EE161 Spring 2006

Name: Grade:

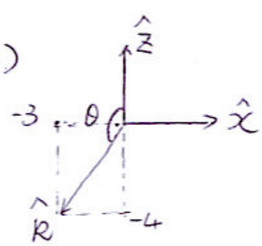
Problem #1 (25pts). The electric field of a plane wave propagating in air has the following expression given by $\mathbf{H}(t) = \hat{y}3\sin(\omega t + 3x + 4z)$. (1) (6 pts) find the frequency of the wave. (2) (6 pts) find the angle between the propagation direction and the Z-axis. (3) (6 pts) write down the phasor expression of the magnetic field. (4) (7 pts) find the associated electric field in phasor.

(1) $\vec{k} = -3\hat{x} - 4\hat{z}$, $k = |\vec{k}| = 5$

$$\omega = \frac{k}{\sqrt{\mu_0 \varepsilon_0}} = 5 \times 3 \times 10^8 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = \boxed{2.39 \times 10^8 \text{ Hz}}$$

(2)



$$\theta = 180^\circ - \tan^{-1}\left(\frac{3}{4}\right) = \boxed{143^\circ}$$

(3) $H(t) = \hat{y}\, 3\cos(\omega t + 3x + 4z - \frac{\pi}{2})$

$$\tilde{H} = \hat{y}\, 3\, e^{j(3x+4z-\frac{\pi}{2})} = \boxed{\hat{y}\,(-3j)e^{j(3x+4z)}}$$

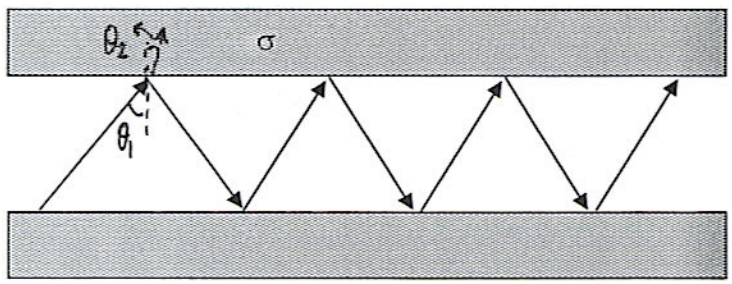
(5) $\tilde{E} = -\eta_0\, \hat{k} \times \tilde{H}$

$$= -377 \times \left(-\tfrac{3}{5}\hat{x} - \tfrac{4}{5}\hat{z}\right) \times \hat{y}\,(-3j)e^{j(3x+4z)}$$

$$= \boxed{\hat{x}\, 904.8j\, e^{j(3x+4z)} - \hat{z}\, 678.6j\, e^{j(3x+4z)}}$$

or $-\hat{x}\, 904.8\, e^{j(3x+4z-\frac{\pi}{2})} + \hat{z}\, 678.6\, e^{j(3x+4z-\frac{\pi}{2})}$

Problem #2 (25 pts). A plane wave can propagate in the region confined by two parallel conductor plates through bouncing between these two plates, as shown in the following figure. The middle region is air and the frequency of the wave is 1MHz. If the conductor is not perfect, a small portion of the electromagnetic power may be transmitted inside the conductor. (1) (8pts) If the conductivity is very large, what is the direction of the transmitted wave approximately? (2) (8pts) If the conductor is silver with conductivity $\sigma=3\times10^6$ S/m, what is its skin depth? (3) (9pts) If one wants to make sure the transmitted energy attenuates at least 40dB when it reaches to the outer boundary of the silver plate, what is the minimum thickness the plate has to have.



(1) : $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_2 = \sqrt{\frac{\mu\varepsilon_2}{\ }} \simeq \sqrt{\frac{\mu\sigma}{\pi f}} \cdot \frac{1}{1+j} \Rightarrow n_2 \propto \sigma$

If $\sigma \to \infty \Rightarrow n_2 \to \infty \Rightarrow$ $\boxed{\theta_1 \to 0}$

(2) $\delta_s = \frac{1}{\sqrt{\pi f \mu \sigma}} = \boxed{0.29 \text{ mm}}$

(3) $\frac{P_2}{P_1} = 10^{-4} \Rightarrow e^{-2\alpha d} = 10^{-4} \Rightarrow$ $\boxed{d = 1.34 \text{ mm}}$

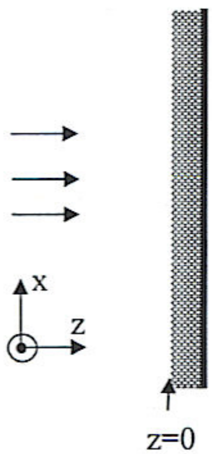
Problem #3 (25 pts). A 1GHz plane wave is normally incident in air onto the plane surface onto a dielectric coated infinite perfect electric conductor (PEC) plate, as shown in the following figure. The thickness of the coating is *d=2.5 cm,* the dielectric property is $\varepsilon_r = 9, \mu_r = 1$. Assume the electric field is right-hand circularly polarized. Please find

(1) (6 pts) the phasor expression of the incident field.

(2) (6 pts) the total complex reflection coefficient at the reference plane z=0.

(3) (8 pts) the phasor expression and the polarization state of the reflected wave.

(4) (5 pts) the position of the first maximum E field observed in the air region.



(1) $\tilde{E} = E_0(\hat{x} - j\hat{y})e^{-jkz}$ $\quad k = \frac{2\pi}{\lambda}$, $\lambda = 30$ cm

(2) $\lambda_{dielectric} = \frac{30}{\sqrt{\varepsilon_r}} = 10$ cm

$d = 2.5$ cm $= \frac{\lambda_{diel}}{4}$ ⇒ The dielectric slap acts as a $\frac{\lambda}{4}$ Transformer

$\Gamma|_{z=\frac{\lambda}{4}} = -1$ (PEC plate) ⇒ $\Gamma|_{z=0} = 1$

$Z_{in} = 0$ (short circuit) $\quad Z_{in} = \infty$ (Open Circuit)

(3) RHCP $\xrightarrow{\Gamma=-1}$ LHCP ⇒ $\tilde{E} = E_0(\hat{x} + j\hat{y})e^{-jkz}$

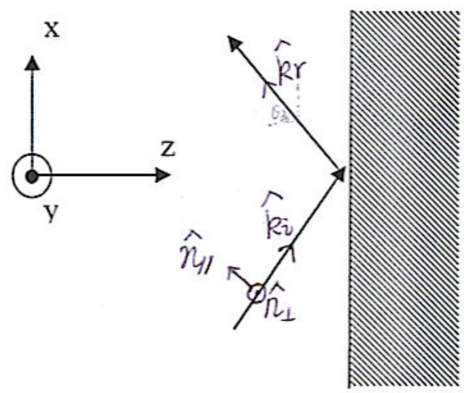
(4) $\theta_r = 0$ ⇒ Firs maximum @ $\boxed{z = 0}$

Problem #4 (25 points). In the oblique incidence case of a plane wave in air onto a lossless non-magnetic dielectric material, the dielectric constant is $\varepsilon_{r2} = 9$,

(1) (8 pts) Find out the Brewster angle for the boundary.

(2) (8 pts) If a left-hand circularly polarized wave incident at the Brewster angle to the boundary, what is the polarization state of the reflected wave?

(3) (9 pts)Write down the phase expression of the reflected field.



(1) $\theta_B = \tan^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right) = \tan^{-1}(3) = \boxed{71.565^\circ}$

(2) LHCP ⇒ Incident wave has both parallel and perpendicular component of E field.

$$\vec{E}^i = \hat{n}_{//} E_{//} e^{-j\vec{k}_i\cdot\vec{r}} + j\,\hat{n}_\perp E_\perp e^{-j\vec{k}_i\cdot\vec{r}} \qquad (E_{//} = E_\perp)$$

Incident angle = brewster angle ⇒ no reflection for paralled polarized component

⇒ $\vec{E}^r$ contains ⊥ component only ⇒ E field is always in $\hat{n}_\perp$ direction (y direction).

⇒ Relflected wave is $\boxed{\text{linearly polarized}}$

(3) $\vec{E}^r = \boxed{\hat{y} E^r e^{-j\vec{k}_r\cdot\vec{r}}}$

$$= \hat{y}\,(\Gamma_\perp)(jE_\perp)\,e^{-jk_0(x\sin\theta_B - z\cos\theta_B)}$$

↑ reflection coef. ↑ incident

where $\Gamma_\perp = \dfrac{\cos\theta_B - \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_B}}{\cos\theta_B + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2\theta_B}} = -0.8$