi r^2} D$$

$$= \frac{100}{4\pi (10)^2} (1.5)$$

$$\approx 0.12$$

$$\boxed{|E| = 9.49 \text{ V/m}}$$

$\leftarrow$  same for dipole & loop because far field

$$e) N = 10$$

$$R_{loss} = \frac{R_s}{2\pi a} (2\pi a b) N^2$$

$$= 6.5$$

$$R_r = 31200 \left(\frac{NS}{\lambda^2}\right)^2$$

$$= 2.38 \times 10^{-4}$$

$$L = \mu b \left(\ln \frac{8b}{a} - 2\right) N^2$$

$$= 7.52 \times 10^{-4}$$

$$\boxed{Z = 6.5 + j 2212 \Omega}$$

$$e_r = \frac{R_r}{R_r + R_{Ls}}$$

$$= \frac{2.38 \times 10^{-4}}{2.38 \times 10^{-4} + 6.5}$$

$$= \boxed{3.66 \times 10^{-5}}$$

$$S = \frac{P}{4\pi r^2} D$$

$$= 0.12$$

$$\Rightarrow \boxed{|E| = 9.49 \text{ V/m}}$$

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Problem #2 (33 points) A 20Km long communication link is setup with two aperture antennas with all components properly matched. If  $P_t=1W$  and  $f=4GHz$  and the transmitted antenna has a gain of 20dB. Determine

- (a) What is the power density at the receiving antenna assuming proper alignment? +8  
 (b) If the noise temperature of the receiver is 500K and the receiver bandwidth is 20MHz, in order to achieve a signal to noise ratio of 30dB, how much gain is required for the receiving antenna? +8  
 (c) What is the minimum area required for the receiving antenna? +8  
 (d) If frequency is changed to 12GHz, with everything else staying the same including the area of the receiving antenna, how far can this communication link be constructed now? +6

$$a) f = 4 \text{ GHz} \Rightarrow \lambda = 0.075 \text{ m}$$

$$S = \frac{P_t}{4\pi R^2} G_t \quad G_t = 20 \text{ dB} \Rightarrow G_t = 100$$

$$= \frac{1}{4\pi (20k)^2} (100)$$

$$= 1.99 \times 10^{-8}$$

$$P_r = S A_{er}$$

$$\frac{P_r}{A_{er}} = 1.99 \times 10^{-8}$$

$$b) T = 500 \text{ K} \quad B = 20 \text{ MHz} \quad \text{SNR} = 30 \text{ dB} \Rightarrow \text{SNR} = 10^3$$

$$P_n = kTB$$

$$= 1.38 \times 10^{-23} (500) (20 \text{ MHz})$$

$$= 1.38 \times 10^{-13}$$

$$\text{SNR} = \frac{P_r}{P_n}$$

$$10^3 = \frac{P_r}{1.38 \times 10^{-13}}$$

$$P_r = 1.38 \times 10^{-10}$$

$$P_r(\text{dB}_m) = P_t(\text{dB}_m) + G_t(\text{dB}) + G_r(\text{dB}) - 20 \log R(\text{km}) - 20 \log f(\text{MHz}) - 32.44$$

$$G_r(\text{dB}) = 10 \log (1.38 \times 10^{-7}) - 10 \log 1000 - 20 + 20 \log 20 + 20 \log 4000 + 32.44$$

$$= 11.9 \text{ dB}$$

$$= 15.5$$

Continued on  
back ↘

$$c) A_e = \frac{\lambda^2}{4\pi} G_r$$

$$= \boxed{6.93 \times 10^{-3} \frac{\text{m}^2}{\text{sr}}}$$

$$d) f = 126 \text{ Hz} \Rightarrow \lambda = 0.025 \text{ m}$$

$$P_r = P_t \frac{1}{(R\lambda)^2} A_e A_c$$

$P_r, P_t, A_e, A_c$  all stay the same

$\Rightarrow$  Take ratio of  $\frac{P_{r, \text{old}}}{P_{r, \text{new}}}$

$$1 = \frac{(R_{\text{new}} \lambda_{\text{new}})^2}{(R_{\text{old}} \lambda_{\text{old}})^2}$$

$$R_{\text{new}}^2 = \left( \frac{R_{\text{old}} \lambda_{\text{old}}}{\lambda_{\text{new}}} \right)^2$$

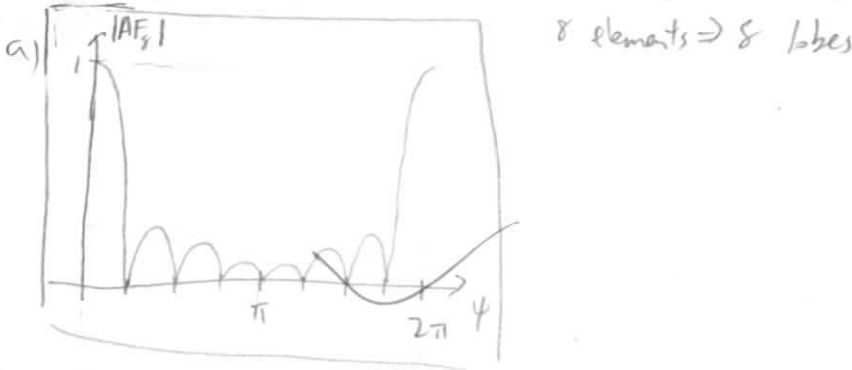
$$= \left( \frac{20 \text{ km} (0.075)}{0.025} \right)^2$$

$$\boxed{R_{\text{new}} = 60 \text{ km}}$$

+33

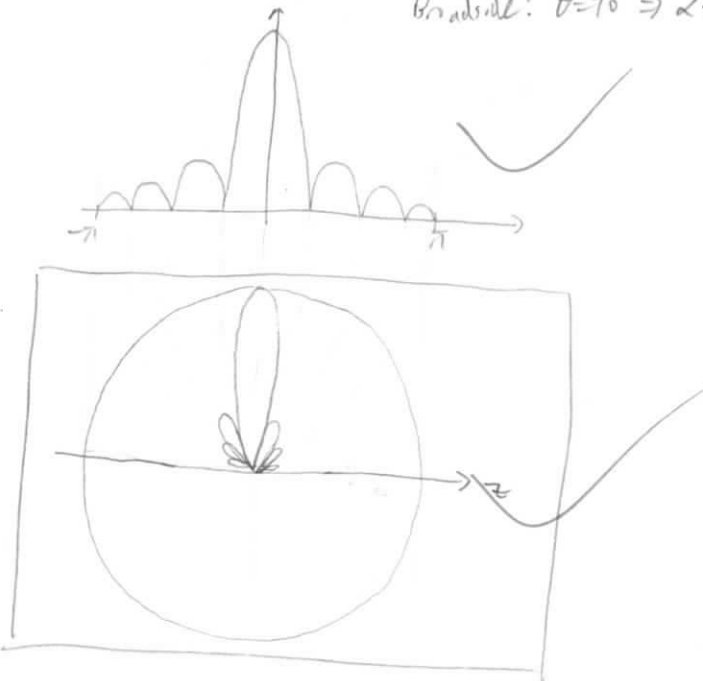
Problem #3 (33 points). For a linear, uniform array of 8 elements.

- (a) Plot the universal array factor. +8
- (b) Assume that the spacing between adjacent elements is half-wavelength spacing, plot the array factor of the radiation pattern in polar plot when the antenna beam is pointing to the broadside. +8
- (c) What if the beam is pointing to +60 degree away from broadside? What is the progressive phase shift in this case? Show the radiation pattern in polar plot derived from universal factor. +8
- (d) Design the array in the manner of an ordinary endfire array and a Hansen-Woodyard array. +9



b)  $d = \frac{\lambda}{2} \Rightarrow \beta d = \pi$

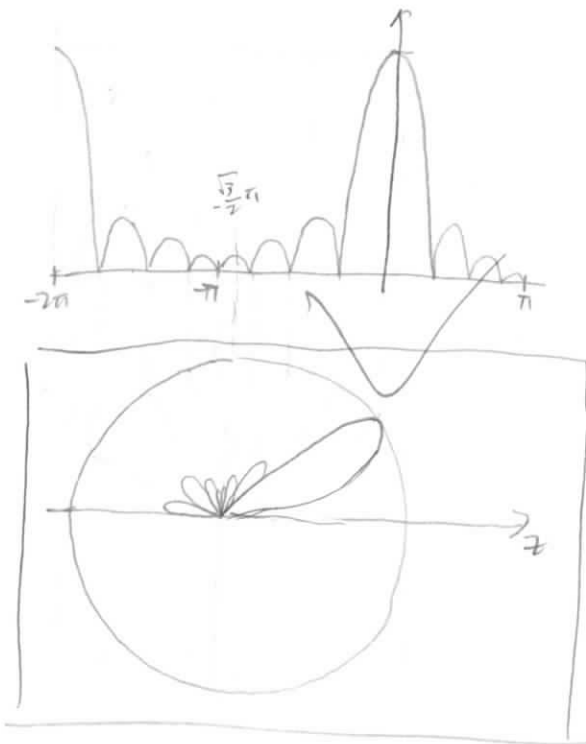
Broadside:  $\theta = 90^\circ \Rightarrow \alpha = 0$



c)  $+60^\circ$  away from broadside  $\Rightarrow \theta = 30^\circ$

$$\alpha = -\beta d \cos \theta_0$$

$$= -\frac{\sqrt{3}}{2} \pi$$



d) Ordinary Endfire:  $\theta_0 = 0^\circ, 180^\circ$

$$\alpha = \pm \beta d$$

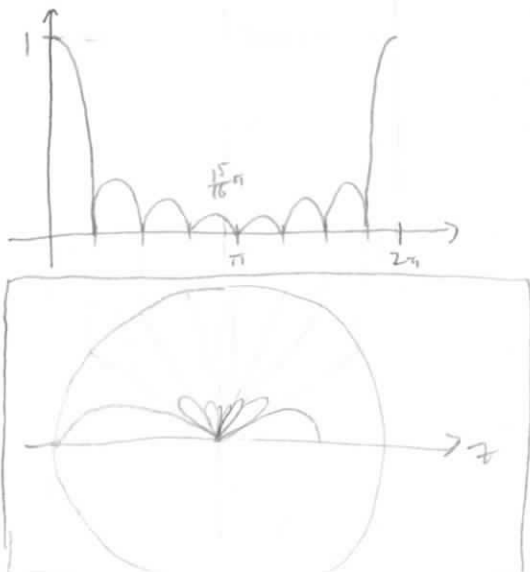
$= \pm \pi \Rightarrow$  grating lobes (decrease  $d$ )

$$z \beta \lambda \leq 2\pi - \frac{\pi}{N}$$

$$d \leq \frac{\lambda}{2} \left(1 - \frac{1}{2N}\right)$$

$$= \frac{15}{32} \lambda \Rightarrow \beta d = \frac{15}{16} \pi$$

$$\Rightarrow \alpha = \pm \frac{15}{16} \pi$$



Hansen-Woodyard Endfire:  $\alpha = \pm(\beta d + \delta)$

$$\delta \approx \frac{\pi}{N}$$

$$= \frac{\pi}{8}$$

$$= \pm \frac{\pi}{8} \pi$$

