EE161 Fall 2005

Name: Solution Grade:

Problem #1 (25 points). The electric field of a plane wave propagating in air has the following expression given by $\mathbf{E}(t) = \hat{y}3\sin(\omega t + 8x + 6z)$. (1) (5 points) find the frequency of the wave. (2) (5 points) find the angle between the propagation direction and the Z-axis. (3) (5 points) write down the phasor expression of the electric field. (4) (10 points) find the associated magnetic field in phasor.

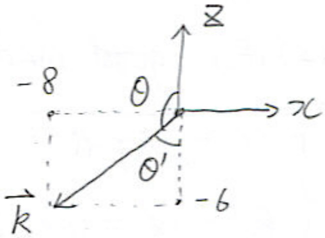
From $\vec{E}(t)$ expression:

$$\vec{k} = -8\hat{x} - 6\hat{y} \Rightarrow k = |\vec{k}| = \sqrt{(-8)^2 + (-6)^2} = 10$$

$$\hat{k} = \frac{\vec{k}}{|k|} = -\frac{4}{5}\hat{x} - \frac{3}{5}\hat{y}$$

(1) $k = \omega\sqrt{\mu_0\varepsilon_0} \Rightarrow f = \frac{1}{2\pi}\cdot\frac{k}{\sqrt{\mu_0\varepsilon_0}} = \frac{k}{2\pi}\cdot c = \frac{10}{2\pi}\times 3\times10^8\ Hz = \boxed{4.77\times10^8\ Hz}$

(2) propagation direction : direction of $\vec{k}$



since: $\theta' = \tan^{-1}\frac{8}{6} = 53.1^\circ$

$\Rightarrow \theta = 180^\circ - \theta' = \boxed{126.9^\circ}$

(3) $\vec{E}(t) = \hat{y}\ 3\sin(\omega t + 8x + 6z)$

$= \hat{y}\ 3\cos(\omega t + 8x + 6z - \frac{\pi}{2})$

$\Rightarrow \tilde{E} = \hat{y}\ 3e^{j(8x+6z-\frac{\pi}{2})} = \boxed{\hat{y}(-3j)e^{j(8x+6z)}}$

(4) $\tilde{H} = \frac{1}{\eta_0}\hat{k}\times\tilde{E} = \frac{1}{377}(-\frac{4}{5}\hat{x} - \frac{3}{5}\hat{z})\times\hat{y}(-3j)e^{j(8x+6z)}$

unit vector!

$= \boxed{\hat{z}(0.00637j)e^{j(8x+6z)} - \hat{x}(0.0047j)e^{j(8x+6z)}}$

Problem #2 (25 points). A 25 MHz plane wave propagates toward positive Z axis. The wave propagates in a infinite semiconductor medium with dielectric constant $\varepsilon_r = 9$ and $\sigma = 70\,(\mathrm{S/m})$. At the reference plane Z=0, the time domain electric field is measured to be $\mathbf{E}(0,t) = \hat{x}10\cos(\omega t + 45^o)$.

(1) (5 points) Identify which category this medium belongs among good dielectric, quasi conductor or good conductor.

(2) (10 points) Find the time-domain expression of both the electric field and magnetic field $\mathbf{E}(z,t)$ and $\mathbf{H}(z,t)$

(3) (5 points) the skin depth of this semiconductor medium

(4) (5 points) the power attenuation in dB scale at z=1mm penetration of the medium

(1) $\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}$, where $\varepsilon' = \varepsilon = \varepsilon_0\varepsilon_r = 7.96\times10^{-11}$

$$\varepsilon'' = \frac{\sigma}{\omega} = \frac{70}{2\pi\times50\times10^6} = 4.456\times10^{-7}$$

$$\frac{\varepsilon''}{\varepsilon'} > 10^2 \Rightarrow \boxed{\text{good conductor}}$$

(2) For good conductor

$$\begin{cases} \alpha = \sqrt{\pi f \mu\sigma} = 83.12 \\ \beta = \alpha = 83.12 \\ \eta_c = (1+j)\frac{\alpha}{\sigma} = (1+j)1.19 \end{cases}$$

General form: $\vec{E} = \hat{x}\,E_0 e^{-\alpha z}\cos(\omega t - \beta z + \varphi)$

Since at $z=0 \Rightarrow \vec{E}(0,t) = \hat{x}\,10\cos(\omega t + 45^\circ)$

Compare two expressions: $E_0 = 10$, $\varphi = 45^\circ$

$$\Rightarrow \vec{E} = \hat{x}\,10e^{-\alpha z}\cos(\omega t - \beta z + 45^\circ)$$

$$= \boxed{\hat{x}\,10e^{-83.12z}\cos(5\pi\times10^7 t - 83.12z + 45^\circ)}$$

Phasor expression

$$\tilde{E} = \hat{x}\,10e^{-83.12z}\,e^{j(-83.12z)}\cdot e^{j(45^\circ)}$$

$$\tilde{H} = \frac{1}{\eta_c}\hat{z}\times\tilde{E} = \frac{10}{1.19\cancel{(1+j)}}\cdot e^{-83.12z}e^{-j83.12z}\cdot\frac{\sqrt{2}}{2}\cancel{(1+j)} = 5.94\,e^{-83.12z}e^{-j83.12z}$$

$\Rightarrow$ Time expresson

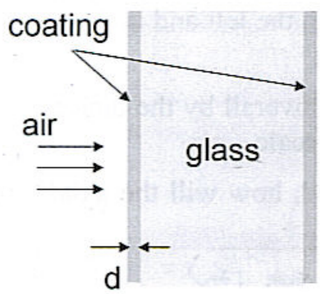
$$\boxed{\vec{H}(t) = 5.94e^{-83.12z}\cos(5\pi\times10^7 t - 83.12z)}$$

(3) $\delta_s = \frac{1}{\alpha} = 0.012\,m$

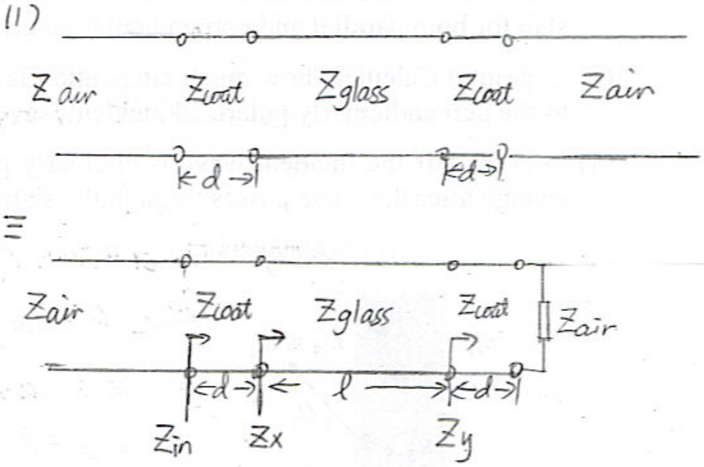
(4) $A = 10\log e^{-2\alpha z}\Big|_{z=1mm}$

$$= \boxed{-0.722\,dB}$$

Problem #3. (25 points) To design the anti-flaring coating of a glass requires the same type of coating is placed in both side of the glass. The thickness of glass is arbitrary. The glass has a dielectric constant of 4. The light wave incidents on the glass normally. (1) (5 points) Draw the transmission line equivalence of the original structure (2) (10 points) If the light wave has a frequency of 10^{15} Hz, how shall we select the dielectric constant and the thickness d of the coating material to make the light pass through the glass without reflection? (3) (10 points) Illustrate your choice of material will cause no-reflection on the first boundary step by step using the transmission line model.



(1)

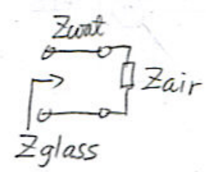


(2) Matching requirement $Z_{in} = Z_{air}$

Z_{in} is Z_x transformed by transmission line with characteristic impedance Z_{coat} and length d. And

$$Z_x = Z_{glass} \cdot \frac{Z_y + j Z_{glass} \tan(\beta l)}{Z_{glass} + j Z_y \tan(\beta l)}$$

l is arbitrary $\Rightarrow$ we have no control $\Rightarrow$ only when $Z_y = Z_{glass}$, Z_x is a fixed number. (Otherwise Z_x is varying with l, we can not match it to Z_{in})



$\Rightarrow Z_y = Z_{class}$.

Using $\frac{\lambda}{4}$ transformer: $d = \frac{\lambda}{4} + n \cdot \frac{\lambda}{2}$, $n = 0, 1, 2 \cdots$

$$Z_{coat}^2 = Z_{glass} Z_{air} \Rightarrow \varepsilon_{r,coat} = \sqrt{\varepsilon_{r,glass} \cdot \varepsilon_{r,air}} = \sqrt{4 \times 1} = 2$$

(3) Check: $Z_y = \frac{Z_{coat}^2}{Z_{air}} = Z_{glass}$; $Z_x = Z_{glass}$; $Z_{in} = \frac{Z_{coat}^2}{Z_x} = \frac{Z_{coat}^2}{Z_{glass}} = Z_{air}$.

Therefore, no reflection!

Problem #4 (25 points). In the oblique incidence case of plane wave in air onto a lossless non-magnetic dielectric slab, the dielectric constant is $\varepsilon_{r2} = 9$, and the thickness of the slab is unknown. Assuming only a single reflection/transmission is considered on each boundary,

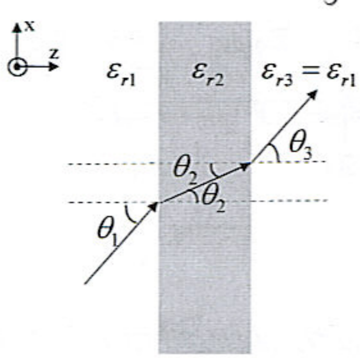
(1) (5 points) Find out the Brewster angle for the first boundary and prove this will be Brewster angle for the second boundary too.

(2) (10 points) If the frequency of the wave is 3 GHz, write down the phasor expression of the electric field in both the air region in the left and in the dielectric slab for both parallel and perpendicular polarizations.

(3) (5 points) Calculate how much attenuation is caused overall by the dielectric slab to the perpendicularly polarized incident wave in dB scale.

(4) (5 points) If the incident wave is circularly polarized, how will the axial ratio R change after the wave passes through the slab?

(1) non-magnetic $\Rightarrow \theta_{B1} = \tan^{-1}\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}} = \tan^{-1}(3) = 71.57^\circ$



when $\theta_1 = \theta_{B1} = 71.57^\circ$

By Snell's Law:

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

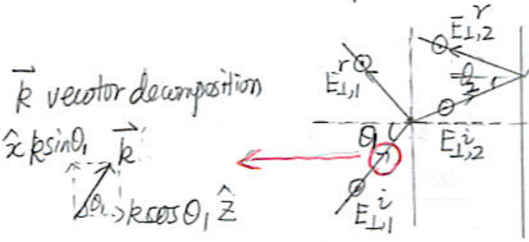
$\Rightarrow \theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}\sin\theta_1\right) = 18.44^\circ$

Again, according to definition $\theta_{B2} = \tan^{-1}\sqrt{\frac{\varepsilon_{r3}}{\varepsilon_{r1}}} = \tan^{-1}\left(\frac{1}{3}\right) = 18.44^\circ$

So $\theta_2 = \theta_{B2}$. Statement proved!

(2) $k_0 = \omega_0\sqrt{\mu_0\varepsilon_0} = \dfrac{3\times10^9}{3\times10^8} = 10$

Perpendicular pol. :



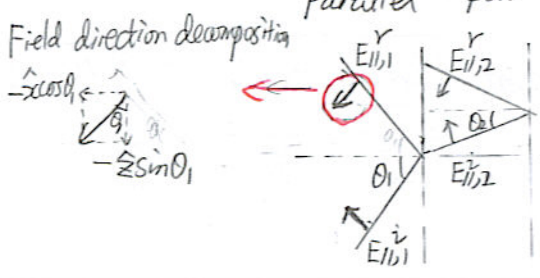
$\vec{E}^i_{\perp,1} = \hat{y}\, E^i_{\perp,1}\, e^{-jk_0(x\sin\theta_1 + z\cos\theta_1)}$

$\vec{E}^r_{\perp,1} = \hat{y}\, E^r_{\perp,1}\, e^{-jk_0(x\sin\theta_1 - z\cos\theta_1)}$

$\vec{E}^i_{\perp,2} = \hat{y}\, E^i_{\perp,2}\, e^{-j3k_0(x\sin\theta_2 + z\cos\theta_2)}$

$\vec{E}^r_{\perp,2} = \hat{y}\, E^r_{\perp,2}\, e^{-j3k_0(x\sin\theta_2 - z\cos\theta_2)}$

Parallel pol. :



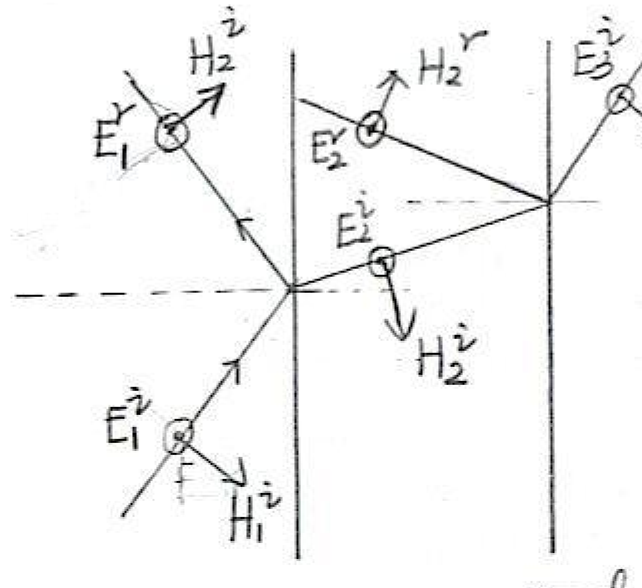
$\vec{E}^i_{\parallel,1} = (\hat{x}\cos\theta_1 - \hat{z}\sin\theta_1)\, E^i_{\parallel,1}\, e^{-jk_0(x\sin\theta_1 + z\cos\theta_1)}$

$\vec{E}^r_{\parallel,1} = (-\hat{x}\cos\theta_1 - \hat{z}\sin\theta_1)\, E^r_{\parallel,1}\, e^{-jk_0(x\sin\theta_1 - z\cos\theta_1)}$

$\vec{E}^i_{\parallel,2} = (\hat{x}\cos\theta_2 - \hat{z}\sin\theta_2)\, E^i_{\parallel,2}\, e^{-j3k_0(x\sin\theta_2 + z\cos\theta_2)}$

$\vec{E}^r_{\parallel,2} = (-\hat{x}\cos\theta_2 - \hat{z}\sin\theta_2)\, E^r_{\parallel,2}\, e^{-j3k_0(x\sin\theta_2 - z\cos\theta_2)}$

(3) For perpendicularly polarized wave:

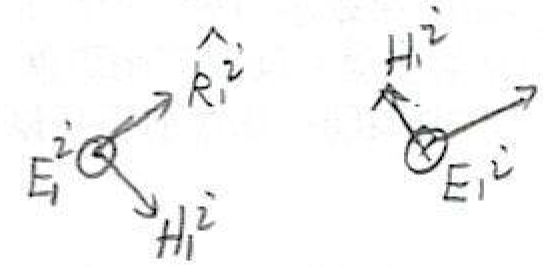


$z=0$ $\quad$ $z=d$

plane wave: $H_n^i = \dfrac{E_n^i}{\eta_n}$, $H_n^r = \dfrac{E_n^r}{\eta_n}$

Note: $\vec{E}$ & $\vec{H}$ can be defined in either of the two opposite direction. But $\vec{E}\times\vec{H}$ should give the propagation direction $\hat{k}$.

For example:



are both correct. You will get the result with a opposite sign in the amplitude. (actually means the same direction)

tangential E (y component)

$$E_{t,1} = E_{t,2}\big|_{z=0} \Rightarrow E_1^i + E_1^r = E_2^i + E_2^r \qquad ①$$

$$E_{t,2} = E_{t,3}\big|_{z=d} \Rightarrow E_2^i\, e^{-j3k_0 d\cos\theta_2} + E_2^r\, e^{+j3k_0 d\cos\theta_2} = E_3^i\, e^{-jk_0 d\cos\theta_1} \qquad ②$$

$$H_{t,1} = H_{t,2}\big|_{z=0} \Rightarrow H_1^i\cos\theta_1 - H_1^r\cos\theta_1 = H_2^i\cos\theta_2 - H_2^r\cos\theta_2\big|_{z=0}$$

tangential H (x component)

$$\Rightarrow \frac{\cos\theta_1}{\eta_0}\left(E_1^i - E_1^r\right) = \frac{\cos\theta_2}{\frac{1}{3}\eta_0}\left(E_2^i - E_2^r\right) \qquad ③$$

$$H_{t,2} = H_{t,3}\big|_{z=d} \Rightarrow \frac{\cos\theta_2}{\frac{1}{3}\eta_0}\left(E_2^i e^{-j3k_0 d\cos\theta_2} - E_2^r e^{j3k_0 d\cos\theta_2}\right) = \frac{\cos\theta_1}{\eta_0} E_3^i e^{-jk_0 d\cos\theta_1} \qquad ④$$

Solve the four equations:

$$E_3^i = \left(\frac{6\cos\theta_2\cos\theta_1}{(\cos^2\theta_1 - 9\cos^2\theta_2)} \cdot \frac{e^{jk_0 d\cos\theta_1}}{j\sin(3k_0 d\cos\theta_2)}\right) E_1^i$$

When there is no slab

$$E_3^i = E_1^i$$

So attenuation should be $A = 20\log\left|\dfrac{6\cos\theta_2\cos\theta_1}{(\cos^2\theta_1 - 9\cos^2\theta_2)\cdot\sin(3k_0 d\cos\theta_2)}\right|$.

(4) circularly polarized $\Rightarrow R=1$

Since there is different transmission coefficient for // and $\perp$ polarized wave, they would have different amplitude after passing through the slab $\Rightarrow R>1$. So R becomes $\boxed{\text{larger}}$