EE161 Electromagnetic Waves

Fall, 2010

October 12th, 2010
10:00 am – 11:30 am

Midterm #1

Name: Solution

Student ID:

Score:

Problem #1 (35 pts). The electric field of a plane wave propagating in air has the following expression:

$$\mathbf{E}(t) = \hat{x}3\cos(\omega t + 8x - 6z) + \hat{y}5\sin(\omega t + 8x - 6z) + \hat{z}4\cos(\omega t + 8x - 6z) \text{ (V/m)}.$$

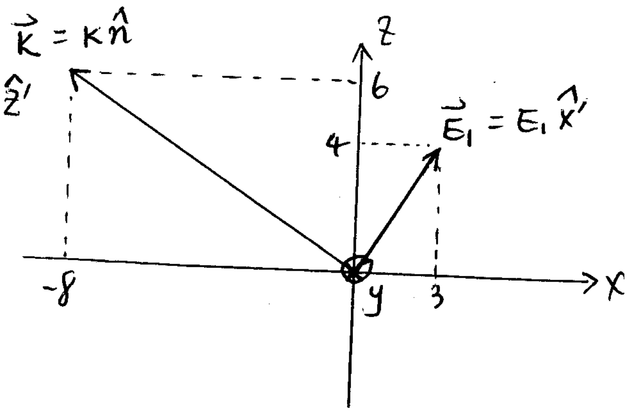
(a) (6 pts) Find the frequency of the wave.

(b) (8 pts) Find the angles between the propagation direction and the x-, y-, z-axis.

(c) (10 pts) Determine the polarization state of the wave.

(d) (11 pts) Find the instantaneous magnetic field.

Solution:



(a) $\vec{k} = k\hat{n} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$

$k_x = -8. \quad k_y = 0. \quad k_z = 6$

$\Rightarrow k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \sqrt{(-8)^2 + 0^2 + 6^2} = 10.$

$\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{10} = \dfrac{\pi}{5}$ (m).

In air $\Rightarrow f = \dfrac{c}{\lambda} = \dfrac{3\times10^8}{\pi/5} = 4.77\times10^8$ (Hz).

(b) $\vec{k} = k\hat{n} = -8\hat{x} + 0\hat{y} + 6\hat{z}$, $k = 10.$

$\Rightarrow \hat{n} = \vec{k}/k = -0.8\hat{x} + 0.6\hat{z}$ (unit vector in wave propagation direction).

x-axis: $\cos^{-1}(\hat{n}\cdot\hat{x}) = \cos^{-1}(-0.8) = 143.13°$

y-axis: $\cos^{-1}(\hat{n}\cdot\hat{y}) = \cos^{-1}(0) = 90°$

z-axis: $\cos^{-1}(\hat{n}\cdot\hat{z}) = \cos^{-1}(0.6) = 53.13°$

(c). Use cosine reference.

$\vec{E}(t) = \hat{x}3\cos(\omega t + 8x - 6z) + \hat{y}5\cos(\omega t + 8x - 6z - 90°) + \hat{z}4\cos(\omega t + 8x - 6z)$

convert to phasor:

$\tilde{E}(x, y, z) = \hat{x}3e^{-j(-8x+6z)} + \hat{y}5e^{-j(-8x+6z)}e^{j(-90°)} + \hat{z}4e^{-j(-8x+6z)}.$

$$\tilde{E} = [3\hat{x} + 4\hat{z}]e^{-j(-8x+6z)} + \hat{y}\,5e^{-j(-8x+6z)}e^{j(-90^\circ)}$$

Define $\hat{x}' = \frac{1}{5}(3\hat{x} + 4\hat{z}) = 0.6\hat{x} + 0.8\hat{z}$.

$\hat{y}' = \hat{y}$.

$\hat{z}' = \hat{n} = -0.8\hat{x} + 0.6\hat{z}$ (propagation direction).

notice that $\hat{x}' \cdot \hat{y}' = 0$.

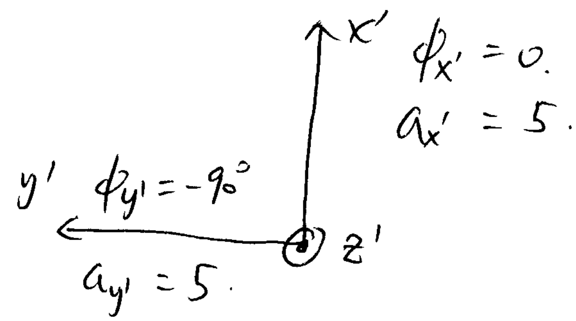
$\hat{x}' \cdot \hat{z}' = (0.6\hat{x} + 0.8\hat{z}) \cdot (-0.8\hat{x} + 0.6\hat{z}) = 0$

$\hat{y}' \cdot \hat{z}' = 0$.

$\Rightarrow$. $\hat{x}'$, $\hat{y}'$, $\hat{z}'$ are a new cartesian coordinate system.

and a plane wave propagates in $\hat{z}'$ direction.

$$\tilde{E} = \hat{x}'\,5e^{-jkz'} + \hat{y}'\,5e^{-jkz'}e^{j(-90^\circ)}.$$



Same magnitude, $\phi_{x'}$ leads $\phi_{y'}$ by 90°. $\Rightarrow$ RHCP.

Right-handed circular polarization.

(d). In x', y', z' coordinates.

$$\tilde{H} = \frac{1}{\eta}\hat{z}' \times \tilde{E} = \frac{1}{\eta}\hat{z}' \times \left[\hat{x}' 5e^{-jkz'} + \hat{y}' 5e^{-jkz'} e^{j(-90^\circ)}\right]$$

$$= \frac{1}{\eta}\left[\hat{y}' 5e^{-jkz'} - \hat{x}' 5e^{-jkz'} e^{j(-90^\circ)}\right]$$

$$= \frac{1}{\eta}\left[\hat{y}' 5e^{-jkz'} + \hat{x}' 5e^{-jkz'} e^{j90^\circ}\right]$$

Convert back to x, y, z coordinates.

$$\tilde{H} = \frac{1}{\eta}\left[\hat{y} 5e^{-j(-8x+6z)} + (0.6\hat{x} + 0.8\hat{z}) 5e^{-j(-8x+6z)} e^{j90^\circ}\right]$$

$$= \frac{1}{\eta}\left[\hat{x} 3e^{-j(-8x+6z)} e^{j90^\circ} + \hat{y} 5 \cdot e^{-j(-8x+6z)} + \hat{z} 4 e^{-j(-8x+6z)} e^{j90^\circ}\right]$$

Convert to time domain:

$$\vec{H}(t) = \frac{1}{377}\left[\hat{x} 3\cos(\omega t + 8x - 6z + 90^\circ) + \hat{y} 5\cos(\omega t + 8x - 6z) + \hat{z} 4\cos(\omega t + 8x - 6z + 90^\circ)\right] \quad (\text{A/m}).$$

$$\omega = 2\pi f = 2\pi \times 4.77 \times 10^8 = 3 \times 10^9 \ \text{rad/s}$$

Problem #2 (25 pts). A 15 MHz plane wave propagates in $+z$-direction within an infinite medium with $\varepsilon_r = 4$, $\mu_r = 1$, $\sigma = 70$ (S/m). At the reference plane $z = 0$, the instantaneous electric field is measured to be 100 mV/m with reference phase of 0° at the moment $t = 0$, and the field is y-polarized.

(a) (15 pts) Find the time-domain expression of both the electric field and magnetic field $\mathbf{E}(z, t)$ and $\mathbf{H}(z, t)$.

(b) (5 pts) The skin depth of this medium.

(c) (5 pts) At what distance z the electrical field intensity is 50 μV/m.

(a) $f = 15\times10^6$ (Hz) $\quad \omega = 2\pi f = 30\pi\times10^6$ (rad/s)

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon_0\varepsilon_r} = \frac{70}{30\pi\times10^6 \times \frac{1}{36\pi}\times10^{-9}\times 4} = 2.1\times10^4 \gg 1.$$

$\Rightarrow$ good conductor!

$$\alpha = \sqrt{\pi f\mu\sigma} = \sqrt{\pi\cdot15\times10^6\times4\pi\times10^{-7}\times70} = 64.38 \ (\text{Np/m})$$

$$\beta = \alpha = 64.38 \ (\text{rad/m}).$$

$$\eta_c = (1+j)\frac{\alpha}{\sigma} = (1+j)\frac{64.38}{70} = 0.92(1+j) \ (\Omega)$$
$$= 1.30\, e^{j45^\circ} \ (\Omega)$$

y-polarized & $E_0 = 100$ mV/m.

$$\tilde{E}(z) = \hat{y} E_0 e^{-\alpha z} e^{-j\beta z} = \hat{y} E_0 e^{-64.38z} e^{-j64.38z}$$

$$\vec{E}(t) = \hat{y}\, 100\, e^{-64.38z}\cos(30\pi\times10^6 t - 64.38z) \ (\text{mV/m})$$

$$\tilde{H}(z) = \frac{1}{\eta_c}\hat{z}\times\tilde{E}(z) = -\frac{1}{\eta_c}\hat{x}E_0 e^{-\alpha z}e^{-j\beta z}$$
$$= -\hat{x}\frac{E_0}{1.30e^{j45^\circ}} e^{-64.38z} e^{-j64.38z}$$
$$= \hat{x}\, 76.92\, e^{j135^\circ} e^{-64.38z} e^{-j64.38z}$$

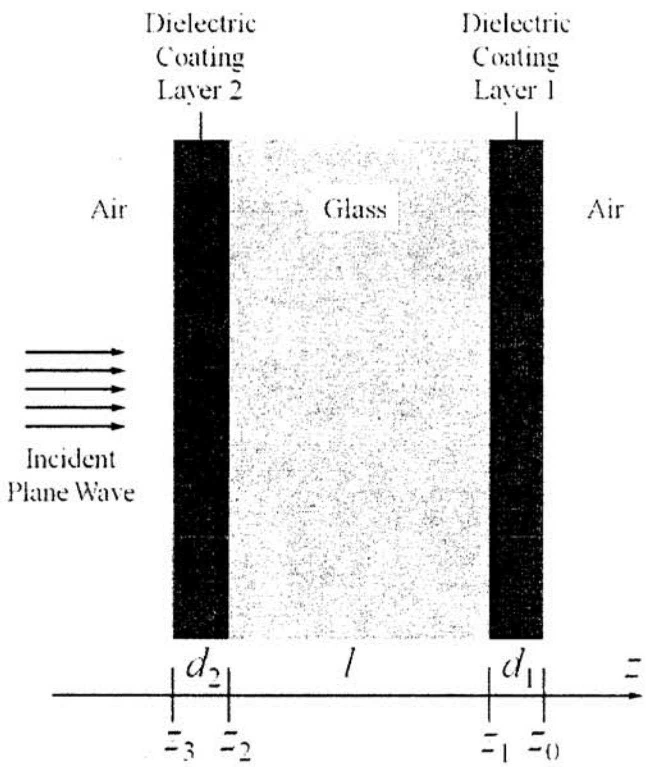
$$\vec{H}(t) = \hat{x}\, 76.92\, e^{-64.38z}\cos(30\pi\times10^6 t - 64.38z + 135^\circ) \ (\text{mA/m})$$

(b). $\delta_s = \frac{1}{\alpha} = 0.0155$ (m).

(C). $\frac{50\,\mu V/m}{100\ mV/m} = \frac{50 \times 10^{-6}}{100 \times 10^{-3}} = 0.5 \times 10^{-3} = -33$ dB

$z = \frac{-33}{-8.68\alpha} = 0.0591$ (m).

Problem #3 (40 pts). To design the anti-flaring coating of a glass for green light spectrum (frequency is 570 Tera Hertz), the same type of dielectric coating is placed at both sides of the glass (see figure below). The glass is a lossless non-magnetic medium with dielectric constant of 9. The thickness of glass l is arbitrary. A plane wave is normally incident from air on the glass (note that air also occupies the region $z > z_0$).

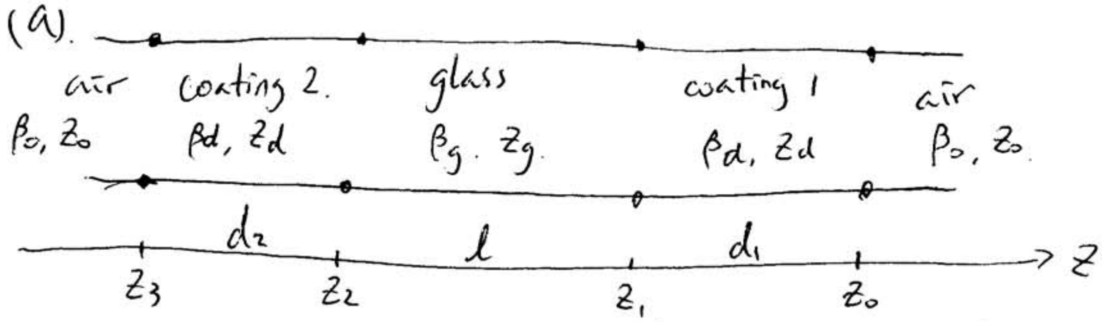


(a) (10 pts) Draw the transmission line equivalence of the original structure (you need to find the phase constants and characteristic impedances).

(b) (15 pts) Determine the dielectric constant of the coating material and the thickness of the coating layer 1 (d_1) and layer 2 (d_2). Assume the coating material is perfect dielectric, and keep the coating layers as thin as possible.

(c) (15 pts) Assume the thickness of the glass is l = 2 mm, and a violet light (frequency is 710 Tera Hertz) is normally incident on the same coated glass. Find the reflection coefficient at $z = z_3$.

(a).



In air: $\beta_0 = \omega\sqrt{\mu\varepsilon_0} = \dfrac{2\pi f}{c} = \dfrac{2\pi \times 570 \times 10^{12}}{3 \times 10^{8}} = 1.194 \times 10^{7}$ (rad/m).

$Z_0 = \eta_0 = 377 (\Omega)$

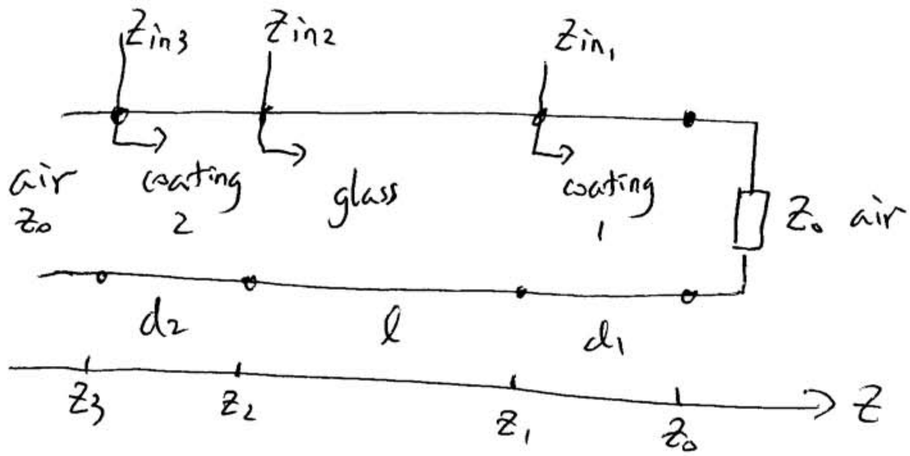
In glass: $\beta_g = \beta_0 \cdot \sqrt{\varepsilon_{rg}} = 1.194 \times 10^{7} \times \sqrt{9} = 3.582 \times 10^{7}$ (rad/m)

$Z_g = \eta_0 / \sqrt{\varepsilon_{rg}} = \dfrac{377}{\sqrt{9}} = 126 (\Omega)$.

In coating: $\beta_d = \beta_0 \cdot \sqrt{\varepsilon_{rd}}$, $Z_d = \eta_0 / \sqrt{\varepsilon_{rd}}$.

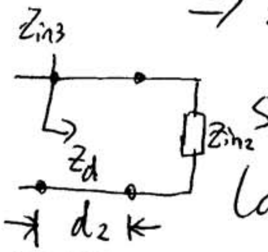
ε_{rd} is the dielectric constant of the coating medium.

(b).

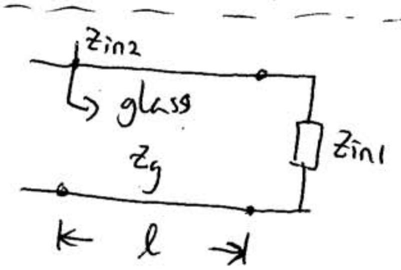


Define the input impedances at $z = z_1, z_2, z_3$ as Z_{in1}, Z_{in2}, Z_{in3}.

Anti-glare $\Rightarrow$ no reflection @ $z = z_3 \Rightarrow Z_{in3} = Z_0$.

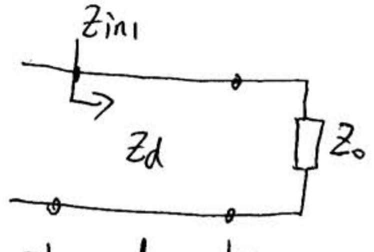


$\Rightarrow Z_{in2}$ must be constant for any glass thickness l so that after the impedance transformation of coating layer #2, Z_{in3} is constant $Z_{in3} = Z_0$.



$$Z_{in2} = Z_g \cdot \frac{Z_{in1} + j Z_g \tan \beta_g l}{Z_g + j Z_{in1} \tan \beta_g l}$$

Only when $Z_{in1} = Z_g$, Z_{in2} is constant for any l. and under this condition, $Z_{in2} = Z_g$.



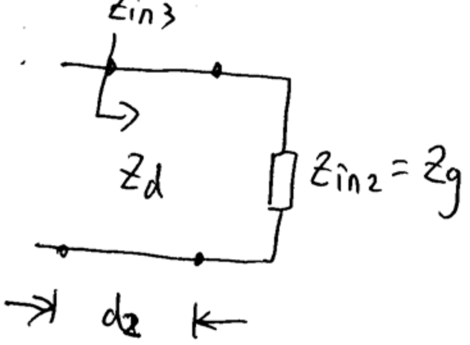
A quarter-wave length impedance transformer can be used to get $Z_{in1} = Z_g$.

$\Rightarrow \varepsilon_{rd} = \sqrt{\varepsilon_{r_{air}} \cdot \varepsilon_{r_{glass}}} = \sqrt{1 \cdot 9} = 3.$

$$\lambda_0 = \frac{c}{f} = \frac{3 \times 10^8}{5.7 \times 10^{14}} = 0.526 \times 10^{-6}\ (m) = 526\ (nm)$$

(wave length in air, in nano meter).

$$d_2 = \frac{\lambda_0}{4\sqrt{\varepsilon_{rd}}} = \frac{526}{4 \times \sqrt{3}} = 75.9\ (nm).$$



Again, a quarter-wave length impedance transformer can be used to get $Z_{in3} = Z_0$

$$\varepsilon_{rd} = \sqrt{\varepsilon_{r\,air} \cdot \varepsilon_{r\,glass}} = \sqrt{1 \cdot 9} = 3$$

$$d_1 = \frac{\lambda_0}{4\sqrt{\varepsilon_{rd}}} = 75.9 \ (nm).$$

In (a), $\beta_d = \beta_0 \cdot \sqrt{\varepsilon_{rd}} = 1.194 \times 10^7 \times \sqrt{3} = 2.068 \times 10^7 \ (rad/m)$

$$Z_d = \eta_0 / \sqrt{\varepsilon_{rd}} = \frac{377}{\sqrt{3}} = 218 \ (\Omega).$$

(c). For violet light, $f = 710 \ THz = 7.1 \times 10^{14} \ (Hz)$

$$\beta_0 = \omega\sqrt{\mu_0 \varepsilon_0} = \frac{2\pi f}{c} = \frac{2\pi \times 7.1 \times 10^{14}}{3 \times 10^8} = 1.487 \times 10^7 \ (rad/m).$$

$$\beta_d = \beta_0 \cdot \sqrt{\varepsilon_d} = 1.487 \times 10^7 \times \sqrt{3} = 2.576 \times 10^7 \ (rad/m).$$

$$Z_d = \eta_0 / \sqrt{\varepsilon_{rd}} = 377/\sqrt{3} = 218 \ (\Omega).$$

$$\beta_g = \beta_0 \cdot \sqrt{\varepsilon_{rg}} = 1.487 \times 10^7 \times \sqrt{9} = 4.461 \times 10^7 \ (rad/m).$$

$$Z_g = \eta_0 / \sqrt{\varepsilon_{rg}} = 377/\sqrt{9} = 126 \ (\Omega).$$

$d_2 = 75.9 \ nm$, $\tan\beta_d \cdot d_2 = \tan(2.576 \times 10^7 \times 75.9 \times 10^{-9}) = -2.472$

$$Z_{in1} = Z_d \cdot \frac{Z_0 + jZ_d \tan\beta_d \cdot d_2}{Z_d + jZ_0 \tan\beta_d \cdot d_2} = 139.1 + j55.7 \ (\Omega).$$

$l = 2\,mm, \quad \tan\beta_g \cdot l = \tan(4.461\times10^{7} \times 2\times10^{-3}) = -2.832$

$$Z_{in2} = Z_g \frac{Z_{in1} + jZ_g \tan\beta_g \cdot l}{Z_g + jZ_{in1} \tan\beta_g \cdot l} = 845 - j16.4 \ (\Omega).$$

$d_1 = 75.9\,nm, \quad \tan\beta_d \cdot d_1 = -2.472.$

$$Z_{in3} = Z_d \frac{Z_{in2} + jZ_d \cdot \tan\beta_d \cdot d_2}{Z_d + jZ_{in2} \tan\beta_d \cdot d_2} = 380.1 - j234.7 \ (\Omega).$$

$$\Gamma(z_3) = \frac{Z_{in3} - Z_0}{Z_{in3} + Z_0} = 0.0914 - j0.282 = 0.296\, e^{j(-72^\circ)}$$