EE161

Electromagnetic Waves

Spring, 2013

Midterm One

Name:

Student ID:

Score:

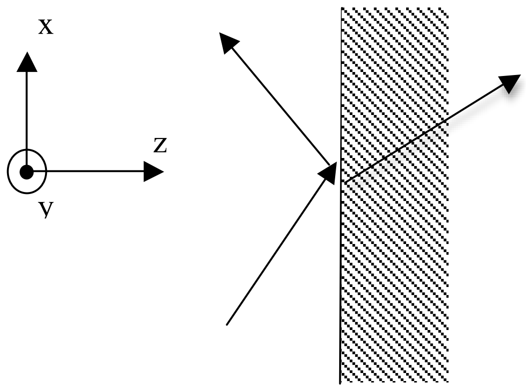
Problem 1. (30pts) The instantaneous electric field intensity vector of a wave traveling in a lossy nonmagnetic medium is given by $\mathbf{E} = e^{\alpha y} \cos\left(6.28 \times 10^9 t + 204 y\right) \hat{x} \ V/m$. The magnetic field of the wave lags the electric field by 21^0. Under these circumstances, find (a) the relative permittivity and conductivity of the medium, as well as (b) the complex propagation coefficient γ. (c) the instantaneous magnetic field intensity vector, and (d) the time-average Poynting vector of the wave.

Problem 2. (40 pts). The magnetic field of a plane electromagnetic wave impinging a PEC plane at z=0 from a nonmagnetic medium is given by $\mathbf{H}_i = [3\cos(\omega t - \beta z)\hat{x} - \sin(\omega t - \beta z)\hat{y}]$ A/m (z<0), where $\omega = 6\pi \times 10^8$ rad/s and $\beta = 4\pi$ rad/m. Determine (a) the polarization state (type and handedness) of the incident wave, (b) complex and instantaneous electric and magnetic field intensity vectors of the reflected wave, (c) the polarization state (type and handedness) of the reflected wave, (d) phasor and instantaneous field vectors of the resultant wave (the total field) in the incident medium, (e) the total time-average Poynting vector in the incident medium, and (f) the surface current density on PEC.

Problem 3. (30 pts) In the oblique incidence case of a plane wave in air onto a lossless non-magnetic dielectric material, where the boundary is at the plane of z=0.

(1) If the wave is with parallel polarization and the measured reflection disappears when the incident angle is 60 degree, what is the dielectric constant of this material?

(2) At the same incident angle, if the transmitted wave has a linear polarization angle of 45 degrees to the x-z plane toward the positive side of the z-axis, what is the polarization angle of the incident wave?



EE 161 Midterm #1 Solution

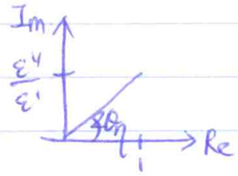
Problem 1.

(a) (10 pts)

$\omega = 6.28 \times 10^{9} \ \frac{rad}{s}$, $\beta_y = 204 = \beta \ \frac{rad}{m}$

$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon'}} \left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2} = |\eta_c| e^{j\theta_\eta}$ $\quad \theta_\eta = 21°$

$\left(\frac{\eta_0}{\sqrt{\varepsilon_r'}}\right)^2 \cdot \left(\frac{1}{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}\right) \cdot \left(1 + j\frac{\varepsilon''}{\varepsilon'}\right) = |\eta_c|^2 e^{j2\theta_\eta}$



$\tan^{-1}\left(\frac{\varepsilon''}{\varepsilon'}\right) = 2\theta_\eta = 42° \Rightarrow \frac{\varepsilon''}{\varepsilon'} = 0.9$ $\therefore$ quasi-conductor.

$\beta = \omega \left\{ \frac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1 \right] \right\}^{1/2}$ $\quad \left(\frac{\beta}{\omega}\right)^2 = \frac{\mu\varepsilon'}{2}\left(\sqrt{1+(0.9)^2} + 1\right)$

$\Rightarrow \varepsilon_r' = 80.9848$ #

$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega \varepsilon_r' \varepsilon_0} = 0.9 \quad \Rightarrow \sigma = 4.0472 \ \frac{S}{m}$ #

(b) (5 pts)

$\gamma = \alpha + j\beta \quad \alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon' \quad \alpha^2 = 6.1281 \times 10^{3}$

$\therefore \alpha = 78.2821 \ \frac{Np}{m}. \quad \Rightarrow \gamma = 78.2821 + j204$ #

(c) (10 pts)

$\eta_c = \frac{\eta_0}{\sqrt{\varepsilon_r'}} (1 - j0.9)^{-1/2} = 33.7194 + j12.9394 = |\eta_c| e^{j\theta_\eta}$

$\Rightarrow |\eta_c| = 36.1168 \ \Omega$

$\tilde{H} = \frac{1}{\eta_c} \underset{\uparrow (-\hat{y})}{\vec{\beta}} \times \tilde{E} = \hat{z}\, 0.0277 e^{78.2821y} \cos(6.28\times10^{9} t + 204y - 21°) \ \frac{A}{m}$ #

(d) (5 pts)

$$S_{av} = \frac{1}{2} Re[\tilde{E} \times \tilde{H}^*] = \frac{1}{2} Re[\hat{x} e^{\alpha y} e^{j\beta y} \times \hat{z} 0.0277 e^{\alpha y} e^{-j\beta y} e^{j\theta_\eta}]$$

$$= (-\hat{y}) \, 0.01385 \, e^{2\alpha y} \cos(21°)$$

$$= (-\hat{y}) \, 0.01293 \, e^{156.5642 y}$$

\#

Problem 2

(a) (5 pts)

$\tilde{H}_i = \hat{x}\, 3 e^{-j\beta z} + \hat{y}\, e^{-j\beta z} e^{j\frac{\pi}{2}}$ $\qquad \frac{|H_y|}{|H_x|} = \frac{1}{3}$

$\delta = \delta_y - \delta_x = 90^\circ \rightarrow \sin\delta > 0,\ \chi > 0$

$\therefore$ left-hand elliptical polarization ※

(b) (10 pts)

$\frac{\omega}{\beta} = u_p = \frac{c}{n} \qquad n = 2,\ \varepsilon_r = 4 \qquad \varepsilon = \varepsilon_r \varepsilon_0 = 3.54 \times 10^{-11}\ \frac{F}{m}$

$\eta_i = \sqrt{\frac{\mu}{\varepsilon}} = 188.41\ \Omega$ or $60\pi\ \Omega$

$\tilde{E}_i = -\eta_i \vec{\beta} \times \tilde{H}_i = \hat{x}\, \eta_i e^{-j4\pi z} e^{j\frac{\pi}{2}} - \hat{y}\, 3\eta_i e^{-j4\pi z}\ \frac{V}{m}$

$\because$ PEC & normal incidence, $\eta_{PEC} = 0$ $\therefore \Gamma = -1$

$\tilde{E}_r(z) = -\hat{x}\, \eta_i e^{j4\pi z} e^{j\frac{\pi}{2}} + \hat{y}\, 3\eta_i e^{j4\pi z}\ \frac{V}{m}$ ※

$\tilde{E}_r(z,t) = \mathrm{Re}[\tilde{E}_r(z) e^{j\omega t}]$

$= -\hat{x}\, \eta_i \cos(6\pi \times 10^8 t + 4\pi z + \frac{\pi}{2})$
$+ \hat{y}\, 3\eta_i \cos(6\pi \times 10^8 t + 4\pi z)\ \frac{V}{m}$ ※

$\tilde{H}_r(z) = \frac{1}{\eta_i} \vec{\beta}_r \times \tilde{E}_r = \hat{x}\, 3 e^{j4\pi z} + \hat{y}\, e^{j4\pi z} e^{j\frac{\pi}{2}}\ \frac{A}{m}$ ※

$\tilde{H}_r(z,t) = \mathrm{Re}[\tilde{H}_r(z) e^{j\omega t}]$
$= \hat{x}\, 3\cos(6\pi \times 10^8 t + 4\pi z) + \hat{y}\cos(6\pi \times 10^8 t + 4\pi z + \frac{\pi}{2})\ \frac{A}{m}$ ※

(c) (5 pts)

$\delta^r = \delta_y^r - \delta_x^r = -90^\circ \rightarrow \chi < 0$

$\therefore$ right-hand elliptical polarization ※

(d) (10 pts)

$$\tilde{E}_1^{tot} = \tilde{E}_i + \tilde{E}_r \quad , \quad \tilde{H}_1^{tot} = \tilde{H}_i + \tilde{H}_r$$

$$\tilde{E}_1^{tot} = \hat{x}\,\eta_i\, e^{j\frac{\pi}{2}} \left(e^{-j4\pi z} - e^{j4\pi z} \right) + \hat{y}\, 3\eta_i \left(e^{j4\pi z} - e^{-j4\pi z} \right)$$

$$= \hat{x}\, 2\eta_i \sin(4\pi z) + \hat{y}\, j\, 6\eta_i \sin(4\pi z) \quad \frac{V}{m}$$

\#\#

$$E_1^{tot}(z,t) = Re\left[\tilde{E}_1^{tot} e^{j\omega t} \right] = \hat{x}\, 2\eta_i \sin(4\pi z)\cos(6\pi \times 10^8 t) \quad \frac{V}{m}$$

\#\#

Similarly,

$$\tilde{H}_1^{tot} = \hat{x}\, 6\cos(4\pi z) + \hat{y}\, j\, 2\cos(4\pi z) \quad \frac{A}{m}$$ \#\#

$$H_1^{tot}(z,t) = \hat{x}\, 6\cos(4\pi z)\cos(6\pi \times 10^8 t) \quad \frac{A}{m}$$ \#\#

(e) (5 pts)

$$S_{av} = \frac{1}{2} Re\left[\tilde{E}_1^{tot} \times \tilde{H}_1^{tot*} \right]$$

$$= \frac{1}{2} Re\left[\hat{z} \left(-j\, 40\eta_i \sin(4\pi z)\cos(4\pi z) \right) \right] = 0 \quad \frac{W}{m^2}$$

\#\#

(f) (5 pts)

$$\vec{J}_s = \hat{n} \times (\vec{H}_1 - \vec{H}_2) = (-\hat{z}) \times \tilde{H}_i(z=0) = \hat{x}\, j - \hat{y}\, 3 \quad \frac{A}{m^2}$$

\#\#

Problem 3

(a) (15 pts)

$\mu_r = \varepsilon_r = 1$

$\because$ no reflected parallel polarized wave.

$\theta_i \longrightarrow$ Brewster's angle

$\tan\theta_{B_{\parallel}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sqrt{3} \Rightarrow \varepsilon_{r_2} = 3. \quad \varepsilon_2 = \varepsilon_{r_2}\varepsilon_0 = 2.655\times10^{-11}\ \frac{F}{m}$ #

(b) (15 pts)

linear polarization angle of $45° \longrightarrow |E_x| = |E_y|$

Assume $\tilde{E}^t = \hat{x}\,E_0\,e^{-j\beta z'} + \hat{y}\,E_0\,e^{-j\beta z'} = \hat{x}\,E^t_{\parallel} + \hat{y}\,E^t_{\perp}$

$\sin(60°) = \sqrt{3}\cdot\sin\theta_t \quad \therefore \theta_t = 30°$

$\eta_2 = \frac{\eta_1}{\sqrt{\varepsilon_{r_2}}} = \frac{\eta_1}{\sqrt{3}} \Rightarrow \frac{\eta_1}{\eta_2} = \sqrt{3}$

Let $\tilde{E}^i = \hat{x}\,E^i_{\parallel} + \hat{y}\,E^i_{\perp}$

$E^i_{\parallel}\,\tau_{\parallel} = E^t_{\parallel}, \quad E^i_{\perp}\,\tau_{\perp} = E^t_{\perp}$

$\frac{E^i_{\perp}\,\tau_{\perp}}{E^i_{\parallel}\,\tau_{\parallel}} = \frac{E^t_{\perp}}{E^t_{\parallel}} = 1 \quad \therefore \frac{E^i_{\perp}}{E^i_{\parallel}} = \frac{\tau_{\parallel}}{\tau_{\perp}}$

$\frac{E^i_{\perp}}{E^i_{\parallel}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \cdot \frac{\eta_2\cos\theta_i + \eta_1\cos\theta_t}{2\eta_2\cos\theta_i} = \frac{\cos\theta_i + \sqrt{3}\cos\theta_t}{\cos\theta_t + \sqrt{3}\cos\theta_i}$

$= \frac{2}{\sqrt{3}} = 1.1547 = \tan\varphi$

$\varphi = \tan^{-1}(1.1547) = 49.1°$ #