EE161. Final Exam

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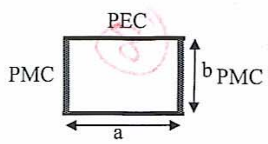
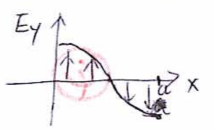
Problem #1. (20 pts) Perfect Magnetic Conductor (PMC) is a counter-part of Perfect Electrical Conductor under the dual relationship of electric field and magnetic field. For PMC, it satisfies the boundary condition that the tangential magnetic field and normal electric field must equal to zero. For an air filled waveguide shown as below, the top and bottom of the waveguide are made of PEC, while the two sides are made of PMC. The dimensions are a=5cm, b=3cm. (1) Find the dominant mode propagating in this waveguide? (2) Draw the E field template for TE_{10} mode in the cross section (3) To keep the single-mode operation, how should one limit the operating frequency?

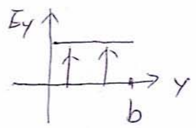


(a>b)

1) TEM is the dominant mode. 6

2) TE_{10} $E_y \propto \cos\frac{\pi}{a}x$ $E_x = 0$ 7





3) The first higher order mode is TE_{10} 7

$$f_{c10} = \frac{c}{2a} = \frac{3\times10^{8}}{2\times5\times10^{-2}} = 3\,Ghz$$

For single-mode operation

$$0 < f < 3\,Ghz$$

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Problem #2. (20 pts) A rectangular waveguide cavity has the dimension a = 5cm and width b = 2cm. (1) what are the lowest five modes and their corresponding cutoff frequencies? (2) If an electromagnetic wave propagates with an operating frequency at 25% below the cutoff frequency of the TM_{11} mode of the waveguide, what are the operating frequency and the wave number of this wave? (3) Find out all the possible values of the phase constant β, guiding wavelengths λ_g, the phase velocity v_p and group velocity v_g at this operating frequency.

1) TE modes : $H_z \propto \cos\frac{m\pi}{a}x \cos\frac{n\pi}{b}y$ $\quad m, n = 0, 1, 2 \cdots$ m, n can't be zero simultaneously

TM modes : $E_z \propto \sin(\frac{m\pi}{a}x) \sin\frac{n\pi}{b}y \quad m, n = 1, 2, \cdots$

The lowest five modes are

1st TE_{10} $\quad f_{c10} = \frac{c}{2a} = \frac{3\times10^8}{2\times5\times10^{-2}} = 3Ghz$

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2nd TE_{20} $\quad f_{c20} = \frac{c}{a} = 6Ghz$

3rd TE_{01} $\quad f_{c01} = \frac{c}{2b} = 7.5Ghz$

4th TE_{11}

5th TM_{11} $\quad \Rightarrow f_{c11} = \frac{c}{2}\sqrt{(\frac{1}{a})^2+(\frac{1}{b})^2} = 8.1Ghz.$

2) The operating frequency is

$$f = f_{c11} \times (1-25\%) = 8.1\times75\% = 6.075\ Ghz$$

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$$k = \frac{2\pi f}{c} = 127.2 \text{ radian/m}$$

3) TE_{10} and TE_{20} can propagate at the operating frequency 6.075 Ghz

TE_{10}

$$\beta = \sqrt{k^2-(\frac{\pi}{a})^2} = \sqrt{(127.2)^2-(\frac{\pi}{0.05})^2} = 110.64$$

$$\lambda_g = \frac{2\pi}{\beta} = 0.0568m$$

$$v_p = \frac{2\pi f}{\beta} = 3.45\times10^8 m/s$$

$$v_g = \frac{c^2}{v_p} = 2.61\times10^8$$

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TE_{20}

$$\beta = \sqrt{k^2-(\frac{2\pi}{a})^2} = 19.93$$

$$\lambda_g = \frac{2\pi}{\beta} = 0.3152m$$

$$v_p = \frac{2\pi f}{\beta} = 1.915\times10^9 m/s$$

$$v_g = \frac{c^2}{v_p} = 4.7\times10^7 m/s$$

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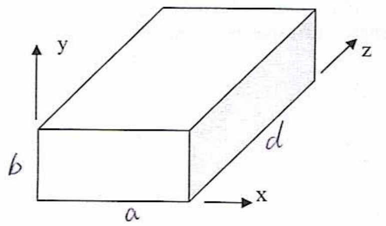
Problem #3. (20 pts) A PEC rectangular waveguide cavity has the dimension a = 5cm and width b = 3cm and length d=8cm (1) Find out the lowest two resonant frequencies of this cavity (2) Draw the H_x and H_z field distribution in the cavity for these two resonant modes. (3) If one replaces the front PEC plate by PMC, what changes will happen to the lowest resonant frequency?



1) 1st mode is TE_{101}

$$f_{r101} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{d}\right)^2}$$

$$= 3.54\ Ghz$$

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2nd mode is TE_{102}

$$f_{r102} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{2}{d}\right)^2}$$

$$= 4.80\ Ghz$$

3) If front PEC plate is replaced by PMC, the 1st mode is $TE_{10\frac{1}{2}}$

$$f_{r10\frac{1}{2}} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{2d}\right)^2}$$

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$$= 3.143\ Ghz$$

$$\Delta f_r = 3.54 - 3.143 = 0.397\ Ghz.$$

the lowest resonant frequency will be 0.397 Ghz lower.

2) For both TE_{101} and TE_{102}

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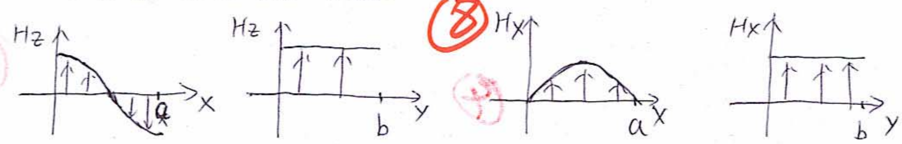
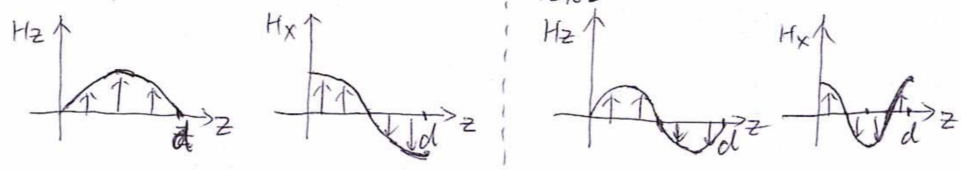


TE_{101} | TE_{102}



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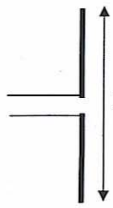
Problem #4. (20 pts) For a half-wave dipole antenna working at 900MHz, as shown in the following figure, (1) Determine the physical length l of the antenna. (2) Find the expression of the radiation pattern and the maximum radiation direction of the antenna. (3) Assuming the radiation resistance of the antenna is 73 Ohm and the loss resistance and reactance component can be neglected, calculate VSWR if one uses a 35 Ohm transmission line to feed it. (4) If the antenna has an Ohmic resistance of 2 Ohm and the feed line to the antenna is matched, what is the radiation efficiency of the antenna? (5) Under the same conditions in (4), if the antenna is used for transmitting, the power output to the antenna is 1 W, how much power will be radiated into the space and what is the maximum power density at a range of 1 km.



1) $f = 900\,mhz$

$\lambda = \frac{c}{f} = \frac{3\times10^8}{9\times10^8} = \frac{1}{3} m$ 4

$L = \frac{\lambda}{2} = \frac{1}{6} m$

2) $f(\theta) = \frac{\cos(\pi/2 \cos\theta)}{\sin\theta}$ 4

$\theta_{max} = \pi/2$

3) $\Gamma = \frac{73-35}{73+35} = 0.352$

$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = 2.086$ 4

4) $\xi = \frac{R_{rad}}{R_{rad} + R_{ohm}} = \frac{73}{73+2} = 97.3\%$ 4

5) The feed line to the antenna is matched, therefore

$P_t = 1W$

$P_{rad} = P_t \cdot \xi = 0.973\,W$

$S_{max} = \frac{P_{rad} \cdot D}{4\pi R^2}$ 4

$= \frac{0.973 \times 1.64}{4\pi (1000)^2}$

$= 1.27\times10^{-7}\, W/m^2$

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Problem #5. (20 pts) A WIMAX communication link operates at 3.5GHz with a bandwidth of 1MHz. The transmitted power should reach to a distance of at least 1 Km away and provide enough signal power for reception. The receiver has a noise temperature of 300K and both the transmitter and the receiver use an half-wave dipole antenna. (1) If the minimum signal to noise ratio for reception is 10dB at the receiver, what should be the minimum transmitter power? (2) If one wishes to cut down the transmitter power to 5dB lower while keeping the same specifications for the system, how much more gain the transmitter antenna has to have now? (3) If one is allowed to use a linear array of dipole antennas for transmitting the signal, how many antenna elements this array needs to have?

1) $f = 3.5\,Ghz$

$\Delta f = 1\,Mhz$

$P_n = k\,T_{sys}\,\Delta f = 1.38 \times 10^{-23} \times 300 \times 10^{6} = 4.14 \times 10^{-15}\,W$

$\frac{P_r}{P_n} \geq 10 \Rightarrow P_r \geq 4.14 \times 10^{-14}\,W$ ~~$3.3085 \times 10^{-4}\,W$~~

$P_t = \frac{P_r}{G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2} \geq 3.3085 \times 10^{-4}\,W$

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where $G_t = G_r = 1.64$ $\lambda = \frac{c}{f}$ $R = 1\,km$

The minimum transmitter power is $3.3085 \times 10^{-4}\,W$

2) 5 dB more gain

$G = \frac{1}{10^{-0.5}} \times 1.64 = 5.186$

6

3) $\frac{5.186}{1.64} = 3.16 = \frac{G_{array}}{G_{dipole}}$

6

Therefore this array needs to have 4 antenna elements