EE162A Midterm Solution
Spring 2016

Name:

Grade:

Problem #1 (34 points). A dipole antenna and a circular loop antenna are both designed to operate at 1MHz. Each antenna has a maximum dimension of 1 meter and is made of 4mm diameter copper wire. (1) What are the directivities of both antennas (2) Compute the impedance of both antennas (3) Find the radiation efficiency of both antennas. (4) If both antennas are radiating the same amount of power of 100Watt, find the strength of the maximum electric field at 10 meter away for both antennas and compare them.

Sol:

(1)

$$D_{dipole} = 1.5 = 1.76dB; D_{loop} = 1.5 = 1.76dB$$

(2)

Dipole length L=1m, $a = 2 \times 10^{-3}m, f = 10^6 Hz$

a) Short dipole:

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{\pi f \mu}{\sigma}} = \sqrt{\pi \times 10^6 \times 4\pi \times \frac{10^{-7}}{5.8 \times 10^7}} = 2.61 \times 10^{-4}\Omega/\square$$

$$R_r = 20\pi^2 \left(\frac{L}{\lambda}\right)^2 = 20\pi^2 \times \left(\frac{1}{300}\right)^2 = 2.19 \times 10^{-3}\Omega$$

$$R_L = \frac{R_s}{2\pi a} \cdot \frac{L}{3} = 2.61 \times \frac{10^{-4}}{2\pi \times 2 \times 10^{-3}} \times \frac{1}{3} = 6.92 \times 10^{-3}\Omega$$

$$X_A = -\frac{120}{\frac{\pi L}{\lambda}}\left[\ln\left(\frac{L}{2a}\right) - 1\right] = -\frac{120}{\pi \times \frac{1}{300}} \times \left[\ln\left(\frac{1}{4 \times 10^{-3}}\right) - 1\right] = -5.18 \times 10^4\Omega$$

$$Z_A = R_r + R_L + jX_A = 2.19 \times 10^{-3} + 6.92 \times 10^{-3} + j(-5.18 \times 10^4)$$
$$= 9.11 \times 10^{-3} - j5.18 \times 10^4\ \Omega$$

If you assume the dipole is ideal,

$$R_L = \frac{R_s}{2\pi a} \cdot L = 2.61 \times \frac{10^{-4}}{2\pi \times 2 \times 10^{-3}} = 20.76 \times 10^{-3}\Omega$$

$$R_r = 80\pi^2 \left(\frac{L}{\lambda}\right)^2 = 80\pi^2 \times \left(\frac{1}{300}\right)^2 = 8.76 \times 10^{-3}\Omega$$

$$Z_A = R_r + R_L + jX_A = 8.76 \times 10^{-3} + 20.76 \times 10^{-3} + j(-5.18 \times 10^4)$$
$$= 29.52 \times 10^{-3} - j5.18 \times 10^4\ \Omega$$

b) Loop: radius b=L/2=0.5m

$$R_r = 31200\left(\frac{S}{\lambda^2}\right)^2 = 31200 \times \left(\pi \times \frac{0.5^2}{300^2}\right)^2 = 2.38 \times 10^{-6}\ \Omega$$

$$R_L = \frac{L_m}{2\pi a} R_s = \frac{2\pi \times 0.5}{2\pi \times 2 \times 10^{-3}} \times 2.61 \times 10^{-4} = 0.065\ \Omega$$

$$L = \mu b\left[\ln\left(\frac{8b}{a}\right) - 2\right] = 3.52 \times 10^{-6}\ H$$

$$Z_A = R_r + R_L + j\omega L = 2.38 \times 10^{-6} + 0.065 + 22.1j = 0.065 + 22.1j\ \Omega$$

If you assume that $= \frac{1}{2\pi}$,

$$R_r = 31200\left(\frac{S}{\lambda^2}\right)^2 = 31200 \times \left(\pi \times \frac{0.5^2/\pi^2}{300^2}\right)^2 = 0.024 \times 10^{-6}\ \Omega$$

$$R_L = \frac{L_m}{2\pi a} R_s = \frac{2\pi \times 0.5/\pi}{2\pi \times 2 \times 10^{-3}} \times 2.61 \times 10^{-4} = 0.02\ \Omega$$

$$L = \mu b\left[\ln\left(\frac{8b}{a}\right) - 2\right] = 0.89 \times 10^{-6}\ H$$

$$Z_A = R_r + R_L + j\omega L = 0.24 \times 10^{-6} + 0.02 + 5.6j = 0.02 + 5.6j\ \Omega$$

(3)

$$\eta_{dipole} = \frac{R_r}{R_r + R_L} = \frac{2.19}{2.19 + 6.92} = 24\%,$$

If you assume the dipole is ideal instead of short,

$$\eta_{dipole} = \frac{R_r}{R_r + R_L} = \frac{8.76}{8.76 + 20.76} = 29.7\%,$$

$$\eta_{loop} = \frac{R_r}{R_r + R_L} = \frac{2.38 \times 10^{-6}}{2.38 \times 10^{-6} + 0.065} = 3.66 \times 10^{-3}\%$$

If you assume that $= \frac{1}{2\pi}$,

$$\eta_{loop} = \frac{R_r}{R_r + R_L} = \frac{0.024 \times 10^{-6}}{0.024 \times 10^{-6} + 0.02} = 1.2 \times 10^{-4}\%$$

(4) The maximum E field for these two antennas are the same.

$$d_{far\ field} = \frac{2D^2}{\lambda} = \frac{1}{150} \ll 10m$$

So this is far field scenario. As a result, simplifications brought about by far field approximation can be applied.

Method I:

In the far field, we have:

$$\left|S_{avg}\right| = \frac{1}{2\eta_0}|E_{max}|^2, \eta_0 = 120\pi$$

$$\left|S_{avg}\right| = \frac{100}{4\pi R^2} \times D = \frac{100}{4\pi 10^2} \times 1.5 = \frac{3}{8\pi}$$

We have

$$|E_{max}| = \sqrt{\frac{3}{8\pi} \times 2 \times 120\pi} = 9.5V/m$$

Method II:

a) dipole:

$$P_r = \frac{1}{2} R_r \cdot |I_A|^2$$

$$|I_A| = \sqrt{\frac{2P_r}{R_r}} = \sqrt{2 \times \frac{100}{2.19 \times 10^{-3}}} = 302.2\ A$$

$$|E_{max}| = \frac{\omega\mu IL}{8\pi r} = \frac{2\pi \times 10^6 \times 4\pi \times 10^{-7} \times 302.2 \times 1}{2 \times 4\pi \times 10} = 9.5\ V/m$$

b) loop:

$$|I_A| = \sqrt{\frac{2P_r}{R_r}} = \sqrt{2 \times \frac{100}{2.38 \times 10^{-6}}} = 9.167 \times 10^3\ A$$

$$|E_{max}| = \eta\beta^2 S \cdot \frac{I}{4\pi r} = 120\pi \times 0.21^2 \times \pi \times 0.5^2 \times \frac{9{,}167 \times 10^3}{4\pi \times 10} = 9.5\ V/m$$

Both methods indicate the same result that they have the same maximum E field at distance of 10m.

Note: the fact that small loop antenna has larger H field (compared with the E field of the same antenna) and small dipole antenna has larger E field (compared with the H field of the same antenna) only applies to near field scenario.

Problem #2 (33 points) A cellphone communication link operates at a center frequency of 2.14GHz and it consists of two vertical half-wave dipole antennas separated by 2 km. The antennas are lossless, the signal occupies a bandwidth of 60 MHz, the system noise temperature of the receiver is 600K, and the desired signal-to-noise ratio is 20dB, (1) what is the minimum transmitter power the link requires? (2) If one uses two horn antennas in place of the dipole antennas for line of sight reception, how large are the typical physical apertures of the horns in order to keep the transmitter power $1/10^{th}$ of the original? (3) If one sticks to half-wave dipole and reduces the frequency to 700MHz, how much transmitter power would it requires now?

Sol:

(1)

$$P_n = kT_{sys}B = 1.38 \times 10^{-23} \times 600 \times 60 \times 10^6 = 4.97 \times 10^{-13}W = -123dBW$$

$$P_r = P_t + G_r + G_t - 20lgR(km) - 20lgf(MHz) - 32.44 = P_t - 100.77$$

$$P_r - P_n = SNR$$

So we have:

$$P_t = P_n + SNR + 100.77 = -123 + 20 + 100.77 = -2.23dBW = 0.6W$$

(2)

$$\Delta G_r + \Delta G_t = 10dB, so\ \Delta G = 5dB$$

$$G_{horn} = G_{dipole} + \Delta G = 2.15 + 5 = 7.15dB$$

$$G \simeq D = 4\pi \cdot \frac{A}{\lambda^2}$$

$$A = \frac{5.188 \times \left(\frac{30}{2.14} \cdot 10^{-2}\right)^2}{4\pi} = 8.11 \times 10^{-3}\ m^2$$

(3)

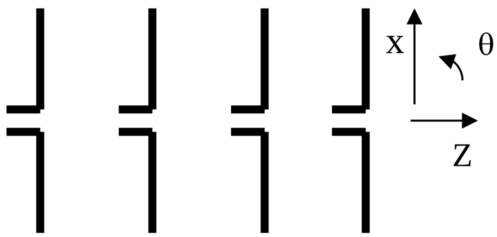
$$L_f = 20lgf, L_{f_1} = 20lg2140 = 66.61dB, L_{f_2} = 20lg700 = 56.9dB$$

$$\Delta L_f = L_{f_1} - L_{f_2} = -9.71dB$$

$$P_t = -2.23 - 9.71 = -11.94dBW = 0.06W$$

Note: $lgF = \log_{10} F$

Problem #3 (33 points). A linear array of four half-wave dipole antennas are placed parallel to each other. (1) find the element pattern in the spherical coordinate system as indicated (2) plot the array factor from the universal factor plot based on uniform amplitude and phase excitation, with half-wavelength spacing. along what direction one sees the maximum radiation? (3) If one desires to steer the antenna beam away from its broadside in the y-z plane by 30 degrees toward the positive Z-axis, what phases one must assign to each antenna element? (4) Stay with the same scanning angle in (3), how much antenna spacing one can increase without seeing the grating lobe?



Sol:

(1)

$$F(\theta_e) = \frac{\cos\left(\frac{\pi}{2} cos\theta_e\right)}{sin\theta_e}$$

According to coordinate transformation (mentioned in discussion class), we have

$\hat{x}cos\theta_e = \hat{x}cos\phi sin\theta$, so $cos\theta_e = cos\phi sin\theta, sin\theta_e = \sqrt{1-(cos\phi sin\theta)^2}$

So we have:

$$F(\theta_e) = \frac{\cos\left(\frac{\pi}{2} cos\phi sin\theta\right)}{\sqrt{1-(cos\phi sin\theta)^2}}$$

(2)

Maximum radiation: y-axis

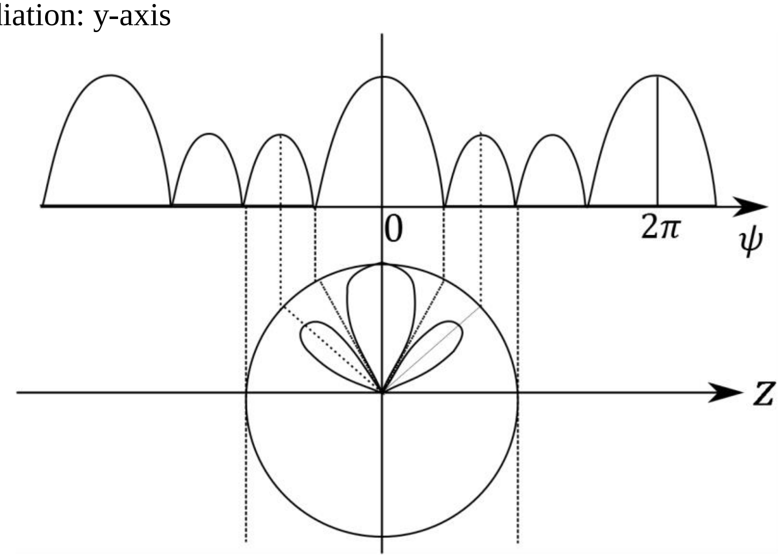


Fig. 3-1: Array factor pattern based on universal array factor

(3)

We have $\theta = 90° - 30° = 60°$, $\psi = \beta d cos\left(\frac{\pi}{3}\right) + \alpha = 0$, so $\alpha = -\pi/2$

The dipole is fed with phase from left to right as: 0, $-\pi/2$, $-\pi$, $-3\pi/2$

(4)

$$d < \frac{\lambda}{1 + |cos\theta_0|} = \frac{\lambda}{1 + \frac{1}{2}} = \frac{2\lambda}{3}$$

$$\Delta d = \frac{2\lambda}{3} - \frac{\lambda}{2} = \frac{\lambda}{6}$$