EE161

Electromagnetic Waves

Spring, 2012

Midterm One

Name:

Student ID:

Score:

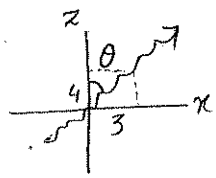
Problem 1. (30pts) The electric field of a plane wave propagating in air has the following expression given by

$\mathbf{E}(t) = -\hat{x}2.4\cos(\omega t - 3x - 4z) + \hat{y}3\sin(\omega t - 3x - 4z) + \hat{z}1.8\cos(\omega t - 3x - 4z)$.

(1) Find the frequency of the wave.
(2) Find the angle between the propagation direction and the positive Z-axis.
(3) What polarization is this wave?
(4) Write down the phasor expression of both the electric field and the magnetic field

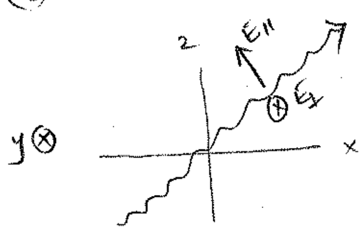
(1) $k_x = 3$; $k_z = 4 \Rightarrow k = \sqrt{k_x^2 + k_z^2} = 5 = \omega\sqrt{\mu\varepsilon} \Rightarrow f = 2.39 \times 10^8\ Hz.$

(2)



$\tan\theta = \frac{3}{4} \Rightarrow \theta = 36.86°$

(3)



$|E_\perp| = |E_y| = 3$

$\vec{E}_{\parallel} = E_x\hat{x} + E_z\hat{z}$

$\Rightarrow |E_{\parallel}| = \sqrt{E_x^2 + E_z^2} = 3.$

Same Amplitudes.

Left hand circularly polarized.

(4) $\tilde{E} = -\hat{x}\,2.4 e^{-j(3x+4z)} + \hat{y}\,3.0\, e^{-j\pi/2} e^{-j(3x+4z)} + \hat{z}\,1.8 e^{-j(3x+4z)}$

$\tilde{H} = \frac{1}{\eta}\hat{k} \times \vec{E} = \frac{1}{377}\frac{[3\hat{x} + 4\hat{z}]}{5} \times \tilde{E}.$

Problem #2 (40 pts). A 2.5 GHz plane wave propagates in air toward positive Z axis. If the wave normally incidents on the surface of a semiconductor material, occupying the half-space of Z>=0, with dielectric constant $\varepsilon_r = 4$ and $\sigma = 100$ (S/m). Assume the wave is linearly polarized in the x-direction, determine

(1) If the semi-conductor is a good conductor or not?

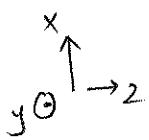
(2) the skin depth of the semi-conductor medium

(3) the reflection coefficient and the transmission coefficient of the wave

(4) the phasor expression of both the electric field and magnetic field $\mathbf{E}(z,t)$ and $\mathbf{H}(z,t)$ inside the semiconductor medium

(5) If a 10cm wide, 5mm thick film made of such semi-conductor is used to conduct this wave on one side of this film, what is the resistance per unit length of this film?

(6) the power attenuation in dB scale at z=1mm penetration of the medium



Air $(z<0)$ $\varepsilon_r = 1;\ \mu_r = 1$ | $\Rightarrow z$ semi-conductor $(z \geq 0)$ $\varepsilon_r = 4;\ \mu_r = 1;\ \sigma = 100$ S/m.

(a) $\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \frac{100}{2\pi(2.5\times10^9)(4\times8.854\times10^{-12})} = 179 > 10^2$ (good conductor)

(b) $\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu\sigma}} = 1\ \text{mm}.$

(c) For good conductor, $\eta_c = (1+j)\frac{\alpha}{\sigma} = 10(1+j) = 14.14 e^{j45°}$

$$\Gamma = \frac{\eta_c - 120\pi}{\eta_c + 120\pi} = 0.948 e^{j177°}$$

$$\tau = 1 + \Gamma = 0.073 e^{j43.5°}$$

(4) $\tilde{E}_t = \tau E_0^i e^{-\alpha z} e^{-j\beta z}$ For good conductor $\alpha = \beta$.

$\Rightarrow \tilde{E}_t = \hat{x}\, 0.073 e^{j43.5°} E_0^i e^{-1000z} e^{-j(1000)z}.$

$$\tilde{H}_t = \hat{y}\, 0.073 e^{j43.5} \frac{E_0^i}{\eta_c} e^{-1000z} e^{-j1000z}$$

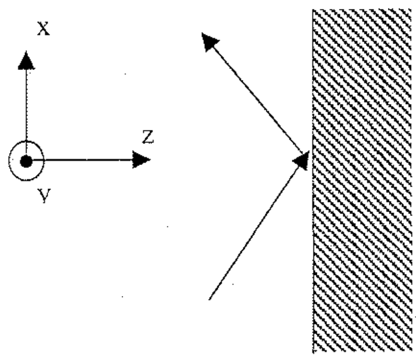
(6) Attenuation $= -8.68\alpha z$

Problem 3. (30 pts). In the oblique incidence case of a plane wave in air onto a lossless non-magnetic dielectric material, the dielectric constant is $\varepsilon_{r2} = 9$,

(1) Find out the Brewster angle for the boundary.

(2) If a left-hand circularly polarized wave incident at the Brewster angle to the boundary, what is the polarization state of the reflected wave?

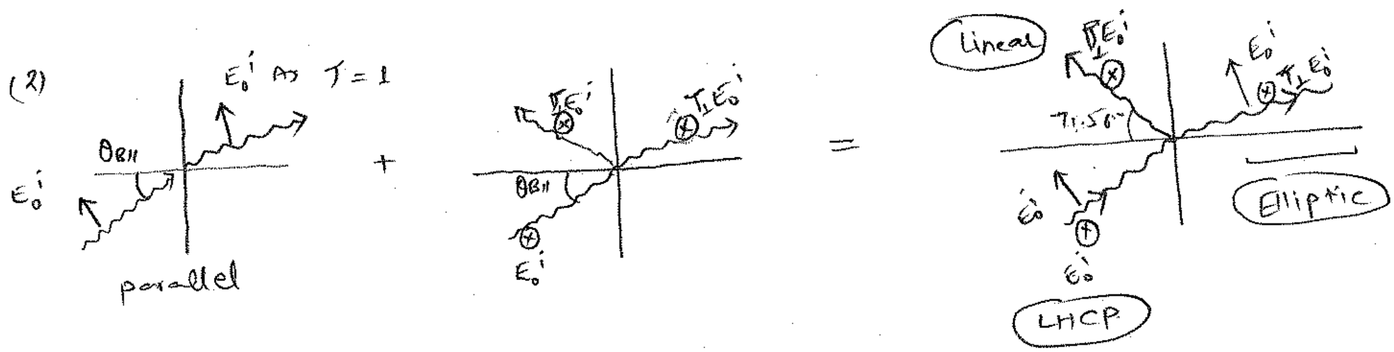
(3) Write down the phasor expression of the reflected wave and the transmitted wave



(1) Brewster angle (For non-magnetic materials, it only exists for parallel polarization)

$$\Rightarrow \theta_{B//} = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \tan^{-1}(3) = 71.56^\circ .$$

(2)



(3) Reflected fields

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\tilde{E}_r = \Gamma_\perp E_0^i (-\hat{y})\, e^{-j[k\cos(71.56^\circ)(-\hat{z}) + k\sin(71.56^\circ)(\hat{x})]}$$

$$\tilde{H}_r = \frac{1}{\eta_0}[\hat{k} \times \tilde{E}_r]$$