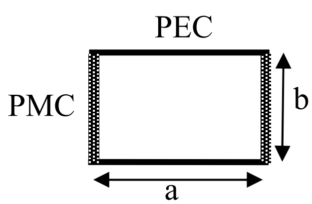
Midterm Exam 2 of EE161

Fall, 2005

Name: _________________

Score: _________________

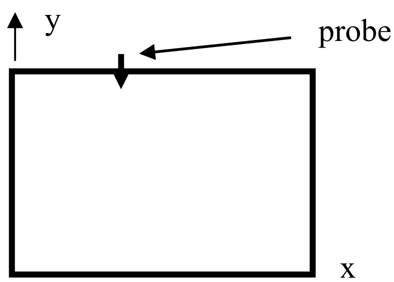
Problem #1. (25 points) Perfect Magnetic Conductor (PMC) is a counter-part of Perfect Electrical Conductor under the dual relationship of electric field and magnetic field. For PMC, it satisfies the boundary condition that the tangential magnetic field and normal electric field must equal to zero. For an air filled waveguide shown as below, the top and bottom of the waveguide is made of PEC, while the two sides are made of PMC. The dimensions are a=5cm, b=3cm. (a) Find the dominant mode propagating in this waveguide? (b) If this waveguide supports TEM mode, what is the characteristic impedance? (c) To keep the single-mode operation, how should one limit the operating frequency? (d) If the waveguide is completely filled with a medium with dielectric constant of 4, answer question (c) again.



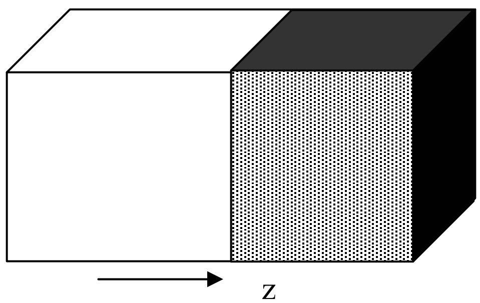
(a>b)

Problem #2. (25 Points) A rectangular waveguide has a length a = 5cm and width b = 3cm. (1) Find the cutoff frequencies for the lowest four modes including both TM and TE modes and sort them in order (2) What is the phase constant β, guiding wavelength λ_g, the phase velocity $\boldsymbol{v}_p$ and group velocity $\boldsymbol{v}_g$ for the dominant mode at 3.5 GHz. (3) Consider a signal propagates along the waveguide with two frequency components at respectively 3.5GHz and 4.0 GHz. They have the same phase at the excitation position; find out how much phase difference between these two components at the observation position which is about 5 cm away from the excitation plane. (4) If a 10ns wide pulse modulated on a carrier of 5.5 GHz is transmitted in the waveguide, at what distance from the excitation plane you may observe two separate pulses?

Problem # 3 (25 points) One needs to understand the waveguide field structure in order to excite a particular waveguide mode or mount semiconductor devices. For example, if we insert a current probe into a rectangular waveguide to excite the TE_{21} mode like in the following figure, we have to make sure the probe is at the maximum E field in the direction of probing. This involves (1) draw the templates for all the E field components on the cross-section for TE_{21} mode. (2) decide where on the top wall of the waveguide one needs to insert the probe, assuming the waveguide dimensions (a=5cm, b=3cm)



Problem #4. (25 points) An air-filled rectangular cavity has a dimension a = 5 cm, b = 3 cm, d=5.5cm, (1) Find the lowest resonance frequency and mode of this cavity. (2) Find the resonant frequency again if the cavity is completely filled with a medium with dielectric constant ε_r=4 (3) What are the guide wavelengths for the above two cases. (4) If the cavity is filled partially with the same medium in Z direction to tune the resonant frequency as shown in the figure, from equal phase analysis, choose the minimum length we should fill in order to have a TE_{10} mode to resonate at 6GHz.



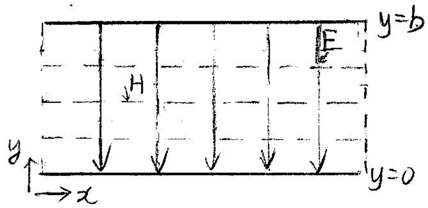
Example Midterm 2 Solution

EE161, Spring 2006

#1. (a) TEM

(b) TEM mode: solve for $\vec{E}$: $\nabla\phi^2 = 0$

$\phi = 0|_{y=0}$, $\phi = V_0|_{y=b}$



$\Rightarrow \phi = \frac{V_0}{b} y$

$\Rightarrow \vec{E} = -\nabla y = -\frac{V_0}{b}\hat{y}$

$\Rightarrow \vec{E}$ field is constant, has only y component

Similarly, $\vec{H}$ field is constant, has only x component.

Shown in the figure.

Express: $\vec{E} = -E_0\hat{y}$, $\vec{H} = H_0\hat{x}$, $Z_{TEM} = \eta \Rightarrow \frac{E_0}{H_0} = \eta$

$$\left.\begin{aligned} V &= -\int_0^b \vec{E}\cdot d\vec{y} = E_0 \cdot b. \\ I &= \int_0^a \vec{H}\cdot dx = H_0 \cdot a \end{aligned}\right\} \Rightarrow Z_0 = \frac{V}{I} = \frac{E_0}{H_0}\cdot\frac{b}{a} = \eta\frac{b}{a}$$

(c) First higher-order mode is TE_{10} mode, since $a > b$

$H_{z,10} = H_{10}\sin\frac{\pi}{a}x$

$$K_c^2 = \left(\frac{\pi}{a}\right)^2 \Rightarrow f_c = \frac{K_c}{2\pi\sqrt{\mu_0\varepsilon_0}} = \frac{C_0}{2a} = \frac{3\times10^8 m/s}{2\times 5cm} = 3\times10^9 Hz$$

(d) $$f_c' = \frac{K_c}{2\pi\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r}} = \frac{f_c}{\sqrt{\varepsilon_r}} = 1.5\ GHz$$

#2. (1) Cuttoff frequency of TEmn / TMmn mode

$$f_c = \frac{K_c}{2\pi\sqrt{\mu_0\varepsilon_0}} = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}} \cdot \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$TE_{10} : \quad f_{c,TE10} = \frac{3.0\times10^8 m/s}{2} \times \frac{1}{5cm} = 3\ GHz$$

$$TE_{01} : \quad f_{c,TE01} = \frac{3.0\times10^8 m/s}{2} \times \frac{1}{3cm} = 5\ GHz.$$

$$TE_{11}/TM_{11} : \quad f_{c,TE11/TM11} = \frac{3.0\times10^8 m/s}{2} \times \sqrt{\left(\frac{1}{5cm}\right)^2 + \left(\frac{1}{3cm}\right)^2} = 5.83\ GHz$$

(2) Dominant: TE_{10} mode

$$\beta^2 = K_0^2 - K_c^2 = \left(\frac{2\pi\times3.5\times10^9}{3.0\times10^8}\right)^2 - \left(\frac{\pi}{5\times10^{-2}}\right)^2 = 1425.6$$

$$\Rightarrow \beta = 37.76 \quad (rad/m)$$

$$\lambda_g = \frac{2\pi}{\beta} = 0.166 \quad (m)$$

$$V_p = \frac{w}{\beta} = \frac{2\pi\times3.5\times10^9 rad/s}{37.76\ rad/m} = 5.82\times10^8 m/s.$$

$$V_g = \frac{C_0^2}{V_p} = \frac{(3.0\times10^8 m/s)^2}{5.82\times10^8 m/s} = 1.55\times10^8 m/s.$$

(3) Phase term $\sim e^{-j\beta z} \Rightarrow \phi = \beta z$.

For 3.5 GHz signal, $\beta_1 = 37.76$

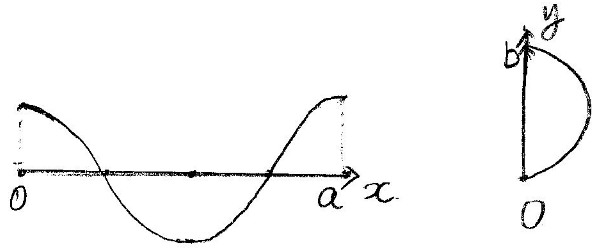
For 4.0 GHz signal, $\beta_2 = \sqrt{\left(\frac{2\pi\times4\times10^9}{3.0\times10^8}\right)^2 - \left(\frac{\pi}{5\times10^{-2}}\right)^2} = 55.41$

$$\Rightarrow \Delta\phi = (\beta_2 - \beta_1)\cdot z = (55.41 - 37.76)\,rad/m \times 5\,cm = 0.8825\,rad$$

Note: for both 3.5 GHz and 4.0 GHz signal, only TE_{10} mode is excited.

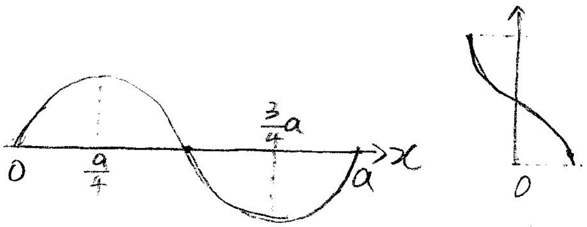
#3 (1) TE_{21} mode has E_x and E_y components.

E_x :



(1) (2)

E_y :



(3) (4)

(2) On the top wall, $E_x|_{y=b} = 0$

Find the maximum E_y position: should be at $x = \frac{a}{4}$ or $\frac{3a}{4}$

$x = 1.25\,cm$, or $x = 3.75\,cm$

#2. (4) Two modes are excited: TE_{10} and TE_{01}, but with different group velocity.

TE_{10} mode: $\beta = \sqrt{\left(\frac{2\pi \times 5.5\times 10^9}{3\times 10^8}\right)^2 - \left(\frac{\pi}{5\times 10^{-2}}\right)^2} = 30.73\pi$

$V_p = \frac{w}{\beta} = 3.58\times 10^8\,m/s$. $\quad V_g = \frac{C_0^2}{V_p} = 2.51\times 10^8\,m/s$

TE_{01} mode: $\beta' = \sqrt{\left(\frac{2\pi \times 5.5\times 10^9}{3\times 10^8}\right)^2 - \left(\frac{\pi}{3\times 10^{-2}}\right)^2} = 15.275\pi$

$V_p' = \frac{W}{\beta'} = 7.2\times 10^8\,m/s$, $\quad V_g' = \frac{C_0^2}{V_p'} = 1.25\times 10^8\,m/s$

The point at which two pulses seperate:

$$\frac{Z}{V_g} - \frac{Z}{V_g'} = 10\,ns \Rightarrow \boxed{Z = 0.40\,m}$$

#4. (1) TE_{101} mode

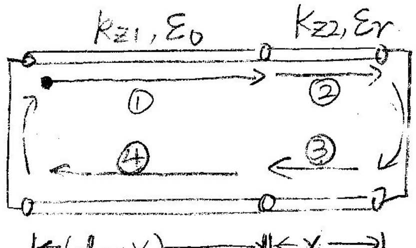
$$f_r = \frac{1}{2\pi\sqrt{\mu_0\varepsilon_0}}\sqrt{\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{d}\right)^2} = \frac{3\times10^8}{2}\times\sqrt{\left(\frac{1}{0.05}\right)^2+\left(\frac{1}{0.055}\right)^2}\ Hz = 4.05\ GHz$$

(2). $$f_r' = \frac{f_r}{\sqrt{\varepsilon_r}} = 2.025\ GHz$$

(3) Guided wavelength in z direction:

$$\lambda_g = 2d = 11\,cm$$

(4) Apply equal phase analysis: when the wave returns to its original point, the phase should be the same!



$\Rightarrow$ The round trip phase delay should be $n\cdot 2\pi$!

$$\underset{①}{k_{z1}\cdot(d-x)} + \underset{②}{k_{z2}\cdot x} + \underset{\Gamma=-1 \text{ for PEC}}{\pi} + \underset{③}{k_{z2}\cdot x} + \underset{④}{k_{z1}(d-x)} + \underset{\Gamma=-1 \text{ for PEC}}{\pi} = n\cdot 2\pi$$

$\Rightarrow 2k_{z2}\cdot x + 2k_{z1}(d-x) = n\cdot 2\pi$ $\quad$ n should be an integer

where $$k_{z1} = \sqrt{k_0^2 - k_c^2} = \sqrt{\left(\frac{6\times10^9\times2\pi}{3\times10^8}\right)^2 - \left(\frac{\pi}{5\times10^{-2}}\right)^2} = 34.64\pi$$

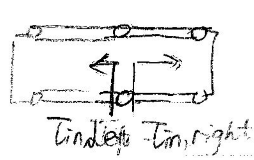
$$k_{z2} = \sqrt{k_0^2\varepsilon_r - k_c^2} = \sqrt{4\times\left(\frac{2\pi\times6\times10^9}{3\times10^8}\right)^2 - \left(\frac{\pi}{5\times10^{-2}}\right)^2} = 77.46\pi$$

$$\Rightarrow 34.64(d-x) + 77.46x = n$$

$$\Rightarrow x = \frac{n-34.64d}{42.82} = \frac{n-1.9052}{42.82}\ (m)$$

when $n=2 \Rightarrow x_{min} = 0.00221\ m = 2.21\ cm$

[Also can be solved by



$\Gamma_{in,left} = \Gamma^*_{in,right}$]