Midterm Exam 2 of EE161

Spring, 2005

Name: ______________

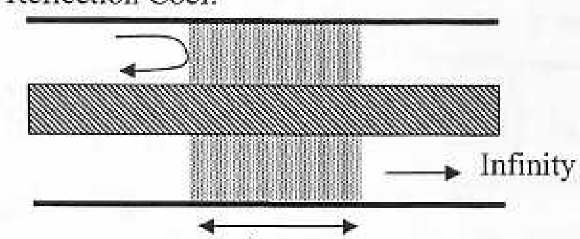
Score: 79

Problem #1. (25 Points) In a coaxial transmission line (TEM mode only), the inner and outer conductors are with radius 0.5mm and 2 mm. The operating frequency is 3 GHz. (1) Find the characteristic impedance Z_0 when the coax is filled with air (2)) Find the wave number k and the guiding wavelength λ_g. (3) Redo (1) and (2) for the case when the coax is filled with Teflon (ε_r=2.55). (4) If a 3cm long section in this coax is filled with Teflon, what is the reflection coefficient when observed on the first air/dielectric surface, considering an infinite long section on the other end?

(21)

$a = .5\times10^{-3}m$ $b = 2\times10^{-3}m$

$f = 3\times10^{9}Hz$



1) $Z_0 = 60 \ln \frac{2\times10^{-3}m}{.5\times10^{-3}m} = 60 \ln 4 = 83.18\,\Omega = Z_0$

$c = f\lambda$

$\lambda_g = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 0.1^m = \lambda_g$

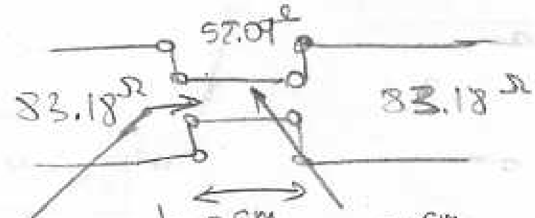
2) $k = \frac{2\pi}{\lambda_g} = \frac{2\pi}{.1} = 62.8\ rad/m = k$

$\varepsilon_r = 2.55$

3) $Z_0 = \frac{60}{\sqrt{2.55}} \ln 4 = 52.09\,\Omega = Z_0$ $\lambda_g = \frac{\lambda_0}{\sqrt{2.55}}$

$\lambda_g = 0.0626^m$

$k = \frac{2\pi}{\lambda_g} = 100.33\ \frac{rad}{m} = k$



(−4)

4) $\lambda_g \approx 6^{cm}$

$\Rightarrow d \approx \frac{\lambda}{2}$

$\Rightarrow Z_{in} = 83.18\,\Omega$

$\Rightarrow$ Approximatly no reflection

I don't have the formulas to get the exact answer b=

Problem #2. (25 Points) For a rectangular waveguide, the length a = 5cm and width b = 2cm. (1) Find the cutoff frequencies for the lowest four modes including both TM and TE modes (2) for single-mode operation, how shall we limit the excitation frequency (3) What is the phase constant β, guiding wavelength λ_g, the phase velocity v_p and group velocity v_g for the dominant mode at 6 GHz. (4) Consider a signal propagates along the waveguide with two frequency components at respectively 6.0GHz and 6.5 GHz. They have the same phase at the excitation position; find out how much phase difference between these two components after they travels about 5 cm.

18

$$f_c = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} \qquad f_{10} = \frac{c}{2a} \qquad f_{01} = \frac{c}{2b} \qquad f_{11} = \frac{2ab}{\sqrt{a^2+b^2}}$$

$$f_{10} = \frac{3\times10^8\times10}{2\times5\times10^{-2}} = 3\times10^9\,Hz = \boxed{3\text{ GHz} = f_{10}}$$

$$f_{01} = \frac{3\times10^8}{2\times2\times10^{-2}} = \boxed{7.5\text{ GHz} = f_{01}}$$

$$f_{20} = \frac{3\times10^8}{2}\sqrt{\left(\frac{2}{5\times10^{-2}}\right)^2} = 6\times10^9\,Hz = \boxed{6\text{ GHz} = f_{20}}$$

$$f_{11} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{5\times10^{-2}}\right)^2+\left(\frac{1}{2\times10^{-2}}\right)^2} = \boxed{8.08\text{ GHz} = f_{11}}$$

1) => Lowest modes are: $TE_{10}, TE_{20}, TE_{01}, TE_{11}, TM_{11}$

2) Single Mode operation → $3\text{ GHz} < f < 6\text{ GHz}$

only TE_{10} mode

3) $f = 6\times10^9\,Hz$ TE_{10}

$$k = \frac{2\pi f}{c} = \frac{2\pi\times6\times10^9}{3\times10^8} = 40\pi$$

$$k_c = \frac{\pi}{a} = \frac{\pi}{5\times10^{-2}} = 20\pi$$

$$\beta = \sqrt{k^2-k_c^2} = \sqrt{1600\pi^2-400\pi^2} = \sqrt{1200\pi^2} = 10\pi\sqrt{12} = 20\pi\sqrt{3} = \boxed{108.83\ \frac{rad}{m} = \beta_{10}}$$

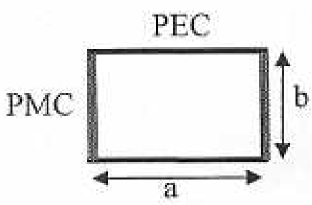
$$\lambda_g = \frac{2\pi}{\beta} = \boxed{0.0577\,m = \lambda_{g10}}$$

$$v_p = \frac{\omega}{\beta_{10}} = \frac{2\pi f}{\beta_{10}} = \boxed{1.73\times10^7\ \frac{m}{s} = v_{p10}} \quad -1$$

$$v_g = \frac{c^2}{v_p} = \boxed{5.20\times10^9\ \frac{m}{s} = v_{g10}} \quad -1$$

23

Problem # 3 (25 points) Perfect Magnetic Conductor (PMC) is a counter-part of Perfect Electrical Conductor under the dual relationship of electric field and magnetic field. For PMC, it satisfies the boundary condition that the tangential magnetic field and normal electric field must equal to zero. For an air filled waveguide shown as below, the top and bottom of the waveguide is made of PEC, while the two sides are made of PMC. (a) What is the dominant mode propagating in this waveguide? (b) What are the first TE mode and its cut-off frequency? (c) What transverse field components exist for this TE mode? (d) Draw the field template (variation) in the cross-section for all the electric field components of this mode.

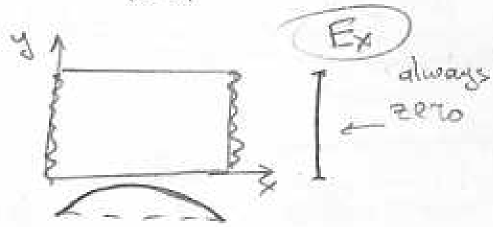


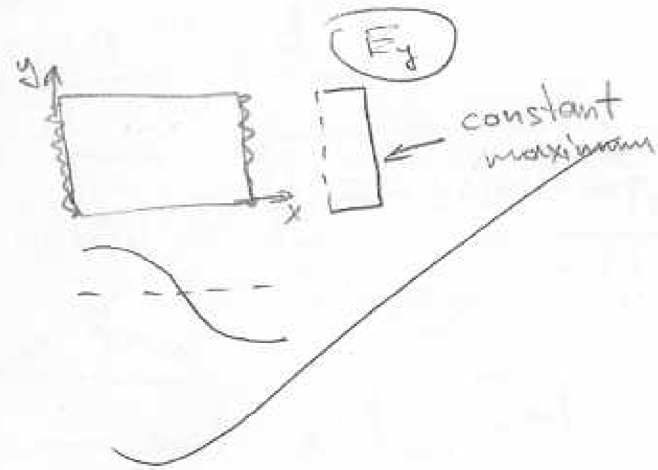
(a>b)

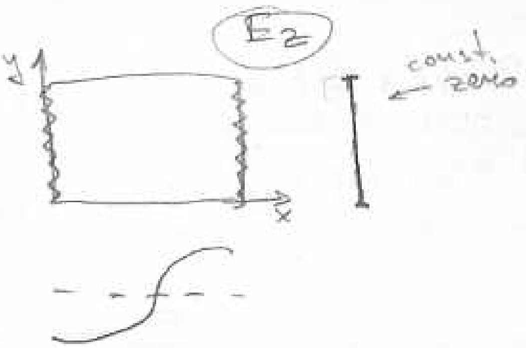
a) Dominant mode here is TEM

b) TE_{10} $f_{10} = \frac{c}{2a}$ -2

d)







c) for TE_{10} mode there exist E_y, H_x

E_x and H_y are always zero

17

Problem #4 (25 points) For an air-filled rectangular waveguide with inner dimension 0.9 in by 0.4 in. (1) If one is designing a cavity resonating at 8 GHz by placing shorting plates in both end of the same waveguide, what is the minimum length of the waveguide (2) What is the second and the third lowest resonant frequencies and their associated modes? (3) Redo the above calculations for the same waveguide filled with dielectrics $\varepsilon_r = 4$.

$f_{res} = 8 \text{ GHz} = \frac{c}{\sqrt{\varepsilon_r}\, 2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$ ($\sqrt{\varepsilon_r} \to 1$)

for TE_{101} $\left(2\,\frac{8\times10^9}{3\times10^8}\right)^2 = \frac{1}{(0.9\times2.54\times10^{-2})^2} + \frac{1}{d^2}$

1) $\Rightarrow$ $d = 0.03278$ m $= 1.29$ in $= d$

2) $f_{r_{011}} = \frac{c}{2}\sqrt{\left(\frac{1}{0.4\times2.54\times10^{-2}}\right)^2 + \left(\frac{1}{.03278}\right)^2}$ = ~~1.546×10^{10} Hz $= f_{r_{10}}$~~ 3rd

2nd $\to f_{r_{201}} = 1.390\times10^{10}$ Hz $= f_{r_{201}}$

$f_{r_{111}} =$ ~~1.679×10^{10} Hz~~ $\leftarrow$ not 3rd or 2nd

−4

3) If $\varepsilon_r = 4$, using above formulas

$d = 0.010279$ cm $= 0.405$ in

$f_{101} = 8$ GHz $\leftarrow$ 1st

~~$f_{011} = 1.038\times10^{10}$ Hz~~ $\leftarrow$ 3rd

$f_{201} = 9.81\times10^9$ Hz $\leftarrow$ 2nd

$f_{111} = 1.088\times10^{10}$ Hz

−4