EE161 Midterm II

Name:

Grade:

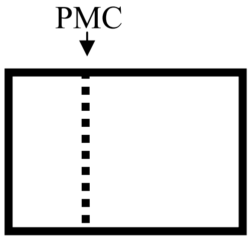
1) Using the parallel plate waveguide model to (1) calculate the dimensions of a quarter-wave microstrip line transformer on a 50mil thick Teflon substrate (εr=2.05). The transformer is supposed to transform an impedance of 25Ohm to 100Ohm. (2) If the voltage of the wave propagating on this line is 1V, find the surface current density on the top and bottom plates and how much power the wave carries (3) Draw field templates for both E and H field for the dominant mode (4) If the parallel plate waveguide is enclosed with PMC on both sides, does it change the boundary conditions? (5) With the PMC model in (4), determine the highest frequency one can use before a second mode appear?

2) A TM wave propagating in a dielectric-filled waveguide of unknown permittivity has dimensions a=5cm and b=3cm. If the x-component of its electric field is given by

$$E_x = -36\cos(40\pi x)\sin(100\pi y)\sin(2.4\pi \times 10^{10} t - 52.9\pi z)$$

Determine: (1) the mode number and the cutoff frequency,
(2) dielectric constant of the material in the guide
(3) the expression for H_y
(4) the power it carries on this mode
(5) other modes which may also propagate
(6) how long does it takes if one transmits information from one end of a 100m long waveguide to the other end with all these modes?

3) Draw the field templates for all the magnetic field components of TE_{20} mode and TM_{12} mode in a rectangular waveguide. If one wishes to insert a thin film of PMC into the waveguide like shown in the following figure, where would he place it without disturbing the original field distribution for either case above?

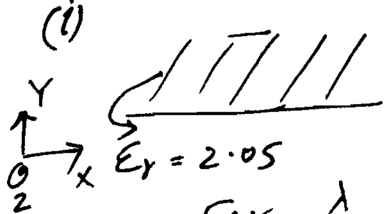


EE 161 Midterm II Solution

problem #1

to use parallel plate waveguide Model

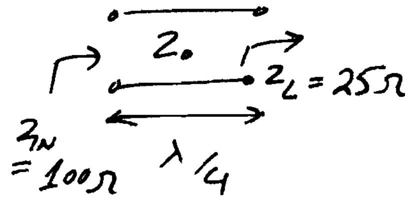
(i)



$d = 50\,\text{mil} = \frac{50}{1000} \times 25.4 = 1.27\text{ mm}.$

$\varepsilon_r = 2.05$

For $\frac{\lambda}{4}$ section, $Z_{IN} = \frac{Z_0^2}{Z_L}$



$\Rightarrow Z_0 = \sqrt{Z_{IN} \times Z_L} = 50\Omega.$ (1)

As, $Z_0 = \frac{\eta_0 d}{\sqrt{\varepsilon_r}\, W} \Rightarrow W = \left(\frac{377 \times 1.27\text{mm}}{\sqrt{2.05} \times 50}\right)^{-1} = 6.688\text{mm}$ (1)

(ii) $V_0 = 1$ volts.

$\vec{J}_{s,top}$, $\vec{J}_{s,bottom}$, $P = ?$

$(\vec{J}_s)_{top} = \hat{n} \times \vec{H} = \hat{Y} \times \hat{x} \frac{V_0}{\eta d} = -\hat{z} \frac{V_0 \sqrt{\varepsilon_r}}{\eta_0 d} = -2.99\,\hat{z}$ (A/m)

(1)

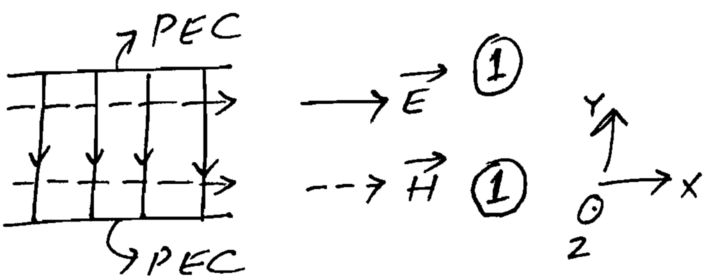
sim

$(\vec{J}_s)_{bottom} = +2.99\,\hat{z}$ (A/m)

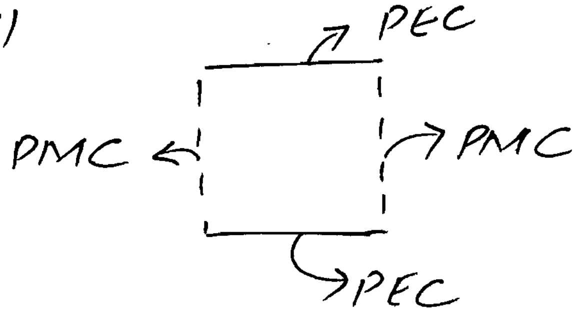
$P = \frac{1}{2} |V_0|^2 \frac{\sqrt{\varepsilon_r}\, W}{\eta_0 d} = \frac{1}{2} \frac{|V_0|^2}{Z_0} = \frac{1}{2} \frac{(1)^2}{50} = 10\text{ mW}.$ (1)

(iii) field templates for both $\vec{E}$ & $\vec{H}$ field for Dominant Mode

Dominant mode in parallel plate waveguide: TEM



(iv)



For PMC,

$$\vec{H}_{tangential} = 0$$

for parallel plate waveguide only H_x exists. 1 why

As H_x is normal to PMC, therefore PMC doesnot Impose any boundary condition on H_x.

Hence field distribution will remain <u>unaffected</u>. 1

(v) First Mode: Dominant Mode: TEM.

Second Mode: TE_{10}

$$f_{c,10} = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{1\pi}{6.688\times10^{-3}}\right)^2 + \left(\frac{0\,\pi}{1.27\times10^{-3}}\right)^2}$$

$$= \frac{1}{2\pi} \cdot \frac{1}{\sqrt{4\pi\times10^{-7}\times2.05\times8.85\times10^{-12}}} \times \left(\frac{1000\,\pi}{6.688}\right)$$

$$= 15.66\ GHz. \quad 1$$

Hence operating frequency should be below $f_{c,10} = 15.66$ GHz, So that only Dominant mode i.e; TEM mode propagates. 1 → operating frequency range=?

Problem #2: (Out of 10)

$a = 0.05\,m$

$b = 0.03\,m$

$E_x = -36\cos(40\pi x)\sin(100\pi y)\sin(2.4\pi\times 10^{10}t - 52.9\pi z)$

(1)

$\frac{m\pi}{a} = 40\pi \Rightarrow m = 40\times 0.05 = 2$ 1

$\frac{n\pi}{b} = 100\pi \Rightarrow n = 100\times 0.03 = 3$ 1

(2)

$\beta = 52.9\pi = 166.2, \mu_r = 1 \quad , \omega = 2.4\pi\times 10^{10} \Rightarrow f = 12\,GHz$

$\frac{\omega}{c}\sqrt{\mu_r\varepsilon_r} = \sqrt{\beta^2 + \left(\frac{2\pi}{0.05}\right)^2 + \left(\frac{3\pi}{0.03}\right)^2} \Rightarrow \varepsilon_r = \frac{(3\times 10^8)^2}{(2.4\pi\times 10^{10})^2}\left((52.9\pi)^2 + (40\pi)^2 + (100\pi)^2\right)$

$\Rightarrow \varepsilon_r = 2.25$ 1

$f_{23} = \frac{3\times 10^8}{2\sqrt{2.25}}\sqrt{\left(\frac{2}{0.05}\right)^2 + \left(\frac{3}{0.03}\right)^2} = 10.77\,GHz$ 1

(3)

$Z_{TM} = \eta\sqrt{1-(f_c/f)^2} = \frac{120\pi}{\sqrt{2.25}}\sqrt{1-(10.77/12)^2} = 110.84$

$H_y = \frac{E_x}{Z_{TM}} = -0.3248\cos(40\pi x)\sin(100\pi y)\sin(2.4\pi\times 10^{10}t - 52.9\pi z)$ 1

(4)

$\vec{S}\cdot\hat{z} = \frac{1}{2}E_x H_y^* - \frac{1}{2}E_y H_x^*$,

$E_x = -36\cos(40\pi x)\sin(100\pi y)e^{-j52.9\pi z}$

$H_y = -0.3248\cos(40\pi x)\sin(100\pi y)e^{-j52.9\pi z}$ 1.5

$E_y = \underbrace{\left(\frac{0.05\times 3}{2\times 0.03}\right)(-36)}_{-90}\sin(40\pi x)\cos(100\pi y)e^{-j52.9\pi z}$

$H_x = -\frac{E_y}{Z_{TM}}$

$\vec{S}\cdot\hat{z} = \frac{1}{2}\frac{36^2}{110.84}\cos^2(40\pi x)\sin^2(100\pi y) + \frac{1}{2}\frac{90^2}{110.84}\sin^2(40\pi x)\cos^2(100\pi y)$

$P_{ave} = \int_0^b\int_0^a \vec{S}\cdot\hat{z}\,dx\,dy = \frac{0.03\times 0.05}{4}\frac{1}{2}\frac{1}{110.84}(36^2+90^2) = 15.89\times 10^{-3}\,W$ 0.5

(5)

just three propagating made:

$$f_{mn} = \frac{3\times10^8}{2\sqrt{2.25}} \sqrt{\left(\frac{m}{0.05}\right)^2 + \left(\frac{n}{0.03}\right)^2} \quad 1$$

(6)

$d = 100m$

$$V_{g_{10}} = \frac{3\times10^8}{\sqrt{2.25}} \sqrt{1 - \left(f_{10}/f\right)^2} \quad 1$$

$$f_{10} = 2\ GHz$$

$$V_{g_{10}} = 1.972\times10^8\ m/s$$

$$T = \frac{d}{V_{g_{10}}} = 509.1\ ns \quad 1$$

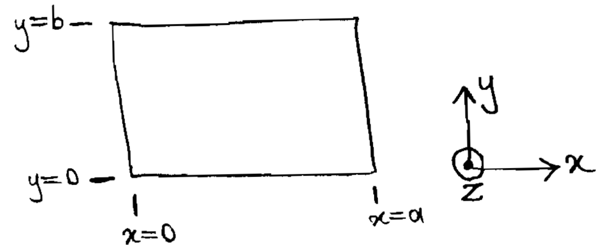
Problem #3:

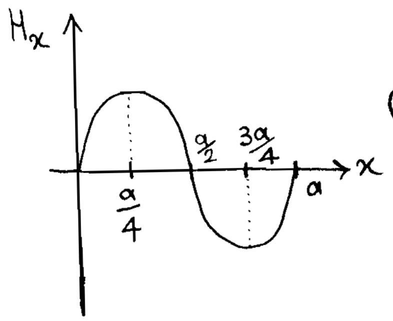
TE_{20} mode: (Out of 10)

$$H_x = H_{x_0}\sin\left(\frac{2\pi x}{a}\right)\cos\left(\frac{0\pi y}{b}\right)e^{-j\beta z} = H_{x_0}\sin\left(\frac{2\pi x}{a}\right)e^{-j\beta z}$$

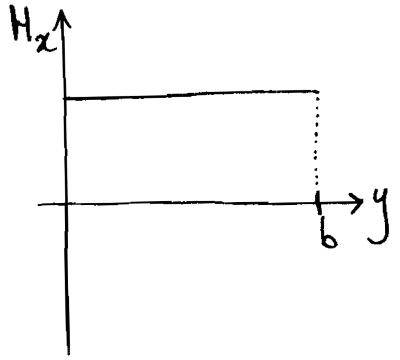
$$H_y = H_{y_0}\cos\left(\frac{2\pi x}{a}\right)\sin\left(\frac{0\pi y}{b}\right)e^{-j\beta z} = 0$$

$$H_z = H_{z_0}\cos\left(\frac{2\pi x}{a}\right)\cos\left(\frac{0\pi y}{b}\right)e^{-j\beta z} = H_{z_0}\cos\left(\frac{2\pi x}{a}\right)e^{-j\beta z}$$

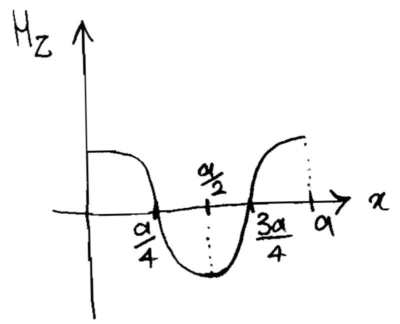




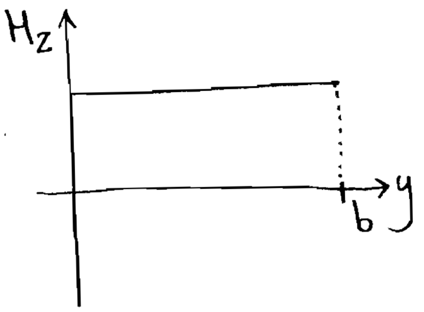
1



1



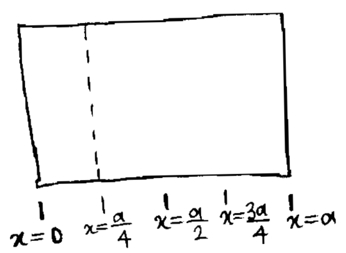
1



1

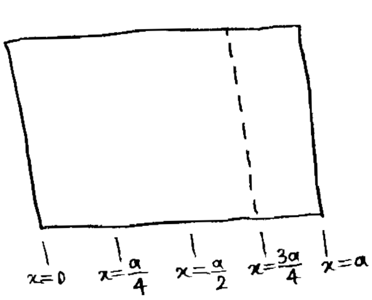
For PMC $H_{tan} = 0$ and $|H_n| = max$:

$$\left.\begin{array}{l} H_{tan} = H_z \\ H_n = H_x \end{array}\right\} \Rightarrow x = \frac{a}{4} \text{ or } x = \frac{3a}{4}$$



1

or

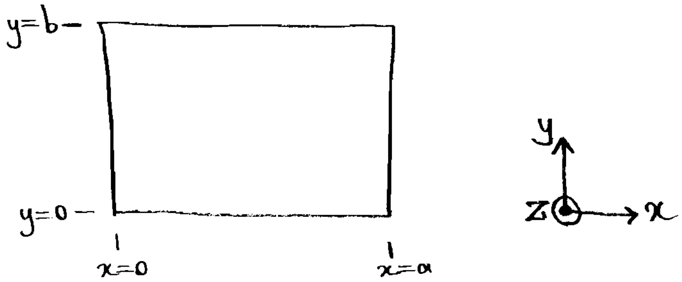


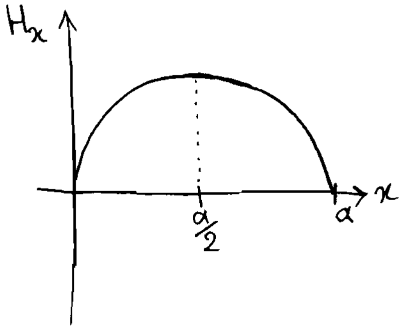
TM_{12} mode: ~~(Out of 10)~~

$$H_x = H_{x_0} \sin\left(\frac{1\pi x}{a}\right) \cos\left(\frac{2\pi y}{b}\right) e^{-j\beta z}$$

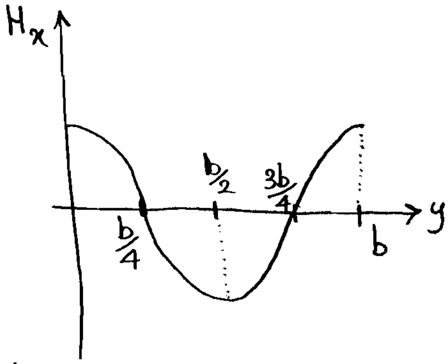
$$H_y = H_{y_0} \cos\left(\frac{1\pi x}{a}\right) \sin\left(\frac{2\pi y}{b}\right) e^{-j\beta z}$$

$$H_z = 0$$

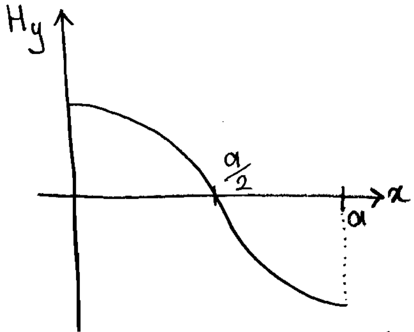




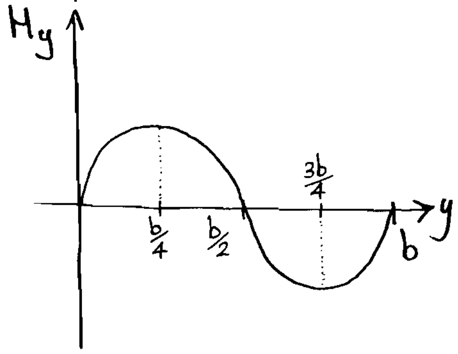
1



1



1

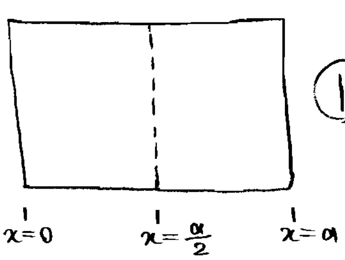


1

For PMC $H_{tan} = 0$ and $|H_n| = max$:

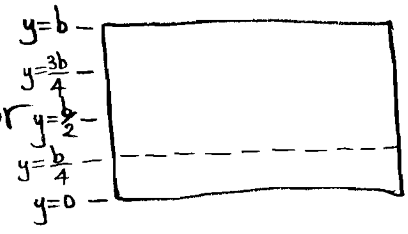
$$\left.\begin{aligned} H_{tan} &= H_y \\ H_n &= H_x \end{aligned}\right\} \Rightarrow x = \frac{a}{2}$$

$$\left.\begin{aligned} H_{tan} &= H_x \\ H_n &= H_y \end{aligned}\right\} \Rightarrow y = \frac{b}{4} \text{ or } y = \frac{3b}{4}$$



1

or



or