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2	25
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EE161 Spring 2005

Name: [redacted] Grade: 88

Problem #1 (25 points). The electric field of a plane wave propagating in air has the following expression given by $\mathbf{E}(z,t) = \hat{x}4\cos(\omega t + 6z) + \hat{y}3\sin(\omega t + 6z)$. (1) (5 points) Find the operating frequency of the wave. (2) (5 points) Write down the phasor expression of the electric field. (3) (5 points) Find the associated magnetic field. (4) (10 points) Identify the wave polarizations states with rotation directions and find the axial ratio.

$$E(z,t) = \hat{x}\,4\cos(\omega t+6z) + \hat{y}\,3\sin(\omega t+6z) = \hat{x}\,4\cos(\omega t+6z) + \hat{y}\,3\cos(\omega t+6z-\tfrac{\pi}{2})$$

$k = -6 \qquad E = E_0$

$c = f\lambda \qquad f = \frac{c}{\lambda} = \frac{c}{2\pi}k = \frac{3\times10^8 \times \cancel{6}^{3}}{\cancel{2}\pi} = \frac{9}{\pi}\times10^8 \text{ Hz}$

1) $\boxed{f = 286 \text{ MHz}}$ $\qquad \omega = 2\pi\,\frac{9}{\pi}\times10^8 = 18\times10^8 \text{ rad/s}$

$$\tilde{E}(z,t) = \hat{x}\,4e^{j6z} + \hat{y}\,3e^{j(6z-\frac{\pi}{2})} = \hat{x}\,4e^{j6z} + \hat{y}(-j)3e^{j6z}$$

2) $\boxed{\tilde{E}(z,t) = (\hat{x}4 - \hat{y}3j)\,e^{j6z}\ \tfrac{V}{m}}$

$$\tilde{H} = -\frac{1}{j\omega\mu}\nabla\times\tilde{E} = -\frac{1}{j\omega\mu}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4e^{j6z} & -3je^{j6z} & 0 \end{vmatrix} =$$

$$= -\frac{1}{j\omega\mu}\left[\hat{x}\left(+\frac{\partial}{\partial z}(+3)j\,e^{j6z}\right) + \hat{y}\left(\frac{\partial}{\partial z}4e^{j6z}\right)\right] =$$

$$= -\frac{1}{j\omega\mu}\left(-\hat{x}\,18e^{j6z} + \hat{y}\,24j\,e^{j6z}\right) =$$

$$= j\frac{1}{2\cancel{\pi}\frac{9}{\cancel{\pi}}\times10^8\ 4\pi\times10^{-7}}(-\hat{x}18 + \hat{y}24j)e^{j6z} =$$

$$= (-\hat{x}\,7.96j - \hat{y}\,10.61)\,e^{j6z}\ \tfrac{mA}{m} = \left[\hat{x}\,7.96e^{j(6z-\frac{\pi}{2})} - \hat{y}\,10.61e^{j6z}\right]\tfrac{mA}{m}$$

$$\Rightarrow H(z,t) = \left[(-\hat{x}\,7.96j - \hat{y}\,10.61)\cos(18\times10^8t + 6z)\right]\tfrac{mA}{m}$$

3) $\boxed{H(z,t) = \left[\hat{x}\,7.96\times10^{-3}\sin(18\times10^8t+6z) - \hat{y}\,10.61\times10^{-3}\cos(18\times10^8t+6z)\right]\tfrac{A}{m}}$

Problem #2 (25 points). A 25 MHz plane wave propagates toward positive Z axis. If the wave propagates in a semiconductor material with dielectric constant $\varepsilon_r = 9$ and $\sigma = 100$ (S/m). At the reference plane Z=0, the time domain electric field is measured to be $\mathbf{E}(0,t) = \hat{x}10\cos(\omega t)$. Determine

(1) (10 points) the time-domain expression of both the electric field and magnetic field $\mathbf{E}(z,t)$ and $\mathbf{H}(z,t)$

(2) (5 points) the skin depth of the semi-conductor medium

(3) (10 points) the power attenuation in dB scale at z=1mm penetration of the medium

$f = 25$ MHz $\qquad E(0,t) = \hat{x}\, 10\cos(\omega t)$

$\omega = 2\pi \times 25 \times 10^6 = 5\pi \times 10^7$

$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \frac{100 \times 36\pi \times 10^{9}}{5\pi \times 10^{7}} = 72 \times 10^3 = 72000 > 100$

=> Good conductor

$\alpha = \sqrt{\pi f \mu \sigma} = \beta = \sqrt{\pi \times 25 \times 10^6 \times 4\pi \times 10^{-7} \times 100} = \pi\, 10\sqrt{10} = \alpha = \beta$

$\tilde{E}(z) = \hat{x}\, 10\, e^{-\pi 10\sqrt{10} z}\, e^{-j\pi 10\sqrt{10} z}$

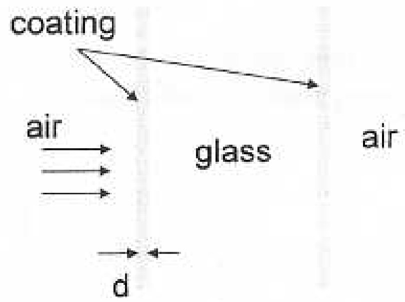
$\eta_c = (1+j)\frac{\pi 10\sqrt{10}}{\sqrt{10}\, 10} = (1+j)\frac{\pi}{\sqrt{10}}$

$\tilde{H}(z) = \hat{y}\left(\frac{10}{1+j}\,\frac{\sqrt{10}}{\pi}\right) e^{-\pi 10\sqrt{10} z}\, e^{-j\pi 10\sqrt{10} z} = \hat{y}\, 7.12\, e^{-j45}\, e^{-\pi 10\sqrt{10} z}\, e^{-j\pi 10\sqrt{10} z}$

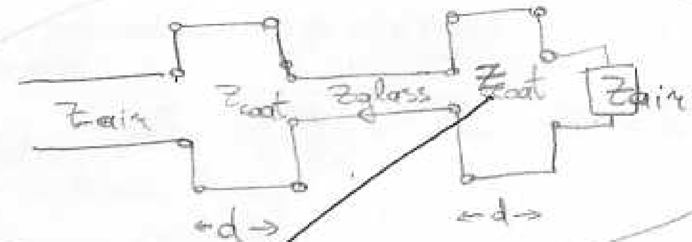
1) $\boxed{E(z,t) = \hat{x}\, 10\, e^{-99.4 z} \cos(5\pi \times 10^7 t - 99.4 z)\ \frac{V}{m}}$

$\boxed{H(z,t) = \hat{y}\, 7.12\, e^{-99.4 z} \cos(5\pi \times 10^7 t - 99.4 z - 45^\circ)\ \frac{A}{m}}$

Problem #3. (25 points) To design the anti-flaring coating of a glass requires the same type of coating is placed in both side of the glass. If the glass has a dielectric constant of 4 and the thickness of the glass can be arbitrary. The light wave incidents on the glass normally. (1) (10 points) Draw the transmission line equivalence to this structure (2) (15 points) If the light wave has a frequency of 10^{15} Hz, how shall we select the dielectric constant and the thickness d of the coating material to make the light pass through the glass without reflection?



1)



$Z = \eta$

$\Gamma = 0$

$\varepsilon_{r_{glass}} = 4$ $\quad Z_{glass} = \eta_{glass} = \sqrt{\frac{\mu}{\varepsilon}} = \eta_0 \frac{1}{\sqrt{\varepsilon_r}} = \frac{120\pi}{2} = 60\pi$

$Z_{air} = 120\pi$, $\quad Z_{coat} = \frac{120\pi}{\sqrt{\varepsilon_{r_{coat}}}}$

Quarter wave transformer:

$$Z_{in} = \frac{Z_0^2}{Z_L} = Z_{air} = \frac{(Z_{coat})^2}{Z_{glass}}$$

$$(Z_{coat}) = \sqrt{60\pi \times 120\pi} = 60\pi\sqrt{2} = \frac{120\pi}{\sqrt{\varepsilon_{r_{coat}}}}$$

$$\Rightarrow \varepsilon_{r_{coating}} = \left(\frac{\overset{2\sqrt{2}}{\cancel{120\pi}}}{\cancel{60\pi}\sqrt{2}}\right)^2 = 2$$

2) $\varepsilon_{r_{coating}} = 2$

$$\beta_{coat} = \frac{2\pi}{\lambda_{coat}} = \omega\sqrt{\mu\varepsilon} = \frac{2\pi\, 10^{\cancel{15}\,7}}{3\times\cancel{10^8}}\sqrt{2} = \frac{2\sqrt{2}\pi}{3}\times 10^{7}$$

$$\lambda_{coat} = \frac{\cancel{2\pi}}{\frac{2\sqrt{2}\cancel{\pi}}{3}\times 10^{7}} = \frac{3}{\sqrt{2}}\times 10^{-7}$$

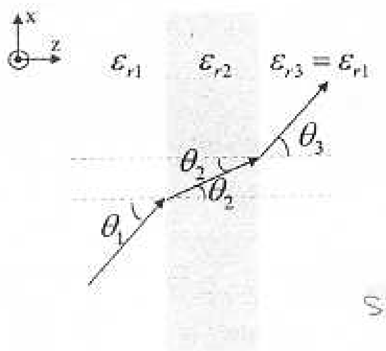
2) $d = \frac{\lambda_{coat}}{4} = \frac{3}{4\sqrt{2}}\times 10^{-7} = 5.3\times 10^{-8}\ \text{m} = d$

Problem #4 (25 points). Consider the oblique incidence case of plane wave in air onto a lossless non-magnetic dielectric slab. The dielectric constant is $\varepsilon_{r2} = 3$, and the thickness is unknown. Therefore

(1) (8 points) for parallel polarization, choose an incident angle so that the wave can pass through the slab without any loss.

(2) (12 points) if the frequency of the wave is 1 GHz, please write down the phasor expression of the electric field in both the air and the dielectric slab for parallel polarization

(3) (5 points) if the incident wave is circularly polarized, how will the axial ratio R change after the wave passes through the slab?



$\theta_{B_{\parallel}} = \tan^{-1}\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}} = \tan^{-1}\sqrt{3} = 60^\circ$

1) $\boxed{\theta_{B_{\parallel}} = 60^\circ = \theta_1}$

$\Gamma = 0$ ← no reflections

Snell's Law $\frac{\sin\theta_2}{\sin\theta_1} = \frac{\sqrt{\varepsilon_{r1}}}{\sqrt{\varepsilon_{r2}}}$

$\Rightarrow \theta_2 = 30^\circ$

$f = 1\times10^9$ Hz $\quad k_1 = \frac{2\pi\times10^9}{3\times10^8} = \frac{20}{3}\pi$

$\tilde{E}^i_{\parallel} = (\hat{x}\,0.5 - \hat{z}\frac{\sqrt{3}}{2})\, E_0^i\, e^{-j\frac{20}{3}\pi(x\frac{\sqrt{3}}{2} + z\frac{1}{2})}$

$k_2 = k_1\sqrt{\varepsilon_{r2}} = \frac{20}{\sqrt{3}}\pi$

2) $\boxed{\tilde{E}_1 = (x\,0.5 - \hat{z}\frac{\sqrt{3}}{2})\, E_0^i\, e^{-j(\frac{10}{\sqrt{3}}\pi x + \frac{10}{3}\pi z)}}$

$\tilde{E}^t_{\parallel} = (\hat{x}\frac{\sqrt{3}}{2} - \hat{z}\frac{1}{2})\, E_0^i\, e^{-j\frac{20}{\sqrt{3}}\pi(x\frac{1}{2} + z\frac{\sqrt{3}}{2})}$

$\boxed{\tilde{E}_2 = T(\hat{x}\frac{\sqrt{3}}{2} - \hat{z}\frac{1}{2})\, E_0^i\, e^{-j(\frac{10}{\sqrt{3}}\pi x + 10\pi z)}}$ −4

circularly polarized → $R_1 = \boxed{1}$ old axial ratio

$\eta_2 = \frac{120\pi}{\sqrt{3}} = 40\sqrt{3}\pi$

after passing

$a_{\parallel_1} = a_{\parallel_2}$ but $a_{\perp_2} = \tau a_{\perp_1}$ new axial ratio

$\tau = \frac{2\times 40\sqrt{3}\pi \cdot \frac{1}{2}}{20\sqrt{3}\pi + 60\sqrt{3}\pi} = \frac{40\sqrt{3}\pi}{80\sqrt{3}\pi} = \frac{1}{2} \Rightarrow a_{\perp_2} = \frac{1}{2}a_{\perp_1}$

3) $\Rightarrow R_2 = \frac{a_{\parallel_2}}{a_{\perp_1}} = \boxed{2}$ −2