EE161 Midterm II

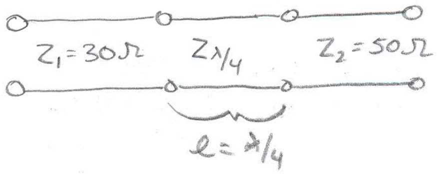
Name: Solution

Grade:

ID:

1) (35pts) Using the parallel plate waveguide model to (1) calculate the dimensions of a quarter-wave microstrip line transformer on a 50mil (1mil= 1/1000inch) thick Teflon substrate (εr=2.05). The transformer is supposed to transform an impedance of 30Ohm to 50Ohm. (2) If the voltage of the wave propagating on this line is 1V, find the surface current density on the top and bottom plates and how much power the wave carries (3) If the parallel plate waveguide is enclosed with PMC on both sides, does it change the boundary conditions of the one without PMC? (4) With the PMC model in (3), determine the highest frequency one can use before a second mode appear? (5) If the second mode appears, plot the field template of the longitudinal field component for this mode.

(1) Quarter-Wave Transformer (10 points)



$$Z_{\lambda/4} = \sqrt{Z_1 Z_2}$$
$$= \sqrt{30 \cdot 50}$$
$$= 38.73\ \Omega$$

Parallel-Plate Waveguide Model

$$Z = \frac{\eta h}{W}$$

$$h = 50\text{mil} = 1.27\text{mm}$$

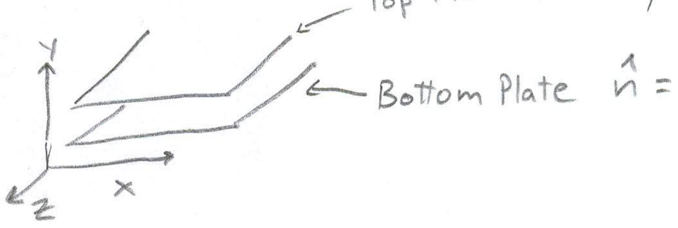
$$\Rightarrow W = \frac{\eta h}{Z} = \frac{\eta_0 h}{\sqrt{\varepsilon_r}\, Z}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377$$

$$W = \frac{(377)(0.00127)}{\sqrt{2.05}\ (38.73)} = 0.00863 = \boxed{8.63\text{ mm}}$$

(2) Surface Current Density

$$J_s = \hat{n} \times \vec{H}$$



For a parallel-plate waveguide:

$$\vec{H} = \hat{x}\frac{V_0}{\eta h} e^{-jkz} \quad \text{(TEM wave)}$$

Top surface:

$$\vec{J_s} = \hat{y} \times \vec{H} = -\hat{z}\frac{V_0}{\eta h} e^{-jkz} = -\hat{z}\frac{(1)\sqrt{2.05}}{(377)(0.00127)} e^{-jkz}$$

$$= \boxed{-\hat{z}\, 3 e^{-jkz}} \quad \text{or } @\ z=0 \quad \boxed{\vec{J_s} = -3\hat{z}}$$

Similarly,

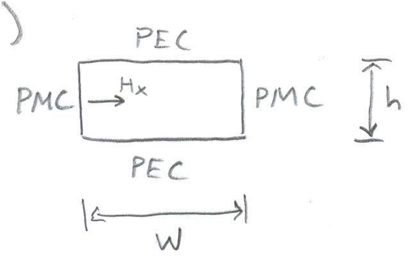
Bottom Surface:

$$\vec{J}_s = -\hat{y} \times \vec{H} = \boxed{\hat{z}\, 3 e^{-jkz}} \text{ or } @\, z=0 \; \boxed{\vec{J}_s = 3\hat{z}}$$

Power

$$P = \frac{1}{2}\frac{V_0^2}{Z_0} = \frac{1}{2}\frac{1}{38.73} = 0.0129 \text{ W} = \boxed{12.9 \text{ mW}}$$

(3)



For the dominant TEM mode, only Hx exists. Since this is normal to the PMC boundary, it does not affect the fields.

(4) TM waves

$$E_z \propto \cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y \rightarrow TM_{01} \text{ can exist.}$$

TE waves

$$H_z \propto \sin\frac{m\pi}{a}x \cos\frac{n\pi}{b}y \rightarrow TE_{10} \text{ can exist}$$

$$f_{c,TM_{01}} = \frac{c}{2\pi\sqrt{\varepsilon_r}}\sqrt{\left(\frac{\pi}{b}\right)^2} = \frac{(3\times10^8)}{2\pi\sqrt{2.05}}\left(\frac{\pi}{0.00127}\right) = 82.5 \text{ GHz}$$

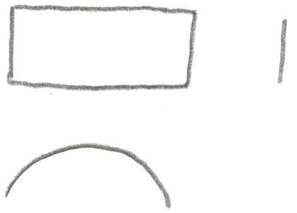
$$f_{c,TE_{10}} = \frac{c}{2\pi\sqrt{\varepsilon_r}}\left(\frac{\pi}{a}\right) = 12.14 \text{ GHz}$$

Next mode would be the $\underline{TE_{10}}$ mode at $\underline{12.14 \text{ GHz}}$.

(5) For the TE_{10} mode, the longitudinal component, H_z, will be:

$$H_z \propto \sin\frac{m\pi}{W}x \cos\frac{n\pi}{h}y$$

H_z



2) (35pts) A TE wave propagating in a dielectric-filled rectangular waveguide of unknown permittivity has dimensions a=5cm and b=3cm. If the x-component of its electric field in time domain is given by

$$E_x=-36\cos(40\pi x)\sin(100\pi y)\sin(2.4\pi \times 10^{10}t-52.9\pi z)$$

Determine: (1) the mode number and the cutoff frequency,
(2) dielectric constant of the material in the guide
(3) the mathematical expression for the complete H components
(4) the power it carries on this mode
(5) other modes which may also propagate
(6) how long does it takes if one transmits information from one end of a 100m long waveguide to the other end with all these modes?

(1). Mode number

$$40\pi = \frac{m\pi}{a} = \frac{m\pi}{0.05} \qquad 100\pi = \frac{n\pi}{b} = \frac{n\pi}{0.03}$$

$$\boxed{m=2} \qquad \boxed{n=3}$$

$$\boxed{TE_{23}}$$

We meed to know ε_r to determine the cutoff frequency:

$$k^2 = \omega^2\varepsilon_r\varepsilon_0\mu_0 = \beta^2 + k_c^2 \qquad k_c^2 = (40\pi)^2 + (100\pi)^2$$

$$\beta^2 = (52.9\pi)^2$$

$$\varepsilon_r = \frac{1}{\varepsilon_0\mu_0}\left(\frac{\beta^2+k_c^2}{\omega^2}\right) \qquad \omega = 2.4\pi\times10^{10}$$

$$= 2.25$$

$$f_{c,TE_{23}} = \frac{c}{2\pi\sqrt{\varepsilon_r}}\sqrt{(40\pi)^2+(100\pi)^2} = \boxed{10.77\ GHz}$$

(2) From part (1), $\boxed{\varepsilon_r = 2.25}$

(3) Complete H-fields

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y} \qquad \text{where } H_z = A_{mn}\cos\frac{m\pi}{a}x\cos\frac{n\pi}{b}y$$

so $$|E_x| = \frac{\omega\mu n\pi}{k_c^2\, b}A_{mn} = 36 \Rightarrow A_{23} = \frac{36k_c^2 b}{3\omega\mu\pi} = \frac{(40^2\pi^2+100^2\pi^2)(0.03)}{3(2.4\pi\times10^{10})(4\pi\times10^{-7})\pi}$$

$$= 0.1385$$

TE_{23}

$$\boxed{H_z = -0.1385 \cos 40\pi x \cos 100\pi y \sin(2.4\pi \times 10^{10} t - 52.9\pi z)}$$

$$H_x = \frac{j\beta m\pi}{k_c^2 a} A_{mn} \sin(\sim) \cos(\sim) \sin(\sim)$$

$$\boxed{H_x = -0.0253 \sin(40\pi x) \cos(100\pi y) \sin(2.4\pi \times 10^{10} t - 52.9\pi z)}$$

$$H_y = \frac{j\beta n\pi}{k_c^2 b} A_{mn} \cos(\sim) \sin(\sim) \sin(\sim)$$

$$\boxed{H_y = -0.0631 \cos(40\pi x) \sin(100\pi y) \sin(2.4\pi \times 10^{10} t - 52.9\pi z)}$$

(4)

$$P = \frac{1}{2} \mathrm{Re} \int_S \vec{E} \times \vec{H}^* \cdot d\vec{S}$$

S is the cross-section of the waveguide

$\rightarrow d\vec{S} = \hat{z}\, dx\, dy$

For a TE mode

$$\vec{E} \times \vec{H}^* = \hat{x} E_y H_z^* + \hat{z}(E_x H_y^* - E_y H_x^*)$$

$$\Rightarrow P = \frac{1}{2} \mathrm{Re}\left\{ \int_0^b \int_0^a (E_x H_y^* - E_y H_x^*)\, dx\, dy \right\}$$

$$E_y = \frac{\omega\mu n\pi}{k_c^2 b} A_{mn} \sin(\sim) \cos(\sim) \sin(\sim)$$

$$= -14.4 \sin 40\pi x \cos 100\pi y \sin(2.4\pi \times 10^{10} t - 52.9\pi z)$$

$$E_x H_y^* = (-36)(-0.0631) \cos^2(40\pi x) \sin^2(100\pi y)$$

$$E_y H_x^* = (-14.4)(-0.0253) \sin^2(40\pi x) \cos^2(100\pi y)$$

$$\int_0^b \int_0^a E_x H_y^*\, dx\, dy = \left[\frac{x}{2} - \frac{1}{4(40\pi)} \sin 80\pi x\right]_0^a \left[\frac{y}{2} - \frac{1}{4(100\pi)} \sin 200\pi y\right]_0^b (-36 \cdot -0.0631)$$

$$= \left(\frac{a}{2}\right)\left(\frac{b}{2}\right)(-36 \cdot -0.0631)$$

Likewise, $\displaystyle \int_0^b \int_0^a E_y H_x^*\, dx\, dy = \left(\frac{a}{2}\right)\left(\frac{b}{2}\right)(-14.4 \cdot -0.0253)$

$$P = \frac{1}{4}\left[ab(2.2716 - 0.364)\right]$$

$$= 2.86 \times 10^{-3}\ W = \boxed{2.86\ mW}$$

(5) All modes that have $m < 2$ and $n < 3$ propagate.
Other modes that propagate:

TE_{40}, TE_{41}, TE_{42}, TM_{41}, TM_{42}

TE_{33}, TM_{33}

(6) One must find the slowest group velocity of all propagating modes:

$$V_g = \frac{c\beta}{k_0}$$ → To minimiz V_g, k_c must be maximum since $\beta = \sqrt{k^2 - k_c^2}$.

Mode TE_{33} or TM_{33} has largest k_c of the propagating modes

$$V_g = \frac{(3\times10^8)\sqrt{k^2 - k_{c,TM33}}}{k} = 4.714\times10^7\ m/s$$

$$t = \frac{100\ m}{V_g} = \boxed{2.12\times10^{-6}\ s = 2.12\ \mu m}$$

3) (30pts) An air filled rectangular waveguide cavity has a width (a) to height (b) to length (d) ratio of 4:3:5. (1) If the cavity is designed to resonate at 10GHz, what is the smallest possible dimension of this waveguide cavity? (2) Draw the field template of this mode (3) Calculate the quality factor of this cavity if it is made of copper with conductivity of 5.8x10⁷ S/m. (4) Redo (1) and (3) for the resonating frequency of 60GHz.

(a) Using the fundamental mode, one can minimize the dimensions of the cavity:

$$f_{TE_{101}} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2}$$

The ratio of 4:3:5 will result in $d = 1.25a$:

$$\rightarrow 10\times10^9 = \frac{c}{2\pi}\sqrt{\pi^2\left[\left(\frac{1}{a}\right)^2 + \left(\frac{1}{1.25a}\right)^2\right]}$$

$$10\times10^9 = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{1.25a}\right)^2}$$

$$\left(\frac{20\times10^9}{c}\right)^2 = \frac{(1.25)^2a^2 + a^2}{a^4(1.25)} = \frac{(1.25)^2 + 1}{(1.25)^2 a^2}$$

$$a = 0.0192 = 1.92\text{ cm}$$

So,

$$\boxed{\begin{aligned} a &= 1.92\text{ cm} \\ b &= 1.44\text{ cm} \\ d &= 2.4\text{ cm} \end{aligned}}$$

(b) Field Templates

TE modes:

$$H_z \propto \cos\frac{m\pi}{a}x \cos\frac{n\pi}{b}y \sin\frac{p\pi}{d}z$$

$$E_x \propto \cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y \sin\frac{p\pi}{d}z$$

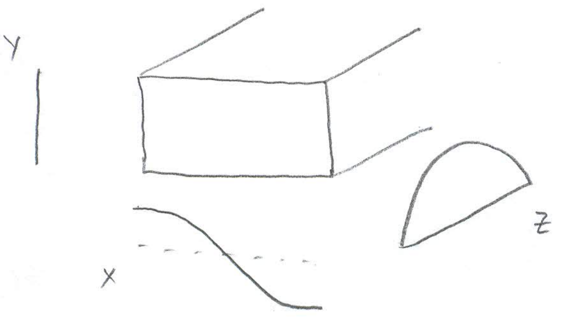
$$E_y \propto \sin\frac{m\pi}{a}x \cos\frac{n\pi}{b}y \sin\frac{p\pi}{d}z$$

$$H_x \propto \sin\frac{m\pi}{a}x \cos\frac{n\pi}{b}y \cos\frac{p\pi}{d}z$$

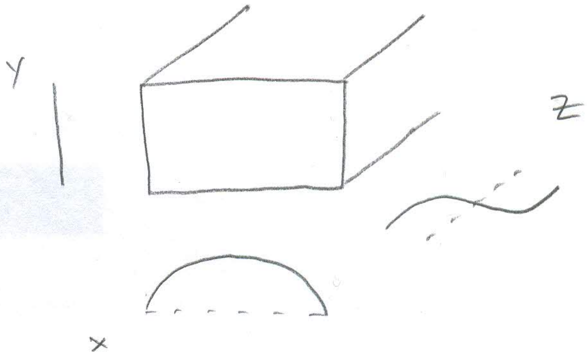
$$H_y \propto \cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y \cos\frac{p\pi}{d}z$$

For the TE_{101} mode

$H_z \propto \cos\frac{\pi}{a}x \, \sin\frac{\pi}{d}z$



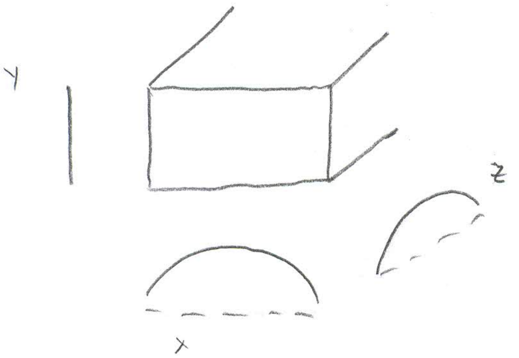
$H_x \propto \sin\frac{\pi}{a}x \, \cos\frac{\pi}{d}z$



$E_x = 0$

$H_y = 0$

$E_y \propto \sin\frac{\pi}{a}x \, \sin\frac{\pi}{d}z$



(3) Quality Factor

$$Q = \frac{(kad)^3 b Z}{2\pi^2 R_s} \; \frac{1}{(2a^3b + 2bd^3 + a^3d + ad^3}$$

$$R_s = \sqrt{\frac{\pi f \mu_0}{\sigma}}$$

$$= \boxed{9147.8}$$

(4) Same as (1) and (3) except at 60GHz

a = 3.2 mm
b = 2.4 mm
d = 4mm

$\boxed{Q = 3,735}$